NUMERICAL SOLUTIONS OF FLUID MECHANICS AND HYDRAULIC MACHINES

of Dr R .K. Bansal

ANUP KUMAR SEN

BlueRose ONE.com
Stories Matter
NewDelhi • London

BLUEROSE PUBLISHERS
India | U.K.

BlueRose ONE.com
Stories Matter
NewDelhi • London

For permissions requests or inquiries regarding this publication, please contact:

BLUEROSE PUBLISHERS
www.BlueRoseONE.com
info@bluerosepublishers.com
+91 8882 898 898
+4407342408967

ISBN: 978-93-7825-678-3

Cover Design: Sharvi Singh

First Edition: June 2026

Fluid Mechanics and Hydraulic Machines.

Properties of Fluids

Q 1 CHAPTER 1

One liter of crude oil weigh 9.6 N. Calculate its specific weight specific gravity.

Volume of 1 liter = 1/1000 m -cube = 1000 cm cube
Weight of liquid = 9.6 N
I) specific weight = W= weight / volume = 9.6 N/ 1/1000 m-cube
= 9600 N/cube

I I) Density of liquid = w/g = 9600/ 9.81
= 978 .6 kg/ m -cube.

III) specific gravity

Density of liquid / Density of water
= 978. 6/ 1000
= . 9786

Q 2

The velocity distribution for flow over a flat plate is given by

.u = 3/ 2 y – y 3/2), where u is the point velocity in meter per second at a distance y meter above the plate. Determine the shear stress at y = 9 cm. Assume dynamic viscosity as 8 poise.

Shear stress T = μ du/ dy

Given y = 9 cm = .09 m

. u = 3/2 y -- y3/2

.du/ dy = 3/2 –3/2 y (3/2—1)

= 3/2(1---3/2 y ½)

(du/dy) at y=0 = 3/2 .= 1.5

(dy/du) at y= 9 = 3/2—3/2 (.09) ½

=(3/2) -3/2x .3

= 1.5 -- .9/2

= 1.5 -- .45

= 1.05

Now $\mu = 8\, poise$ given,

So Shear stress T =$\mu du/dy$= .8 Ns/ m 2 x 1.05

= .840 Ns/ m 2

Q 3.

A plate 0.025mm distant from fixed plate moves at 50 cc /s and a force of 1.471 N/m2 to maintain this speed. Determine the fluid viscosity between plates and poise.

[7.357 x 10^{-4}]

_______________Fixed plate

……plate.ato.o25 mm away………

Let force F is required to maintain the speed of plate ,u = 50 cm/s = .5 m / s

F= 1.471 N/m2 given

Sheer stress = $\mu \frac{du}{dy}$ = F = 1.471 N/m2

1.471 = μx (.5 -- 0)/ dy

= μ.5 / .025 x 1/1000

μ=(1.471 x 0 .025) /.5 x 1/ 1000 N s/ m2

= 7.355 x 1/ 10000 Ns/ m2

Q 4

Determine the intensity of shear of an oil having viscosity =1.2 poise and is used for lubrication in the clearance between a 10 cm diameter shaft and its journal bearing . The clearance is 1 .0 mm and shaft rotates at 200 r p m.

solution ___journal____________

Shaft

____

Shaft and journal sleeve clearance = 1 mm

D = Dia of shaft =.01 m

Shaft and sleeve clearance = .001 m = dye clearnce

Speed of shaft = 200 rpm

μ = viscosity =1.2 poise = 1.2/ 10 Ns/ m2

tangential velocity = Π D n / 60 = 22/7 x 10/100 x 200/60

= 1.0362 m/s

= du

d y = 1/1000 = dye clearance.

Shear stress = T= (μ x 1.0362) /(1 / 1000)

= 1.2/10 x 1.0362/ 1/1000

= 1.2 x 1.0362/ 1/100

= 100x 1.2x1.036

= 124.32 N/m2 **

Note ** because of more numbers after decimal in calculations, there may be small difference in final answers some time in general .Find force and power

Q 5

Two plates are placed at a distance o.15 mm apart. The lower plate is fixed while the upper plate having surface area 1.0 m2 is pulled at .3 m/s Find the force and power required to maintain this speed if the fluid separating them having viscosity 1.5 poise.

solution

Given

A= surface area = 1 m2

. u = speed =0.3 m/s = du

μ= viscosity = 1.5 poise = 1.5/10 Ns/m2

, d y =distance between plates = .15 mm =.15/ 1000 m

Shear stress = μ du/dy = 1.5/10 x.3 / .15 / 1000 =300 N

Force = Shear stress x area = 300x 1.0 m2

Power required to move the plate at a speed of .3 m/s

= force x speed

= 300 Nx.3 m/s= 90 W

Type equation here.

Q 6.

An oil film of thickness 1.5 mm is used for lubrication between a square plate of size 0.9m x 0.9 m and an inclined plane having an angle of inclination 20 º.The weight of the square is 392 .4 N and it slides down the plane with a uniform velocity of 0.2 m/s.

Find the dynamic viscosity of the oil .

Ans:

Given

,u = velocity of plate = .2 m/s = d u

A = Plate size = .9 m x .9m = .81 m2 = area of plate

, d y = film thickness 1.5 mm = 1.5 / 1000 m

Inclination of plane = 20º

F = Weight of plate along the inclined plane

= W cos 70º =392.4 N x .342

= 134.2 N

μ = F/A X d y/ du

= 134.2 x 1.5/ 1000 x.2 x.81

= 134.2 x 1.5 / 162

= 1.240 Ns / m2

= 12.40 poise. = dynamic viscosity of oil.

Q 7

In a stream of glycerin in motion at a certain point the velocity gradient is 0.25 per sec. per meter. The mass density of fluid is 1268.4 kg / cubic meter and kinematic velocity 6.30 x 1/10000 sq. m/s . Calculate the shear stress at the point.

solution

Given

Velocity gradient =du/d y = 0.25 m/s/m

Mass density Þ of the fluid =1268.4 kg/m3

T =μ du/dy

Kinematic viscosity = v = μ/Þ = 6.30 x1/10000

So μ= v x Þ = 6.30 x1/10000x 1268.4

= .799

=.8 kg /m-s

T = shear stress = μ x velocity gradient.

= '8 x.25

=.2 N/m2

Q 8 Find kinematic velocity of an oil having density 980 kg /m2 when at certain point in the oil ,the shear stress is 0.25 N/m2 and velocity gradient is .0.3 /s.

solution

Given

Kinematic vis co. = v ?

Þ=density =980kg/ m2

Sherar stress T = 0.25N/m2

Velocity gradient = du/ d y = 0.3m/s

. d y/du = 1/.3Ns/m2

Shear stress = T = μ du/d y

μ = TX d y/ du

= 0.25 x 1/ .3

Þ= .83/ 980

= .0008469 m2/s or 8.469 stokes.

Q9

Determine the specific gravity of fluid having viscosity 0.07 poise and kinematic viscosity 0.042 stokes .

Given viscosity = μ= .07 poise = .07x 1/10 = .007 Ns /m2

Kin. Vis = 0.042 stokes = .042 / 10000 m2/s

Kin vis = v = μ/Þ

So Þ= mass density = μ/kin vis

= .007 / .042/ 10000 = 10000/6 kg/m3

So, specific gravity = density of liquid/ density of water

= 10000/6/1000

= 10/6 =1.667

Q 11.

If velocity distribution of fluid over a plate is given by u=3/4 y—y , where u is the velocity in m/s at a distance of y m above the plate , determine the shear stress at y = 0.15 m ,take dynamic viscosity of fluid as 8.5 x 1/100000 kg-s/m2.

Ans:

Given , dynamic viscosity =μ = 8.5 x 1/100000 kg-s/m2

Velocity distribution = u = ¾ y –y2

, du/d y = ¾ --2 y

Shear stress T = ?

When y = .15 m

, du / d y = ¾ - 2 y

= ¾ --2 x .15

= .75 --.3

= .45

Shear stress (at y =.15 m) = μ du/d y

= 8.5 x (1/ 100000) x .45

= .3.825 x 1/ 100000 kgf/m2

Q12

An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve . The diameter of shaft is 0.5 m and it rotates at 200 rpm . Calculate the power lost in the oil for sleeve length 100 mm. the thickness of oil film is 1 mm.

Ans [2.15kw]

Given

Dia of shaft =D= .5 m

Oil viscosity = 5 poise = 5/10 =.5 kg -s/m2

Shaft speed = 200rpm

L= Sleeve length = 100 mm =100/1000 = .1 m

Thickness of oil film = 1 mm

Power loss in the oil sleeve length

= Torque (T) X angular speed

Torque = shear force x D/2

= shear stress x Area

Shear stress = μdu/d y= 5/10 x(tangential velocity)/d y

=.5 x (π DN/60)/oil thickness

$= .5 \times (\pi \times .5 \times 200/60)/ 1/1000$

$= 2615$ N/m

So , Torque = shear force x D/2

= (shear stress x area) x D/2

$= (2615 \times (\pi \times D \times L) \times (D/2)$

$= 2615 \times (\pi \times .5 \times 100/1000) \times .5/2$

=102.75Nm

Power lost = Torque x ω, where ω is angular vel of shaft.

$= T \times 2\pi N/60$

$= 102.75 \times 2 \times 22/7 \times 200/ 60$

=2152.85 W

= 2.152 KW.

Q 13

The velocity distribution over a plate is given by $u = 2/3$
$y—y\ 2$

Where u is the velocity in m/s at a distance of y m above the plate. Determine the shear stress at y =0, y = 0. 1 and y = 0 .2 m . take μ =6 poise.

Ans.[2.15kw]

Vel distribution $u = 2/3\ y – y2$. when μ = 6 p = 6/10
= = 6 Ns/m2

Differentiating u , du / d y = 2/3 – 2y -------(i)

Now when y=0, d u/dy = 2/3 = .66

when Y= .1 ,d u / dy= 2/3 –2x .1 = .46

when y = .2 , d u/dy = 2/3 --.4 = .26

shear stress $T = \mu \; du / dy$

case 1, $T = .6 \times .66 = .396$ Ns/m2

case ii $T = .6X \; .46 = .276$ Ns/m2

case iii $T = .6 \; x.26 = .156$ Ns/m2

Q 14

In question 13 ,find the distance in meter above the plate ,at the shear stress is 0.

Ans [.333m]

Given

In question 13 we find

,du / d y =0 ,i.e velocity gradient is 0.then the shear stress is 0.

After diff we get $du/dy = 2/3 - 2y$.

$O = 2/3 - 2y$

$Y = 2/6 = .333$m

So at a distance of .333 m the shear stress will be 0.

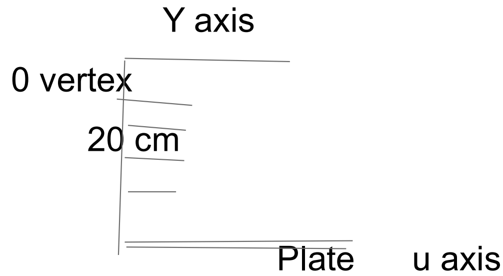
Q 15.

The velocity profile of a viscous fluid over a plate is parabolic with vertex 20 cm from the plate ,where the velocity is 120 cm /s. Calculate the velocity gradient and shear stress at distances of 0,5 and 15 cm from the plate, viscosity of fluid is 6 poise,

Ans [12/s,7.18N/m2, 5.385N/m2 ,3/s,1.795N/m2]

Given

parabola



0 vertex to plate = 20 cm

. u = velocity at vertex = 120 cm /s = 1.2 m/s

.μ = 6 poise = .6 Ns/m^2

To find

1. du/d y
2. shear stress =τ
   due to parabolic nature of velocity the equation is
   u = a y^2+by +c
   a, b, c are constant
   to solve above for following distances –

1. y=0, u=0 2. Y= 20cm

,u =120 cm/s

3. Y = 15 cm.

1.for du/dy y=0. U=0

So, U = ay^2+ b y +c

0 = 0 +0+ c

C =0

2 u = ay^2+ by +c

120 = a$(20)^2$+ b.20+c

= 400 a + 20 b + c

$12 = 40a + 2b$ ……………….. (I)

$6 = 20a + b$ so $b = 6 - 20a$ --------(-2)

Boundary condition

,u $= ay^2 + by + c$

By diff above, $du/dy = 2ay + b$

$0 = 2 \times a \times 20 + b$

$= 40a + b$

. $b = -- 40a$ ------------(-3)

From(2) and(3) we get

, $a = -- 6/20 = -- .3$

, $b = --40a = 12$

.u $= -- .3\,y^2 + 12y$

,$du/dy = --.6y + 12$ = velocity gradient

1. When $y = 0$, $du/dy = --.3 \times 0 + 12 = 12$
2. $Y = 5$, $du/dy = -- .6 \times 5 + 12 = 9/s$
3. $Y = 15$, $du/dy = -- .6 \times 15 + 12 = 3[s$

Shear stress $= \tau$

1. $\mu\,(du/dy) = .6 \times 12 = 7.2$ N/m2
2. –do-- $= .6 \times 9 = 5.4$ N/m2
3. – do -- $= .6 \times 3 = 1.8$ N/m2

Q 16

The weight of a gas is given as 17.658 N/m3 at 30°C and at an absolute presser of N/cm2. Determine the gas constant and also the density of gas, [Ans ; 1.8 kg/m3, 539.55 Nm/kg° k]

Solution:--

W = weight of gas =17.658 N/m3 at 30° C

P = Absolute presser = 29.43N/cm2 = 29.43 x 10^4 N/m2

Absolute temperature = 30 +273 = 303 ° k

To determine

.a gas constant

.b density of gas

a) Gas constant

W = ρ x g

ρ =w/g

= 17.658/9.81

= 1.8 kg/ m3

, b = density of gas

$= R = \frac{P}{\rho X\ T} = \frac{29.43X1\ \ ^4}{1.8X303}$

= 539.6 Nm/kg° k

Q 17 A cylinder of 0.9 m3 in volume contains at 0 °C and 39.24 N/cm2 abs.pr. the air is compressed to .45m3. Find I)Pr. inside cylinder assuming isothermal process. II) Pr. and Temp. assuming adiabatic pro. Take k =1.4 for air. [Ans. 78.48 N/cm2ii 103.5N/m2 140° C]

Solution : Given

V1 =Volume of air cylinder = .9 m3

V2 = -- do -- after compression = .45m3

T1 = 0°+ 273 = 273 k

P1 = 39.24 N/cm2= 39.24x 10^4 N/m2

K = specific heat for air = 1.4

(A) isothermal process

P/ρ = Constant

Or PV=Constant

P1V1= P2V2

$P_{2=} \frac{P1V1}{V2} = \frac{39.24X}{.45} 10^4 X.9$ N/m2

= 78.48 N/cm2

(B) Adiabatic process

$1/\rho^k$= volume

P/ρ^K=Constant

PV^K=Constant

$P_1 v_1^k = p_2 V_2^K$

$p_{2=p_1 X} (\frac{V_1}{V_2})^K$

= 39.24X$(\frac{.9}{,45})^{1.4}$

= 39.24X2.64 = 103.6 N/m2

For finding temp,

PV = RT, PV^K = Constant.

P = RT/V , So RT/VX V^K=Constant.

RT.V^{K-1} = Constant

T.V^{K-1} =Constant/R= Constant

So, $T_1 V_1^{K-1} = T_2 V_2^{K-1}$

$T_2 = T_1 (\frac{V_1}{V_2})^{K-1}$

= 273$(\frac{.9}{.45})^{1.4-1}$

=273X $(2)^{.4}$

=273X 1.32

=360° K

So t_2 = 360—273

= 83°C

Q 18 Calculate the presser exerted by 4 kg mass of nitrogen gas at a temp of 15° C if the volume is .35 m3 molecular weight of nitrogen is 28. [97.8 N/cm2.]

Given

. m = mass of nitrogen =4 kg

Absolute temp = 273 +15 =288° k

Vol of gas = 0.35 m3

Molecular wt of nitrogen =28

Volume of nitrogen = 0.35 m3

Using equation (1.9) we have PV = n x M X R T

Where M XR = Universal gas constant

=8314 Nm/kg-mole° k

One-kg -mole=kg-mass x molecular wt =kg mass x 28

R for nitrogen = 8314/28 =296.9 Nm/kg° k

Now the gas law for nitrogen

PV= m R T

P X .35 =(4 X 296.9 X 288)

P = 977225.14 N/m2

= 977225.14/ 10^4 2N/cm

=97.7 N/cm2.

Q19 The presser of liquid is increased from 60 N/cm2 to 100 N/cm2 and volume decreases by.2 percent. Determine Bulk Modulus of elasticity.

[Ans 2x 10^4 N/cm2]

Solution

Given

Initial presser = 60 N/cm2

Increased pressure =100 N/cm2

Difference of pr = 40 N/cm2

Decrease in volume = .2 %= .2/100

Bulk Modulus of elasticity = $\frac{increase\ in\ presser}{decrease\ in\ volume}$

$=\frac{40}{\frac{,2}{100}}$

$=\frac{40x1}{\frac{2}{10}}$

$=\frac{40\ x100\ x10}{2}$

$=\frac{40000}{2}$

=2 x 10^4 N/cm2

Q 20

Determine the bulk modulus of elasticity of a fluid which is compressed in a cylinder from a volume of 0.009 m3 at 70 N/cm2 presser to a volume of 0.0085 m3 at 270N/cm2 presser.

[Ans 3.6 x 10^3N/cm2]

Solution;

Given

v_1 = .009m3 at pr 70 N/cm2

v_2 = .0085 m3 at pr 270 N/cm2

.dv = v_{1-V_2} = .005m3= .0005 X 100X100X100

=.005 X10^6N/cm2

k = Bulk modulus = $\frac{dp}{\frac{dv}{v}}$ = $\frac{270-70=100}{\frac{.0005}{.009}}$

=$\frac{2x9x10^3}{5}$ = 3.6x 10^3N/cm2

Q 21

The surface Tension of water in contact with air at 20° C is given as 0.0716 N/m. The presser inside a droplet of water is to be 0.0147N/cm2 greater than the outside presser. Calculate the diameter of droplet of water. [1.94 m m]

Solution

Given

Surface tension of water =σ = 0.0716N/m

Presser inside droplet = 0.0147 N/cm2

For droplet

P =$\frac{4\sigma}{d}$

0.0147 =$\frac{4x0.0716\ N/m}{d}$

. d = $\frac{4x\ 0.0716}{0..0147x100}$

= .194 cm

$= 1.94$ mm

Q22

Find the surface tension in a soap bubble of 30 mm diameter when inside presser is 1.962 N/m2 above atmosphere .[Ans .00735N/m.]

Solution

Given

To find surface tension σ of soap bubble

Dia of soap bubble = d =30 mm= 30x 10^{-3}m

Inside presser of bubble =P =1.962 N/m2

Using relation $P = \frac{8\sigma}{d}$

$$\frac{8\,\sigma}{1.962} = 30x10^{-3}$$

$$\sigma = \frac{30x10^{-3}x1.962}{8}$$

= .00732 m

Q 23

The surface tension of water in contact with air is given as 0.0725N/m. The pressure outside the droplet of water of diameter 0.02 mm is atmospheric (10.32N/cm2) .Calculate the presser within the droplet of water. [11.77 N/cm2]

Solution

Given ; diameter of water droplet = d =.02x10^{-3} m

Pressure outside droplet =P= 10.32 N/cm2

.σ =S.T=0.0725N/m

Using the relation

Pressure inside = $\frac{4\sigma}{d}$

$= \frac{4x.0742}{'02x \quad ^{-3}}$

= 14500 N/m2

=1.45 N/cm2

Total pressure inside the droplet

= outside pressure on droplet +inside pr of droplet

= 10.32 N/cm2 + 1.45 N/cm2= 11.77 N/cm2.

Q24

Calculate the capillary rise in glass tube of 3 mm diameter, when immersed vertically in a) water b) mercury .Take surface tension for mercury and water as 0.0725N/m and 0.52 N/m respt.in contact with air . Specific gravity of mercury is given 13.6.

[Ans .966cms,.3275 cm]

Solution

Given

Glass tube diameter = d =3 mm=.003 m

. σ =S.T =.0725

Capillary rise = h = ?

a) for water = $\frac{4\sigma}{\rho g d}$ $=\frac{4x\,.0725}{1000x\ .81\ \ .003}$

$= \frac{.29}{29.43}$ = .00985m =.985 cm

(b) For Mercury

Capillary rise =h = $\frac{4\sigma cos}{\rho g d}$

Given

,ρ = 0.52

, g = 9.81

, d = .003 m

, θ =128° , cos 128° = for mercury =.6928

, ρ =13.6 x1000 kg/m3

, h = $\frac{4x.52 \quad .6928}{13.6x1000x9.81x.003}$

=.0036 m

= .36 cm

Q 25

Capillary rise in the glass tube used for measuring water level is not to exceed .5 mm. Determine its minimum size, given that surface tension for water in contact with air =0.07112 N/m[Ans=5.8cm]

Solution

Given

Capillary rise = h =.5 mm=.0005 m

Surface tension =σ =.07112 N/m

Θ = 0 because of water

Cos θ =1

. ρ =1000

. g = 9.81

We have to find diameter of tube =d

Using the relation for water

Capillary height h = $\frac{4\sigma}{\rho g d}$ =$\frac{4x.07112}{1000 \quad .81xd}$ =

.0005=$\frac{.28448}{9810d}$

. d = $\frac{.28448}{9810 \quad .0005}$ =$\frac{.28448}{4.905}$ =. 058 m

. d =5.8 cm

Q 26

One liter of crude oil weighs 9.6 N. Calculate the specific weight ,density and specific gravity. [Ans; 9600N/m3,979.6 kg/m3,0.9786].

Solution

Given

Volume of liquid =1 lit = $\frac{1}{1000}$ m3

Weight of 1 lit of liquid= 9.6 N

(i) specific weight ω = $\frac{wei}{volume}$

$$= \frac{9.6N}{\frac{1}{1000}m3} = 9600\ N/m3$$

(ii) Density ρ = $\frac{\omega}{g}$ = $\frac{9600}{9.81}$ = 978.59 kg/m3

(iii)Specific gravity= $\frac{dencity\ of\ liquid}{dencity\ of\ water.}$

=$\frac{\rho}{1000}$ =$\frac{978.59}{1000}$ =.9786 .

Q28

Find the capillary rise of water in a tube 0.03 cm diameter .The surface tension of water is 0.0735 N/m .[Ans 9.99 cm]

Solution

Given

. h = capillary rise = ?

Diameter of glass tube =d = .03 cm. = ,03 x10^{-2} m

Density of water =ρ =1000 kg/m3

Surface tension = σ = 0.0735 N/m

The relation h = $\frac{4\sigma}{\rho x g x d}$

=$\frac{4x.0735}{1000x \ \ .81 \ \ .03x10^{-2}}$

=$\frac{2.940}{29.43}$

= 0.09989 m

=9.989 cm.

Q29

Calculate the specific weight ,density ,and specific gravity of two liter of liquid which weighs 15 N. [Ans; 7500N,/m3,764.5 kg/m3,0.764

Solution

Given

Weight of liquid =15N

Volume of liquid = $\frac{2}{1000m}$

Density of water ρ = 1000 kg/m3

(i) specific weight = w = $\frac{weight}{volume} = \frac{15N}{\frac{2}{1000}}$ = 7500N/m3

(ii) Density $\rho = \frac{w}{g} = \frac{7500}{9.81}$ =765.5 kg/m3

(iii) specific gravity $= \frac{density\ of\ liquid}{density\ of\ water}$

$= \frac{765.5}{1000}$

=0.765

Q 30

A 150mm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 151 cm. Both the cylinders are 250 mm high. The space between the cylinders is filled with liquid of viscosity 10 poise. Determine the torque required to rotate the inner cylinder at 100 rpm. (Ans; 13.87Nm)

Solution

Given

Diameter of outer cylinder =d =150 mm= .15 m

Diameter of outside cylinder=D =151 mm =.151m

Speed required for inner cylinder = 100 rpm.

Height of cylinders = 250 mm = .250 m

Gap between cylinders = (151-150)/2 = .5 mm =.005 m

Viscosity =μ =10 poise = .1 Ns/m2

To find Shear stress (τ) to move inner cylinder to 100 rpm.

. τ = μ d u/d y

. u = tangential velocity = $\frac{\pi DN}{60}$

$= \frac{\pi X .15\ X 100}{60}$ =,7857 m/s

Surface area of inner cylinder = circumference x length

Of inner cylinder

$= \pi x\ d\ x\ .25$

$= \pi x\ .15x.25 = .1178\ m2$

.u = .7857 =11/14

. du = u—0= 11/14 =.7857

total Gap between cylinders (y)=151 mm—150 mm=1 mm

single gap d y = 1mm/2 =.5mm= .0005 m

μ = 1 Ns/m2

Shear stress =τ = μ d u/d y

= 1.x11/14 x 1/.0005

= 1571.4 N/m2

Torque T = Force x D/2

=Shear stress x surface area x D/2

= (1571.4 x .1178)x .15/2

$= \frac{27.76}{2}$

=13.88 Nm

Q31

A shaft of diameter is rotating inside a journal bearing of diameter 122 mm at a speed of 360 rpm . the space between shaft and bearing is filled with lubricating oil of viscosity of 6 poise. Find the power absorbed in oil if the length of bearing 100mm .

Ans;- [115.73W.]

Solution

Given

Bearing Dia = d =122mm=.122 m

Shaft Dia = D = 120mm =.120 m

, y =122—120 =2 mm =.002

, d y = y/2 =.002/2 =.001m

N = Speed = 360 rpm.

, μ =6 poise = .6 Ns/m2

,u = tangential velocity = πxD N/60

$= \frac{\pi X.122X320}{60}$

= 2.27 m/s

, d u =u – 0

= 2.27 m/s

Shear stress =μ $\frac{du}{dy}$

=.6 $\frac{2.27}{.001}$ =1362 N/m2

Shear force = shear stress x area

=1362 x πx shaft dia. x length of shaft

= 1362x π x .120 x .1

=50.6 N

Torque = shear force x D/2

= 50.6X .12/2

= 3.03 Nm

Power lost in the lubricating oil

$= \frac{2\pi NT}{60}$

$$=\frac{2\pi X360X3.03}{60}$$

= 111.6 W

Q32

A shaft of diameter 100 mm is rotating inside a journal bearing of 102 mm at a speed of 360 rpm .The space between shaft and bearing is filled with lubricating oil of viscosity 5 poise,. The length of bearing 200 mm. Find the power absorbed in lubricating oil.

[Ans; 111.58 w]

Solution

Given

Speed of shaft =360 rpm

Viscosity μ= 5 poise = ½ Ns/m2

, d y=Dia of journal bearing –Dia of shaft

= 102 mm – 100mm

,d y = 2mm = .002 m

Tangential velocity u = $\frac{\pi DN}{60}$ =$\frac{\pi X.1X36}{60}$ = 1.88 m/s

, du = u—0

=1.88 – 0

=1.88

Shear stress = μ d u/d y

= ½ x 1.88/.001

=1880/2= 940 N/m2

Shear force = shear stress x area

$= 940x\ \pi\ DL$

$= 940X\ \Pi\ x\ .1x\ .2$

= 58.28 N

Torque = Shear force x D/2

= 58.28 X .1/2

= 2.914 Nm

Power absorbed by oil= $\frac{2\pi N}{60}$ = $\frac{2\pi 360 \quad .914}{60}$

= 109.885 W

Q33 Assuming that the bulk modulus of elasticity of water is 2.07x10^6 Kn/m2 at standard atmospheric condition ,determine the increase of pressure necessary to produce 1% reduction in volume at the same temperature.

Solution Given K= bulk modulus of elasticity =2.07x 10^6=$\frac{dv}{v}$

= $\frac{1}{100}$

=.01

Increase in pressure (d p) =k x ($\frac{-dv}{v}$)

=2.07x 10^6x .01

= 2.07 x10^4 kN/m2

Q34.

A square plate of size 1mx1m and weighing 350 N slides down an inclined plane with an uniform velocity of 1.5 m/s. The inclined plane is laid on a slope of 5 vertical to 12 horizontal and has an oil film of 1mm thickness. Calculate the dynamic viscosity of oil.

Solution

Given

Plate area =1mx 1 m = 1 m2

Weight of plate = W=350 N

. u =tangential velocity = 1.5 m/s

Slope of plane

Perpendicular = p =5

Horizontal =12

Tan θ = 5/12

Component of weight along the plane = W x sin θ

=WX 5/13

= 350 x5/13 =134.615

= F

Shear stress = τ = μ du/d y = F/A

[Where u =1.5m/s

. du= u—0= 1.5

, d y =oil thickness of oil film

=1 mm =.001 m]

So μ = $\frac{F}{A}$ X$\frac{dy}{du}$ = $\frac{134.6}{1m^2}$ x$\frac{.001}{1.5}$ =0.0897Ns/m2

= 0.897 poise.

END OF CHAPTER 1.

Fluid Mechanics

Chapter 2

Pressure and its Measurement

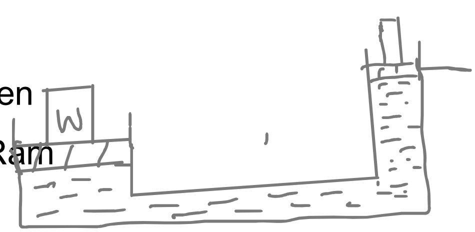
(Numerical Problems)

Q 1

A hydraulic press has a ram of 30 cm diameter and a plunger of 5 cm diameter. Find weight lifted by the hydraulic press when the force applied at the plunger is 400N. [Ans;14.4 k N]

Solution

Given



Ram diameter =30cm=.3 m

Plunger diameter =5cm=.05 m

Force applied at plunger =400N

Let the weight lifted Ram = W

Area of Ram =π/4 d^2 =$\pi/4.3^2$= π/4x .09 m2

Area of plunger =π/4$(.05)^2$ = π/4x.0025 m2

Presser intensity due to plunger = Force/Area

$$=\frac{400}{\frac{\pi}{4}X\ .0025}=\frac{400X4}{\pi X.0025}$$

Pressure intensity at Ram =W/Area of Ram

$$=\frac{w}{\frac{\pi x.09}{4}}=\frac{wx4}{\pi x.09}$$

As per Pascal law intensity of pressure will be equally transmitted.

Hence $\frac{400x4}{\pi x.0025}=\frac{Wx4}{\pi x.09}$

So W = $\frac{400X.09}{.0025}$ = 14400 N=14.4 k N .

Q2

A hydraulic press has a ram of 20 cm diameter and a plunger 4 cm plunger .[Ans; 800N].

Given

W=weight on ram= 20 k N=20000 N

Solution

P =Force applied in plunger. = ?

Diameter of ram= 20 cm = .2 m

Area of ram = $\frac{\pi}{4}$ .2^2=$\frac{\pi}{4}.04$

Area of plunger= $\frac{\pi}{4}.04^2 = \frac{\pi}{4}.0016$

Presser intensity at ram = $\frac{W}{\frac{\pi}{4}x.04}$ =$\frac{20000}{\frac{\pi}{4}x.04}$ (i)

Presser intensity at plunger =$\frac{P}{\frac{\pi}{4.}}$.0016 (ii)

By Pascal law intensity of presser will be equal in all direction

(i) =(ii)

$$\frac{P}{\frac{\pi}{4}x.0016}=\frac{20000}{\frac{\pi}{4}x.04}$$

$$P = \frac{20000X.0016}{.04} = 800 \text{ N}$$

Q 3

Calculate the presser due to a column of .4 m of a) water b)an of specific gr. 0.9and c) mercury of s p gr. 13.6. Take density of water ρ=1000 kg/m3 .[Ans ;a).3924 N/cm2 ,b),353 N/cm2,c) 5.33N/cm2]

Solution

Given

.ρ of water = 1000 kg/m3

.ρ of oil = .9x 1000 = 900 kg/m3

.ρ of mercury = 13.6 x1000 =13600 kg/m3

Putting the relation between Presser energy Z= ρ x g x Z

Here Z =.4 m , g =9.81

For water P = ρ. g , Z

= 1000x 9.81x .4 N/m2

$=\frac{1000x\ \ .81\ \ ,4}{10000}$

= .3924 N/cm2

For oil $P = \rho \times g \times Z$

$= 1000X\ 900X\ 9.81X\ .4$ N/m2

$= \frac{1000x900x9.81\ \ .4}{10000}$

= .353 N/cm2

For mercury ,$P = \rho \times g \times Z$

$= 1000 \times 13.6 \times 9.81 \times .4$ N/m2

$= \frac{13600\ \ \ .81\ \ .4}{10000}$ = 5.33 N/cm2

Q 4

The pressure intensity at a point in a fluid is given 4.9 N/cm2. Find the corresponding height of fluid when itis a) water ,b)an oil of s p gr . 0.8 [Ans a) 5 m of water b) 6.25 m of oil.

Solution

Given

1 Water $P = 4.9 \times 10^4$ N/m2

$= \rho \times g \times Z$

$= 1000 \times 9.81 \times Z$

So $Z = \frac{4.9x10^4}{1000x9.81}$ = 4.994m

2 liquid s p gr .8

$Z = \frac{P}{\rho G} = \frac{4.9\ x\ 10^4}{.8\ x100\ \ \ x9.81}$

$= \frac{4.9x10}{7.848}$ m

= 6.25 m

Q 5

An oil of s p gr .8 is contained in a vessel . At a point height of oil is 20 m Find the corresponding height of water at that

point. [Ans 16 m]

Solution

Given

Height of oil = Z = 20 m

S p gr of oil = .8

Density of oil = .8 x ρ. =,8 x 1000 = 800.kg/m3

pressure at that point = P =ρ x g xZ

= 800 x 9.81 x 20

In case it is water let Z_1 be corresponding height.

P for the same = ρ x g x Z_1

=1000 X 9.81 X Z_1

Equating both we get

1000 x 9.81 x Z_1 = 800 x 9.81 x 20

$$Z_1 = \frac{800X9.81X2}{1000X\ 9.81}$$

= 16 m.

Q 6

An open tank contains water up to a depth of 1.5 m and above it an oil of sp gr .8 for a depth of 2 m. Find the presser intensity

a) at the interface of two liquids, b) at the bottom of the tank.

Solution

.

Pressure intensity in the interface = Z ρ g

= 2 x .8 x 1000 x 9.81 N/m2

= $\frac{2x.8x1000x9.81}{10^4}$ N/cm2

= 1.57 N/cm2

2 P at the bottom of the tank = 1.57 + ρ g Z of water.

= 1.57 + 1000 X 9.81 X 1.5/ 10^4N/cm2

= 1.57 + 1.47

= 3.04 N/cm2

Q 7

The diameter of a small piston and a large piston of a hydraulic jack are 2 cm and 10 cm respectively . A force of 60 N is applied on small piston . Find the load lifted by the large piston when a) when pistons are on the same level b) small piston is 20 cm above the large piston . The density of the liquid of liquid in the jack is 1000kg/m2.[Ans; 1. 1500 N 2. 1520.5 N]

Solution

Given

Dia of large piston = 10 cm

Area = π/4X 10^2

Dia of small piston = 2 cm

Area = π/4 X 2^2

F =force applied on small piston =60 N

F_1 = force lifted at large piston when both are in same level.

Applying Pascal law

$\frac{F_1}{A} = \frac{F}{a}$ or $\frac{F_1}{\frac{\pi}{4}X10^2} = \frac{60}{\frac{\pi}{4}X2^2}$

So $F_1 = \frac{60\ X1\ \ ^2}{2^2} = \frac{6000}{4}$ = 1500 N

2 . When small piston is at 20 cm above the large piston.

Pressure intensity on the small piston

$= \frac{F}{a}$ + ρ g Z (Due to height Z=20cm/100)

$= \frac{60}{\frac{\pi}{4}x4}$ + (1000x9.81x 20/100)/10^4

= 19.099 +.1962

=19.2952 N/cm2 -------(- I)

So equating with $\frac{F_1}{A}$ = (I) = 19.2952

F_1 = 19.2952 X A

=19.2952 X π/4 X $(10)^2$

=19.2952xπ x$\frac{100}{4}$

= 6061.7658/4

=1515.44 N

Q 8

Determine the gage and the absolute pressure at a point which is 2.0 m below the free surface of water of water .Take

Type equation here.

atmospheric as 10.1043 N/cm2. [1.962N/cm2(gage), 12.066N/cm2(abs)]

Solution

Given

To find I gage pressure

2 absolute pressure.

I gage pressure

= absolute pressure –atmospheric pressure.

Gage pressure =pressure at 2m below the water surface

=1000x 2 x 9.81 = 19620 N/cm2

= 19620/10^4

= 1.962 N/cm2

Absolute pressure = atmospheric pressure + gage pr

=(10.1043 + 1.962)

= 12.0663 N/cm2

Q 9

A simple manometer is used to measure the pressure of oil s p gr .8 ,flowing in a pipe line .Its right limb is open to the atmosphere and left is connected to pipe .The center of pipe is 9 cm below the level of mercury s p.gr 13.6 in the right limb . If the difference in mercury level in 2 limbs is 15 cm, determine absolute pressure of in the pipe in N/cm2. [Ans; 12.058 N/cm2]

Solution

Given

. h_1 = center of pipe is 9 cm below the level of mercury

The difference between the mercury level = 15 cm

h_2 = 15 – 9 = 6 cm

S p gr of oil= .8

S p .gr of mercury =13.6

To find absolute pressure of oil in pipe ?

Density of oil = .8 x 1000 =800 kg/m2

Density of mercury =13.6x 1000 = 13600 kg/cm2

Let P be the pressure in pipe

Equating the pressure above the bottom level of mercury

We have pressure in pipe + $\rho_1 g\ h_1 = \rho_2\ g\ h_2$

P + 800x 9.81x .06 =13600x 9.81x .15

P = 20012—470.88

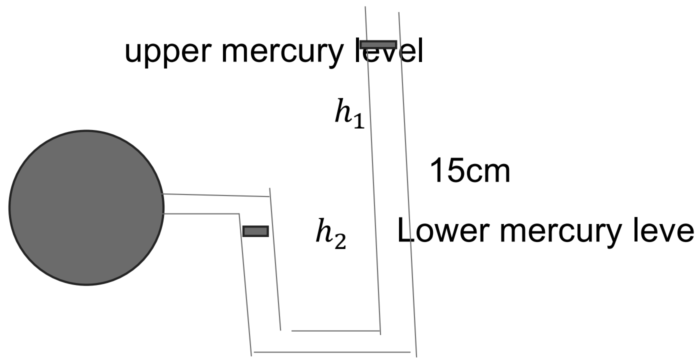
= 19541.12 N/m2 =gage pressure.

= 1.9541 N/cm2

Absolute pressure = Atmospheric pressure – gage pr

= 10.03 + 1.9541

= 11.99 N/cm2



Simple Manometer(Q NO 9)

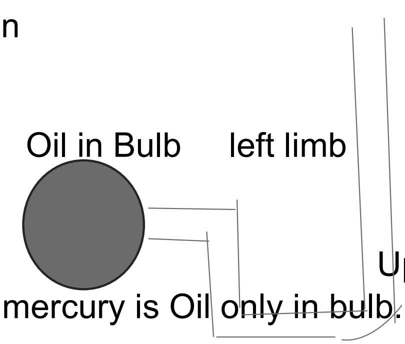
Q 10

A simple manometer (U -tube) containing mercury is connected to a pipe in which an oil of s p gr .8 is flowing .The pressure in the pipe is vacuum .The other end of manometer is open to the atmosphere .Find the vacuum ,pressure in pipe, if the difference in mercury level in two limbs is 20 cm and height in the oil in left limb from the center of pipe is 15 cm below.

[Ans ;27.86N/cm2]

Solution

Given



Upper & Lower level of mercury, above mercury is Oil only in bulb.

Simple manometer

1 difference of mercury level=$h_2 = 20\ cm = ,2m$
2 difference of up .mercury and oil in pipe =$h_1 = .15$ m

ρ_1 of oil = .8 x 1000=800
ρ_2 of mercury =13.6x1000 =13600
Pressure head in right column is = 0

Pressure above upper line of mercury in left column

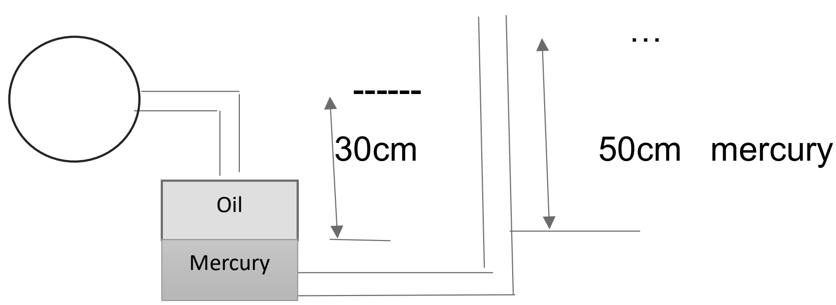
= $\rho_2 g h_2 + \rho_1 g h_1$ + P (Vacuum pr. In pipe)
P = -- (13600x 9.81x.2+800x 9.81x.15)
=-- 27860 N/m2
=-- 27.86 N/cm2 .

Q 11

Single column vertical manometer (micrometer) is connected to a pipe containing oil of s p gr of .9 .The area of reservoir is 80 times the area of manometer tube. The reservoir contains mercury of s p gr 13.6. The level of mercury in reservoir is at a height of 30 cm. below the center of pipe and the difference of mercury level in reservoir and the right limb is 50 cm. Find pressure in the pipe.[Ans ;6.474N/cm2]

Solution

Given



(single column manometer)

s.p. gr of oil =.9, ρ_1=.9x1000 =900 kg/m3 do of mercury=13.6 . ρ_2 = 13.6x1000 =13600 kg/m3

A =area of reservoir = 80x area of manometer tube.

h_1 =30 cm = .3 m

h_2 =50cm =.5 m

Due to fall of liquid in reservoir will cause rise of mercury in right limb.

So change in height due to fall of liquid in reservoir =dh

c/s Area of reservoir(A x dh = area of tube x h_2

dh = axh_2/A (i)

Pressure in right limb above separation line between two liquids = ρ_2x g x (h_2+dh) (ii)

Pressure in left limb above separation line between two liquid = $\rho_1\, xg\, x(dh+h_1)$ (iii)

Equating above i),ii) and iii)

We get pressure in the pipe

$$P_A = \frac{a}{A}.h_2(\rho_2.g - \rho_1 g) + (h_2\rho_2 g - h_1\rho_1 g)$$

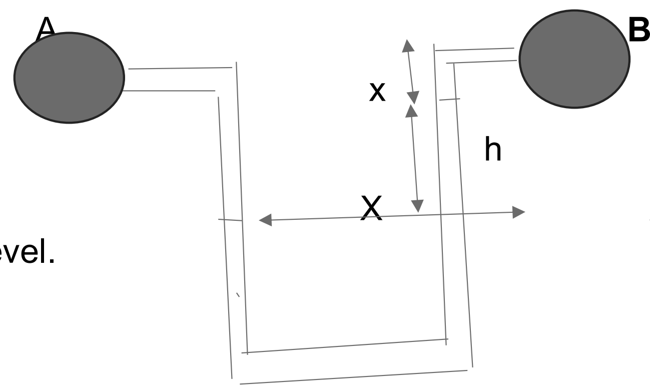
= $\frac{.5a}{80a}$ x.5 (13600x 9.81—900x9.81)+(.5x 13600x 9.81--.3. x 900. x9.81)

= $\frac{.5}{80}$(9.81)(13600—900)+ 9.81(.5x 13600--.3x 900)

= 67486 –2648.7 =64837 N/m2=6.49 N/cm2

Q 12.

A pipe contains an oil of s.p.gr .8 . A differential manometer connected at two points A and B of the pipes shows a difference in mercury level as 20 cm. Find difference in pressure at the two points. [Ans; 25113.6 N/m2]



X mercury lower level.

A and B are in same level and contains same liquid density ρ_1.

Pressures at A and B Bulbs are P_A and P_B

, h = 20cm=.2 m $\rho_1 = .8\, x1000 = 800kg/m2,\ \rho_g = 13.6$

As shown in fig above

Pressure at datum X-X in right limb

$= \rho_g xgxh + \rho_1 . g . x + P_B$ --- 1)

Pressure at datum X-X in left limb

$= \rho_1 g(x + h) + P_A$ -------2)

Equating both above we get

Pressure difference = $P_A - P_B$

$= \rho_g gh + \rho_1 g\, x - \rho_1 g(x + h)$

$= \rho_g gh + \rho_1 gx - \rho_1 gx - \rho_1 gh$

$=gh\ (\rho_g - \rho_1$

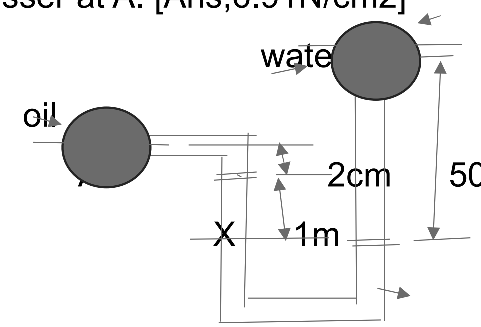
$= 9.81x\ .2\ (13600—800)$

$= 25113.6$ N/m2

Q 14

A differential manometer connects at two points A and B as shown in figure. At B air pressure is 7.848 N/cm2 (abs) , find absolute presser at A. [Ans;6.91N/cm2]

Solution



Diff manometer.

Bulb A and Bulb B

Pressure at B = 7.848 N/cm2 (abs)

To find pressure at A (abs)

Pressure at x-x at right bulb = P_B+ρ g h (water)

$= 7.848\ x10^4 + 1000\ x\ 9.81x\ .5$(i)

Pressure at left bulb = ρ g h(oil) +ρ g h (mercury)+ P_A

= .8 x1000x .12x+13.6x1000x9.81x .10 + P_A(ii)

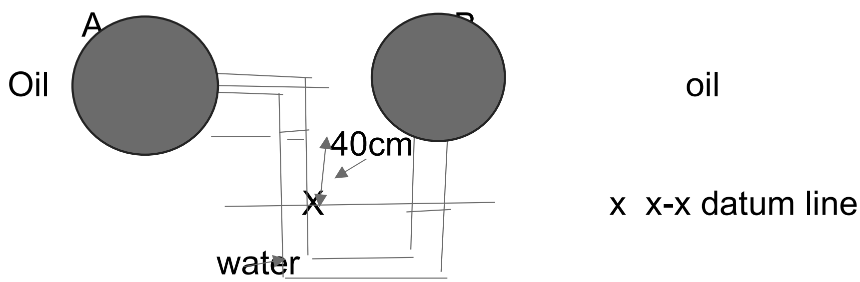
Equating (i) and(ii)

P_A= 7848x10+5x981—8x12x9.81—136x98.1

=70115 N/m2 = 7.0115 N/cm2

Q 15

An inverted differential manometer contains an oil of s.p.gr. .9 is connected to find the difference of pressure at two points of a pipe containing water , If the manometer reading is 40 cm find the difference of pressure .[392.4 N/m2]



Solution

Given

Oil s .p .gr =.

Density = .9x1000 = 900

Height of water from datum x-x= 40 cm= ,4 m

Density of water = 1000. (in pipe)

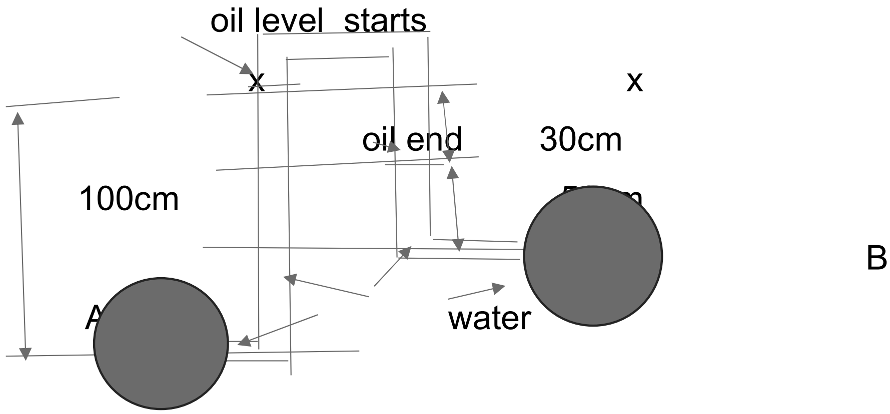
Pressure difference =$(P_B - P_A)$= g x h $(\rho_w - \rho_1)$

=9.81x .4 (1000—900)

= 392.4 N/ m2

Q 16

In the below mentioned figure as per question in the book an inverted differential manometer connected to two pipes A and B containing water .The fluid in manometer is oil of s p gr. .8 .For the manometer reading down in the figure ,find difference in pressure head between A and B. [.26 m of water.]



(Inverted differential manometer)

S p .gr of water =1000

S p gr of oil = .8x1000 =800

x-x = datum , other dimension are mentioned in the figure above.x

Pressure at x-x datum at right limb = P_B-- .50x $\rho_w$xg -- .3x ρ_{oil}xg

=P_B--.5X1000X9.81-- .3X800X9.81

=P_B –9.81X7.6

Pressure at datum in left limb =P_A--1x1000 x9.81

=P_A—9810

Equating above two equation

$P_A - P_B$ = 9810—7259.

= 2550.6

Pressure head at A =$\frac{P_A}{\rho g}$

Pressure head at B = $\frac{P_B}{\rho g}$

Now difference between head = $\frac{P_A}{\rho g} - \frac{P_B}{\rho g}$

$=\frac{P_A - P_B}{\rho g}$

$=\frac{2550.6}{1000 \quad .81}$

=

’26 m of water.

Q 17

If the atmospheric pressure at sea level is 10.143 N/cm2,determine the pressure at height 2000m assuming that the pressure variation follows(i) hydrostatic law(ii) isothermal law . The density of air is given as 1.208 kg/m3 .[7.77N/cm2 and 8.03 N/cm2]

Solution

Given

Atmospheric pressure at sea level =10.143 N/cm2

=10.143 x10^4 Nm2

To find pressure at height 2000m

1 as per hydrostatic law

$\rho = 1.208 kg/m3 \ (given)$

pressure at any point = p =ρ g h

= 10.143x 10^4x9.81x2000

By hydrostatic law : ρ is constant and is given by

$\frac{dp}{dz}$ =--ρ g (integrating both side)

$\int dp$=$\int -\rho g dz$ = [--ρ g z](z =z and z =0 at sea level)

P--P_0 =-- ρ g z

So =P =P_0—ρ g h

= 10.143x 10^4—1.208 x 9.81 x2000

= 101430—23700.96

= 77729 N/m2

= 77729x10^{-4}N/cm2

=7.77N/cm2

2 isothermal law ; -

Pressure at any point as per isothermal law is given by

P =$P_0 e^{-g \ /RT}$

= 10.143x$10^4 e^{\frac{-gz}{RT}}$

= 101430$e^{\frac{-gz\rho_0}{p \quad 0}}$

=101430$e^{\frac{-9.81x2000x1.208}{101430}}$

= 101430 $e^{-.2336}$

=101430x .7917

=80302.13 N/m2

= 8.03N/cm2

Calculate the pressure at a height of 8000 m above sea level if the atmospheric pressure is 101.3 k N/m2 and temp is 15° C at sea level assuming (i) incompressible(ii)pressure variation follows adiabatic law. (iii) pressure variation fallows isothermal Law. take density of air at sea level as equal 1.285 kg/m3. Neglect variation of G with altitude.

Solution

Given

Z= Height =8000 m above sea level

P_0 =Atm pr =101.3k N /m2= 10.13 N/m2 x 10^4

$T_0 = 15 + 273 = 288$ K

$\rho_0 = 1.285 kg/m3$

A when air incompressible

$\frac{dp}{dz}$ =--ρ g

$\int \frac{dp}{dz} = \int -\rho g dz$

$P_{P_0}^{P}$ = -ρ g $z_{z_0}^{z}$

P --P_0=--ρ g z (z =0)

P = P_0—ρ g z

=101300—1.285x8000x9.81

=101300—100846.8

=453.2 N/m2

B Pressure follows adiabatic law (k taken as 1.4)

P = P_0[1--$\frac{k-1}{k}$gz.$\frac{\rho_0}{P_0}$ $]^{\frac{k-1}{k}}$

=101310[1--$\frac{1.4-1}{1.4}$x9.81x8000$\frac{1.285}{101310}]^{\frac{1.4}{1.4-1}}$

=101310[1—2/7 x $\frac{981x8x12.85}{101310}$ $]^{\frac{7}{2}}$

=101310x$(.7156)^{3.5}$

=101310x .3099

=31395/10^3= 31.4 k N/m2

C Pressure follows isothermal laws ;

P =$P_0 e^{\frac{-g}{RT}}$ =101310 .$e^{-9.81x8000/\frac{p_0}{\rho\ \ 0}}$

=101310$e^{\frac{-981x80x1.285}{101310}}$

= 101310$e^{-.995}$

=101310x$\frac{1}{2.7}$ =37522.2 N/m2

= 37.522 k N/m2

Q 19

Calculate the pressure and density of air at a height of 3000m above the sea level where pressure and temp are 10.143 and 15 ° C respectively. The temp lapse rate is given as 0.0065°k/m. Take density of air equal to 1.285 kg/m3.

[Ans; 6.896 N/cm2,0.937 kg/m3]

Solution

Given

Pressure at sea level = 10.143 N/cm2

Temp = T_0 =15+273 = 288° K

Temp lapse rate = .0065°k/m

P_0 =10.143 N/m2

=1.222 =power index.=1.222 =power index. $\rho_{0 for\ air}$= 1.285 kg/m2

Z = 3000m

Let L be temp lapse rate = $\frac{dT}{dZ}$ =$\frac{-g}{R}$ x $\frac{K-1}{K}$ -----(-I)

Where k=power index

R= $P_0 \ \ /\rho_0 x T_0$ where R ,a constant

=$\frac{10143}{1.285X288}$ =274.09

Putting the values in equation (1)

.0065 = $\frac{-9.81x}{274.09}x(\frac{k-1}{k})$

So $\frac{k-1}{k} = \frac{.0065x274.09}{-9.81}$ =0.1815

K =1.222 =power index.

So P at 3000m = by following equation

= $P_0[1 - \frac{k-i}{k} xgz \frac{\rho_0}{P_0}]^{\frac{k}{k-1}}$

=101430[1--$(\frac{1.222-1}{1.222})x\ 9.81x3000\frac{1.285}{101430}]^{\frac{1.222}{.222}}$

=101430 [1--.068]$^{5.5}$

= 101430[.932]$^{5.5}$

= 101430x.6788

= 68960 N/m2

= 6.896 N/cm2

B to calculate the density at 3000 mw

We know $\frac{P}{\rho}$ =RT

So ρ =$\frac{P}{RT}$

T =temp at 3000m

= t_0 –L or $\frac{dT}{dZ}$ = 15--.0065x 3000 [L=lapse rate]

= 15—19.5

= --4.5°C

T = 273 –4.5 =268.5K

So, ρ= $\frac{P}{RT}$ =$\frac{68960}{274.09x26\ \ .5}$

= .937kg/m3

Q 20

A plane is flying at an altitude of 4000 m. Calculate pressure around the plane given the lapse rate in the atmosphere as.0065°k/m. Neglect variation of g with altitude. Take pressure and temp at ground level as 10.143 and 15° C resp. Density of air at ground level as 1.285 kg/m3. [Ans; 6.038 N/cm2]

Solution

Given

Height of plane = Z =4000m

Lapse rate of temp = L = .0065°k/m

Pressure at ground level =P_0=10.143 N/cm2

= 101430

N/m2

Temp at ground level = T_0= 15°c +273=288 k

Density of air at ground level = ρ_0= 1.285 kg/m3

Lapse rate = L =$\frac{dT}{dZ}$ = .0065K/m

Temp. at 4000m = T_0 --- $\frac{dT}{dZ}$x 4000 m height.

= 288—26= 262 k

First we have to power index k

And before that R (gas constant)= $\frac{P_0}{\rho_0 T_0}$

= $\frac{10143}{1.285x2}$ =274.09

L= --g/R x (K—1)/K

K—1/K = --9'81/274.09 x $\frac{1}{.0065}$

K= 1.222 = Power index

Now we will utilize equation (2.19 of chapter 2)

P = $P_0[\ 1 - \frac{K-1}{K} x\ gzx \frac{\rho_0}{P_0}]^{\frac{k}{k-1}}$

= 101430[1—(1.222—1)/1.222 x 9.81x4000x $\frac{1.285}{101430}]^{\frac{1.222}{.222}}$

= 101430[1-- $\frac{9328.329}{101430}]^{5.5}$

= 101430x [.908]$^{5.5}$

=101430x .5881= 59650'98 N/m2

= 5.965 N/cm2

Q22

An oil of s p gr is .8 under a pressure 137.2 k N/m2

A What is the pressure head expressed in meter of water ?

B what is pressor head expressed in meter in oil ?

[Ans A 14m B. 17.5m]

Solution

Oil s p gr =.8

ρ = .8x 1000= 800kg/m3

Z= Pressure head of oil= $\frac{P}{\rho g}$

$= \frac{137.2}{.8x9.81}$

= 137.2/7.848

= 17.5 m

Z = power head for water = Z= $\frac{137.2x1000}{1000x\ .81}$

$=\frac{137.2}{9.81}$

= 13.985 m

Q 23

The atmospheric pressure at sea level is 101.3 k N/m3 ad temp 15°C. Calculate pressure at 8000 m above sea level assuming i) isothermal variation of pressure and density ii)Adiabatic variation of pressure and density. Density of air 1.285 kg/m3 at sea level. Neglect variation of g .

Ans; 37.45 k N/m2,31.5k N/m2

Solution°

Given

Atm pressure =101.3k N/m2=101300 N/m2

Temp =15°C . ρ_0 =Density at sea level =1.285 kg/m2

i) isothermal variation of Pressure and Density

P= $P_0\ e^{\frac{-gz}{RT}}$ (Equation no 2.18)

RT=$\frac{P_0}{\rho_0}$

Hence P=101.3x1000 $e^{\frac{-9.81x8000x1.285}{101300}}$

$=101300e^{\frac{1}{.9955}}$

$=101300\text{x}\frac{1}{2.7}$=37511.39 N/m2=37.5k N/m2

ii) Adiabatic pressure variation

Assuming power factor k=1.4

P= $p_0[1-\frac{K-I}{K}gz\frac{\rho_0}{P_0}]^{\frac{k}{k-1}}$ as equation no (2.19)

= $101300[1—\frac{1.4-1}{1.4}\text{x}9.81\text{x}8000\text{x}\frac{1.285}{101300}]^{\frac{1.4}{1.4-1}}$

$=101300[1\text{-- }.2844]^{3.5}$

$=101300\text{x }(.7156]^{3.5}$ N/m2

=101300x .31/1000 k N/m2

= 31.4 k N/m2

Q 24

What are the gage pressure and absolute pressure at a point 4 m below the free surface of a liquid of s p gr 1.53, if atmospheric pressure is equivalent to 750 mm of mercury . [Ans;60037N/m2,160099 N/m2]

Solution

Given

Z=4m below the free surface.

Gage pressure =P= ρ g h

= 1000 x 9.81x 4

= 60037.2 N/m2

Absolute pressure =Atmospheric pr + gage pr

Atmospheric pr head = z_0=750 mm of mercury

=.75m of mercury.

Atmospheric pressure =$\rho_0\ g\ z_0$

=(13.6x1000)x 9.81x.75 N/cm2

=100062N/m2

Absolute pressure = 60037.2 +100062

= 160099N/m2

Q 26

A tank contains a liquid of specific gravity .8. Find the absolute pressure and gage pressure at a point which is 2m below the free surface of liquid. Atmospheric pressure head is equivalent to 760mm of mercury. [Ans ;117092 N/cm2,15696 N/m2]

Solution

Pressure point 2m below the free surface of liquid=Z

S. p gr of liquid = .8

Density =.8x1000 =800 kg/m3

Density of mercury = 13.6x1000 =13600 kg/m3

Atmospheric pr head = 760mm of mercury

= 760/1000 m of mercury

=.76 m of hg = z_0

1 gage pressure = z ρ g

=2x 800x9.81

= 15696 N/m2

2 atmospheric pressure $= z_0 \rho_0 x g$

= .76x13600x9.81

= 101396.16

N/m2

Absolute pressure =1 +2

=15696 +101396.16

=117092.16N/m2

End of 2nd Chapter

……………………………..

Fluid Mechanics

CHAPTER 3

Hydrostatic Forces On Surfaces

Q 1

Determine the total pressure and depth of center of pressure on a plane rectangular surface of I m wide ad 3m deep when its upper edge is horizontal and (a) coincides with water surface (b) 2m below the water surface .[Ans; a)44145N,2m b)103005N,3.714m]

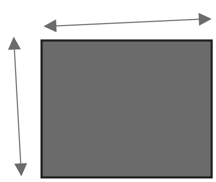
Solution

Given

a) Plane rectangular surface upper edge coincide with water

Upper

edge1m wide



deep

Density of water=1000 kg/m3

-g = 9.81m/m2

,$\bar{h}$ =distance of C,G from upper edge=$\frac{3}{2} = 1.5$

A = 3mx1m=3 m2

F =total pressure

,h$*$ =distance of center of pressure= $\frac{I_G}{A\overleftarrow{h}}$+ $\overleftarrow{h}$

F =ρ g A $\bar{h}$

=1000x 9.81x3 x 1.5=44145 N

a) ii) depth of center of pressure

$h* = \frac{I_G}{\bar{h}} + \bar{h}$, $I_G = \frac{bd^3}{12} = \frac{2x3^3}{12} = \frac{54}{12} = 4.5m^4$

∴ ,$h* = \frac{4.5}{1.5x6}$ =2.0 m

(b) when upper edge of rectangle 2m below free water surface

2m -- - -- -- --Free water surface

G . G= c. g $\bar{h}$= 2m+$\frac{3}{2}$m=3.5m =c g w

surface

3m P P= Center of pressor

. A = Area of rectangle =3mx1m=3 m2

(i) F =total pressure= ρ g A $\bar{h}$

= 1000x9.81x3m2x3.5 N

=103005 N

(ii) center of pressure

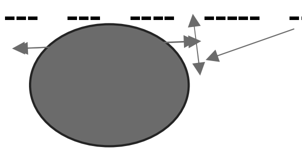
. $h* = \frac{I_G}{\bar{h}} x \frac{1}{A} + \bar{h}$

$= \frac{4.5}{3x3.5} + 3.5$

=3.928 m

Q 2

Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that center of plate is 2 m below the free surface of water. Find the position Center of pressure also. [Ans 34668.54N, 2.07m



Solution

Given

To find a) Total pressure

b) position of center of pressure

c)

,d=diameter of plate = 1.5 m

Area = $\frac{\pi}{4}$ x 1.5^2 =1.767 m2

$\bar{h}$ = c, g of circle from surface water =2m

Total pressure =F =ρ g A $\bar{h}$

= 1000x 9.81x1.767x 2

= 34668.54 N

b) Position of center of pressure

$$h* = \frac{I_G}{\bar{h}A} + \bar{h}$$

$$I_G = \frac{\pi d^4}{64} = \frac{\pi(1.5^4)}{64} = 0.2481 m^4$$

Putting the value

$$,h* = \frac{.2485}{1.767x2} = 2.0769 \text{ m}$$

Q3

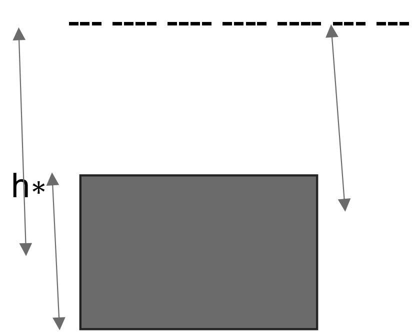
A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of sluice is 6 m in length and depth of centroid of area is 8 m below the water surface. Prove that the depth of center of pressure is given by
8.375 m.

Solution

--water surface

$\bar{h}$ c g

distance =8m



c.g

P

sluice wall

p=center of pressure

sluice vertical wall length=6 m

let us assume the breadth of sluice gate =b m

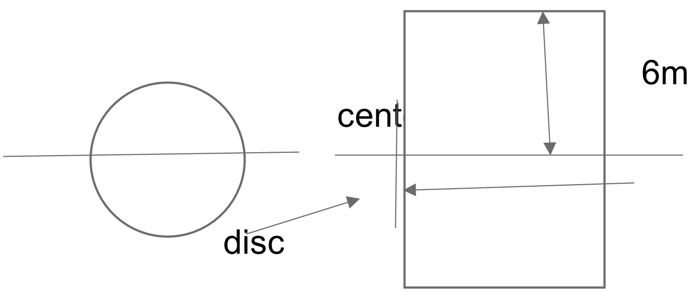
A = 6mx bm =6 b m2

$$I_G = \frac{bd^3}{12} = \frac{b6^3}{12} = \frac{bx216}{12} = 18b$$

$$,h* = \frac{I_G}{\bar{h}} x \frac{1}{A} + \bar{h} = \frac{18b}{8x6b} + 8 = \frac{18}{48} + 8 = 8.375 \text{ m}$$

Q4

A circular opening 3m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which ca rotate about a horizontal diameter. Calculate i. force on the disc ii. torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 6 m. [Ans ;416.05 k N, 39005Nm]



, h∗= pr point P

C.G= 6m from tank top=$\bar{h}$

Diameter of disc=3 m

F = force on the disc on p i. e pressure point

Torque= F x(h∗--$\bar{h}$)

A = area of disc opening = $\frac{\pi}{4}3^2$ =7.0685m2

a) Force on the disc=ρ g A $\bar{h}$

= 1000x 9.81 x 7.0685x6

= 416051.91N

Type equation here.

= 416.051 k N

b) To pressure point from surface water where F will be applied to maintain equilibrium ;

$$= h* = \frac{I_G}{\bar{h}} x \frac{1}{A} + 6 = \frac{\frac{\pi d^4}{64}}{6x\frac{\pi d^2}{4}} + 6 = \frac{\frac{d^2}{16}}{6} + 6$$

$$= \frac{d^2}{96} + 6 = \frac{3^2}{96} + 6 = .09375+6$$

$$= 6.09375 \text{ m}$$

Torque = F x (h*---$\bar{h}$)

= 416051.91(6.09375--- 6)

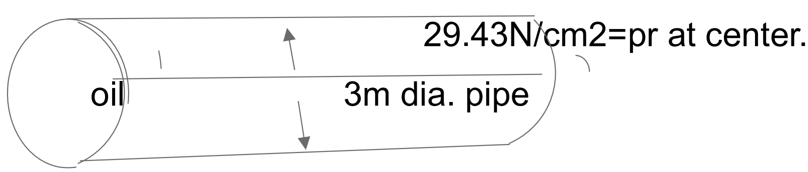
= 39004.7 Nm

Q 5

The pressure at the center of a pipe of diameter 3m is 29.43N/cm2. The pipe contains oil of s p gr 0.87 and is filled with gate valve . Find the force exerted y the oil on the gate and position of center of pressure.[Ans; 2.08MN. .016 m below center of pipe.]

Solution

Given



s p gr of oil =.87

density = .87x1000=870 kg/cm3

1 to find F on the gate

2 position of center of pressure

c/s A of pipe = $\frac{\pi}{4}3^2$ =2. 25π

weight density of oil=870x9.81

pressure at center of pipe =29.43 N/cm2

=29.43x 10^4

Pressure head at the center=$\frac{P}{W_O}$ =$\frac{\Pr atcenrer\ of\ pipe}{weight\ density of\ oil}$

= $\frac{29.43 \quad 10^4}{870x9.81}$ =height of equivalent free oil surface from the center of pipe= 34.48 m= $\bar{h}$

F =force exerted by oil on gate=ρ g A $\bar{h}$

= 870x9.81x2.25π x 34.48

=2080387.4 N

= 2.08 MN.

Position of center of pressure = h∗ = $\frac{I_G}{A\bar{h}}$ + $\bar{h}$

=$\frac{\frac{\pi}{64}a^4}{\frac{\pi}{4}a^2\bar{h}}$ +$\bar{h}$ = $\frac{d^2}{16\bar{h}}$ +$\bar{h}$ =$\frac{3^2}{16x34.48}$ +34.48

= .0163+34.48 =34.4963m

Therefor center of pressure is below the center of pipe by a distance .0163 m

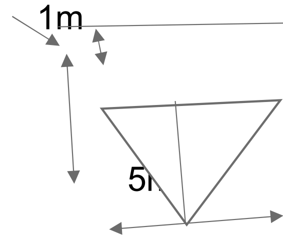
Q 6

Determine total pressure and center of pressure on an isosceles triangular plate of base 5 m and altitude 5 m when the plate is immersed vertically in an oil of s p gr .8. The base of the plate is 1 m below the free surface of water.

[Ans; 261927N, 3.19m] free surface of oil

Solution

Given



Area of triangle $=1/2 \times 5 \times 5=25/2=12.5$ m2

c. g from base= $\bar{h} = \frac{h}{3}$ =5/3

so C.G from oil surface = $\bar{h}$ = 5/3 m +1m =2.666m

density of oil=ρ =.8x 1000=800 kg/m3

A F =total pressure=ρ g A $\bar{h}$

= 800 x9.81 x 12.5x2.666

= 261534.6 N

B $h* = \frac{I_G}{\bar{h}A} + \bar{h}$ = center of pressure

I_G = moment of inertia of triangle = $\frac{bh^3}{36} = 5.\frac{5^3}{36} = 17.36 m^4$

So, $h* = \frac{17.36}{2.666x12.5}$ = .522+ 2.666 =3.19 m

Q 7

The opening in a dam is 3m wide and 2m high. A vertical sluice is used to cover the opening . On upstream of the gate ,the liquid of s the bottom,[Ans; 206010N, 0.964 m above the hinge.]

Solution

Free surface of liquid

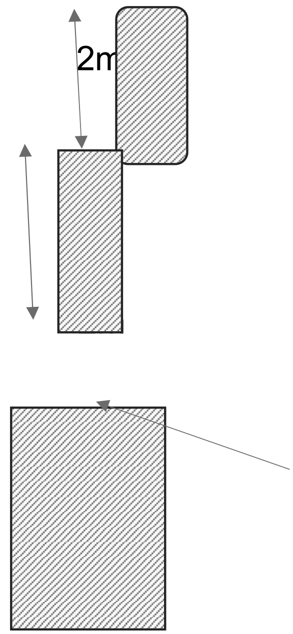
------ ------ ------- -----

upstream down

stream

--- ----- ---

- water side



Hinged at

bottom

Upstream position

Liquid s p gr = 1.5

ρ_1= 1.5x1000 =1500 kg/m3

opening of dam = A = 3m wide x 2m high =6 m2

depth of C.G from surface of liquid= 2/2 + 2m= 3

∴ F_1=force exerted by liquid = $\rho_1\bar{h}g$ A

= 1500x9.81x3x6

=264870 N

Downstream position (water)

A= 6m2

, ρ =1000. Area=6m2 ,g=9.81 kg/m2

$\overline{h_2}$ =depth of C.G=2/2=1 from surface of water,

$\therefore F_2$= force exerted by water on the gate

= $\rho_2 \bar{h}_2$A.g= 1000x 1x6x9.81 =58860N

Therefore R= Resultant Force = $F_1 - F_2$= 264870--- 5886
0

=

206010 N

-- Position of center of pressure of resultant force

F_1 will be acting at a depth of h$*$ from the surface of liquid.

, h$*$=$\frac{I_G}{A\bar{h}}$+ $\bar{h}$

[I_G=$\frac{bd^3}{12}$=$\frac{3x2^3}{12}$ =2m^4]

= $\frac{2}{6x3}$+ 3 =3.111

Distance 0f F_1 from hinge =2+2—3.111

= .889m

Force F_2 will be acting at a depth of $h *_2$from the free surface of water

I_G = 2m^2

$\bar{h}_2$ = 1m

A=6m2

$h *_2$= $\frac{I_G}{\bar{h}_2}$x1/A + $\overline{h_2}$

=$\frac{2x1}{6x1} + 1$

=1.333m

Distance of F_2 from hinge =2—1.333

=.667 m

The resultant force 206010N will be acting at center of pressure point at a distance

$$=\frac{264870x.889--58860x.667}{206010}$$

=.952 m above the hinge.

Q 8

A caisson for closing the entrance of a dry dock is of trapezoidal form 16 m wide at the top 12 m wide at the bottom 8m deep. Find the total pressure and center of pressure on the caisson if the water on is 1m below the top level of the caisson and dock is empty. [Ans; 3.164 MN,4.56 m below water surface.]

ABCD is caisson ,DJ+JI+ IC= 2m+12m+2m

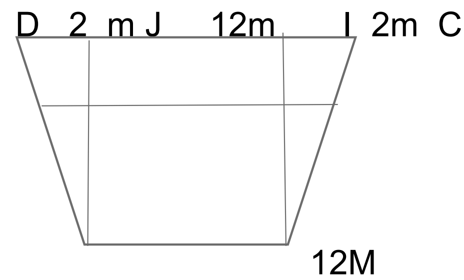
EG +GH+HF=1.75m+12m+1.75m

AB =12m (G and H are crossing points between EF line and AJ and BI lines .We are unable to write G,H at those positions because of laptop problem)

AJ=8m =BI.,

AG=BH=7m and GJ=HI=1m

Dimensions of caisson are detailed above.



EGHF is the water surface line.

Area of trapezoid AEFB = AGHB+AEG +BHF

= 12x7+1/2x 7x

1.75x2

= 96.25 m2

C.G of rectangle = 7/2= 3.5 m = $\bar{h}_1$

C G of 2 triangles = 1/3 x7= 2.33m =$\bar{h}_2$

So C G of submerged portion ABEF = $\bar{h}$

$$= \frac{A_1 x \bar{h}_1 - A_2 x \bar{h}_2}{A_1 + A_2}$$

$$= \frac{84x3.5+2.33x1\ \ .25}{84+1\ \ .55}$$

= 322.54/96.55

=3.35 m

Total pressure F = ρ g A $\bar{h}$

= 1000x 9.81x 96.25x 3.35

= 3163111.87 N

= 3.163 MN

To find center of pressure ;-

I_G of submerged trapezoid=$\frac{a^2+4ab+b^2}{36(a+b)}$

$$=\frac{15.5^2+4.15.5x12 \qquad ^2}{36(15.5+12)}$$

= 387.59 m4

$\bar{h}$ = 3.35 m , A = 96.25

Center of pressure h$*$ $=\frac{I_G}{A\bar{h}}+\bar{h}$

$=\frac{387.59}{96.25x3.35}$+3.35

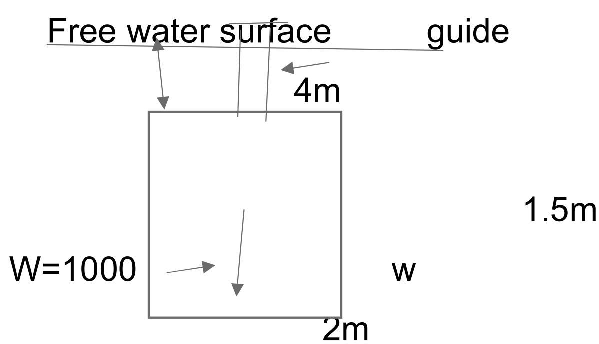
= 1.20 +3.35

= 4.55 m below the water surface.

Q 9

A sliding gate 2m wide and 1.5 m high lies in vertical plane and has a co efficient of friction of 0.2 between itself and guides. If gates weigh 1 ton ,find the vertical force required to raise the gate if its upper edge is at a depth of 4m from the free surface of water.

[Ans; 37768.5



Sliding gate

Due to roller friction vertical force has to counter

Gate self weight and frictional force.

Gate

Wide =2m . height=1.5 m

Area of gate = 2x1.5=3.0 m2

Weight of gate =1000 kg

Co. friction =μ =.2

C G of gate =1.5./2+4= 4.75 m

$\bar{h}$= 4.75m

Total pressure (F) = ρ g A $\bar{h}$

= 1000x9.81 x 3 x 4.75 N

= 139792.5 N

Vertical upward force = weight of gate (W)+Frictional force

= 1000x9.81 +μ x total pressure(F)

= 9810 +.2(139792.5)

=9810 +27958.5

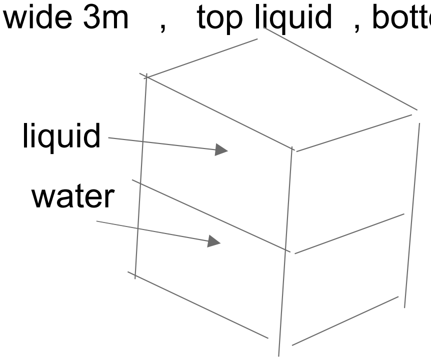
= 37768.5 N

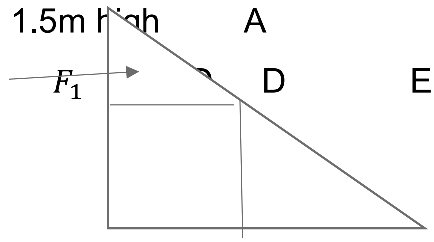
Q 10

A tank contains water up to a height of 1 m above the base . An immiscible liquid of s p gr .8 is filled on top of water up to 1.5m high . Calculate i) total pressure on one side of the tank ii) position of c enter of pressure on one side of the tank which is 3 m wide. [Ans; 76518 N ,1.686 m from top]

Solution

wide 3m , top liquid , bottom water





(I) Total pressure in tank=F=$F_1 + F_2 + F_3$as in pr. diagram.
Liquid height=1.5m water =1m
S p gr liquid =.8
Density of liquid =,8x1000= 800 kg/m3
Density of water=1000 kg/m3
Width of tank =3 m

Ref pr. Diagram

Pressure at A =0
Pressure at base BC=P at DE or BF +P at FC
=ρ g h +ρ g h
= 800 x9.81x 1.5m+ 1000x9.81x1m
=11772+9810 =21582 N
Now force F_1= ½ X ADX DEX3 (Wide)
=1/2 x1.5 x 11772x 3 m
=26487 N
F_2= area DEBF x width of tank
= (1m height x 11772 x 3 m)

=35316 N

F_3 =1/2x EF x FC x width of tank

= ½ x 1 m x 9810 x 3m

= 14715 N

F=Total force =$F_1 + F_2 + F_3$ = 76518 N

Position of center of pressure h∗ for one side of the tank

Taking moment of all above 3 forces about A of pressure diagram figure above, we get

F x h∗ = $F_1 x \frac{2}{3} x AD + F_2\, x \left(AD + \frac{1}{2} BD\right) + F_3 x (AD + \frac{2}{3} x BD$

= 26487x2/3x 1.5+35316x (1.5+1/2x1) +14715(1.5+1/2 x1)

= 26487+70632+31784.

= 128903.4 Nm

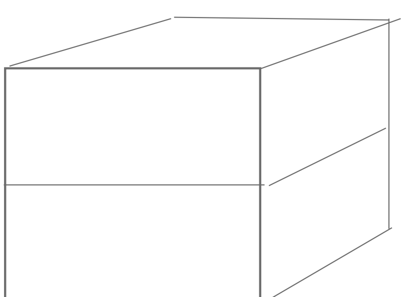
, h∗ =$\frac{128903,4}{76518}$ = 1.684 m from A that is top .

Q 11

A rectangular tank 4m long 1.5m wide contains water up to a height of 2m. Calculate force water pressure on the base of the tank. Find also the depth of the center of pressure from free surface. [117720 N, 2m from free surface.]

Solution

1.5m wide



4m , base

Single tank, half tank up to 2 m of which is with water, other half is empty, obviously top of tank is free surface.

To find

A force due to water pressure on the base of tank

$F = \rho\ g\ h\ A$

$= 1000 \times 9.81 \times 2m\ (4\ m \times 1.5)$

$=117720\ N$

B Center of pressure $=h* = \frac{I_G}{\bar{h}} \times \frac{1}{A} + \bar{h} = 2m$

Since there is empty space the force due to water pressure will have center pressure at 2m from free surface that is at the base of the tank.

Q 12 A rectangular plane surface 1m wide and 3m deep lies in water in such a way that its plane makes an angle of 30°with free surface of water. Determine the total pressure and position of center of pressure when the upper edge of the plate 2m below the free water surface [Ans ;80932.5N ,2.318 m]

Solution free surface of water A

P =Pressure center point

CE is parallel to OA so angle E =30°

$\bar{h}$ =AE+EB=2m+1.5 sin 30°=2 .75m

A =area of plane =1m x3 m=$3m^2$

ρ =1000 kg/m3, g =9.81

1) total pressure = F = ρ x $\bar{h}$ g x A

=1000 x 2.75 x 9.81 x 3

=80932.5 N

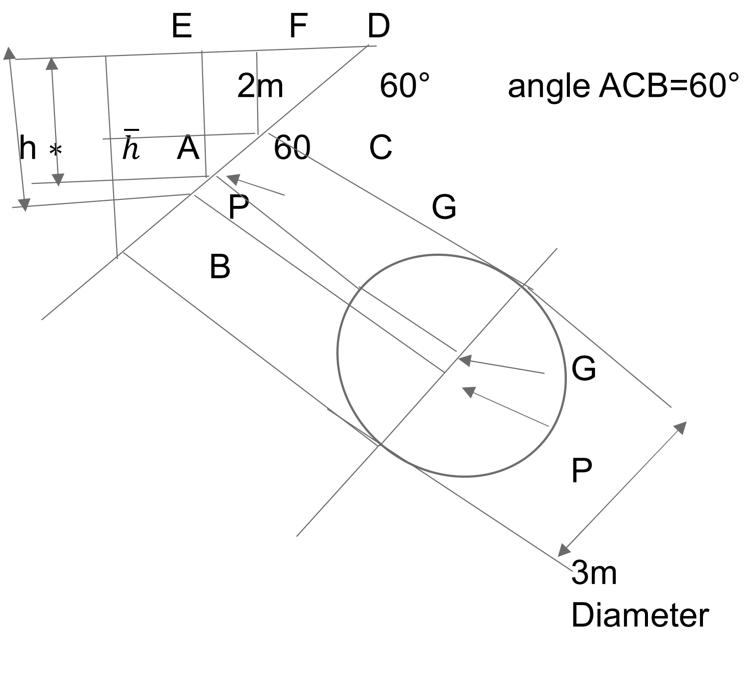
2) Centre of pressure h∗(as per equation 3.10)

$I_G = b\frac{d^3}{12} = \frac{1x3^3}{12} = \frac{27}{12}$=2.25

∴ h∗=$\frac{I_G}{A\bar{h}}$ xsin_θ^2+$\bar{h}$=$\frac{2.25x.5^2}{3x2.75}$+2.75 =.068+2.75=2.818 m

Q13

A circular plate 3m diameter is immersed in water in such a way that the plane of the plate makes a of 60 with free surface of water. Determine the total pressure and position of center of presser when the upper edge of the plate 2 m below the free surface of water [Ans ;228.69k N,3.427m from free surface]



.ρ =1000 kg/m3

Diameter of circle =3 m

Area = $\frac{\pi}{4} 3^2$=7.0685m2

Upper edge 2m below water surface

$\bar{h}$ = 2m + 1.5sin60° =2 +1.299=3.299m

, (1) Total force= F = ρ g A $\bar{h}$

= 1000x 9.81 x 7.0685 x 3.299 N

=228759.2 N

= 228.759 k N

, (2) Center of pressure =h∗ =$I_{G/A\bar{h}}$ + $\bar{h}$

Being a circular plate

Moment of inertia =$\frac{\pi d^4}{64}$= $\frac{\pi 3^4}{64}$ =3.976m4

∴ h∗ =$\frac{I_G}{A\bar{h}}$ +$\bar{h}$ =$\frac{3.976}{7.0685x3.299}$+3.299

= .170 +3.299

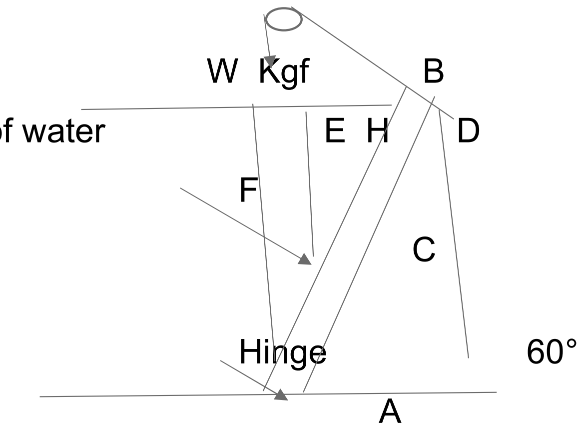
= 3.469 m

Q14

A rectangular gate 6mx2m hinged at its base and inclined at 60° to the horizontal as shown in fig To keep the gate in stable position a counter weight 29430N is attached at the upper end of the gate . Find the depth of water at which gate egis to fall. Neglect the weight of the gate also friction at the hinge and pulley. [Ans 5.43m]

Solution

Free surface of water



AE=h m

AB = length of gate= 6m

.b = width of gate= 2m

D is point on where water surface touch gate.

F force is of water y which gate is likely to fall.

Gate angle with horizon=60°

W = counter w .t to balance the presser of water.

= 29430 N

AD = $\frac{h}{sin60}$ $=\frac{h}{\frac{\sqrt{3}}{2}}$ $=\frac{2h}{\sqrt{3}}$ where h = depth of water I immersed area.

Area of immersed portion of gate = AD x width

$= \frac{2h}{\sqrt{3}}$x 2m

$=\frac{4h}{\sqrt{3}}$

Depth of C.G at the immersed area= $\bar{h}$ =h/2=.5h

Total force F of immersed area =ρ g $\bar{h}$ A

$= 1000 \times 9.81 \times .5h \times \frac{4h}{\sqrt{3}}$

$= \frac{19620}{\sqrt{3}} x h^2$ N ------(-i)

Now we will work out center of pressure h$*$ through which total F will work.

$,h* = \frac{I_G}{A\bar{h}} + \bar{h}$

For this we have to find I G i,e M.O,I of immersed area

$= \frac{bd^3}{12} = \frac{2xAD^3}{12} = 2\,\frac{(\frac{2h}{\sqrt{3}})^3}{12} = \frac{4h^3}{9\sqrt{3}} = I_G$

$h* = \frac{I_G}{\bar{h}} x \frac{sin\theta^2}{A} + \bar{h}$

$= \frac{\frac{4h^3}{9\sqrt{3}} sin\theta^2}{\frac{4h}{\sqrt{3}} x \frac{h}{2}} + \frac{h}{2}$

$= \frac{4h^3/9\sqrt{3}}{\frac{4h^2}{2\sqrt{3}}} x (\frac{\sqrt{3}}{2})^2 + \frac{h}{2} = \frac{6h}{9x4} + \frac{h}{2} = \frac{2h}{3}$ m $= h*$

So force of water will act through h$*$ which is below 2h/3 of free surface water.

Now let us consider triangle CHD, in which CH=2h/3m

Angle HDC=60°

$Sin60° = \frac{CH}{CD}$

$CD = \frac{CH}{SIN60} = \frac{\frac{2h}{3}}{\frac{\sqrt{3}}{2}} = \frac{4h}{3\sqrt{3}} = CD$

AC=Distance of hinge from force F

$= AD—CD = \frac{2h}{\sqrt{3}} -- \frac{4h}{3\sqrt{3}} = \frac{2h}{3\sqrt{3}}$ m=AC

We will take moment of Force F and W (Counter weight) around the Hinge

F X AC =WX AB

$$19620\ h^2/\sqrt{3}\ \text{X}\ \frac{2h}{3\sqrt{3}} = 29430\ \text{NX}\ 6\text{m}$$

$$h^3 = \frac{29430x6x3x\sqrt{3}x\sqrt{3}}{19620x2} = \frac{794610}{19620} = 40.5$$

,h $=(40.5)^{\frac{1}{3}}$ = 3.43 m = depth of water at which the gate begins to fall.

Q 15

An inclined rectangular gate of 5 m and depth 1.5 m is installed to control the discharge of water as show in figure . the end A is hinged . the determine the force normal to get applied at to open It.[Ans 97435-8N] water C D

F 45° O

3m P h∗ $\bar{h}$ A Hinge E 45 ° G

B H 1.5m B

As shown in above figure , F force is applied at h∗ distance and $\bar{h}$ where C.G is existing , both from surface water.

A is the hinged point which is at 1.5 m from B where Force P , normal to the gate, is applied.

B is 3m below the water surface. Gate is at 45° angle and have 5 m width and depth 1.5m.

Area of gate =5x1.57.5m2

$\bar{h}$= depth of C.G=BC—BE

=3m---BGsin45°

$=3m --.75x \frac{1}{\sqrt{2}}$

=2.47m

F =Total force= ρ g A$\bar{h}$

= 1000 x 9.81 x 7.5x 2.47

=181730.25N

This force is working at H which is h$*$ below the surface water.

Before we have to find out M.O.I

$$= \frac{bd^3}{12} = \frac{5x1.5^3}{12} = 1.4m4$$

$$h* = \frac{I_{Gsin45^2}}{A\bar{h}} + \bar{h} = 1.4\frac{(\frac{1}{\sqrt{2}})^2}{7.5x2.47} + 2.47$$

$$= \frac{1.4}{7.5x2.47x\ 2} + 2.47$$

= 2.5078 m

In fig. $\sin 45 = \frac{h*}{OH}$

OH =2.5078X$\sqrt{2}$ =3,546m

Total force will act through H, AH is the distance of F from hinge .

BO=3m/sin 45 =3 x 1.414=4.242m

BH= BO—HO=4.242—3.546 =.696m

AH =AB—BH=1.5 --.696=.804m

Now taking moment about the hinge

PXAB = FX AH

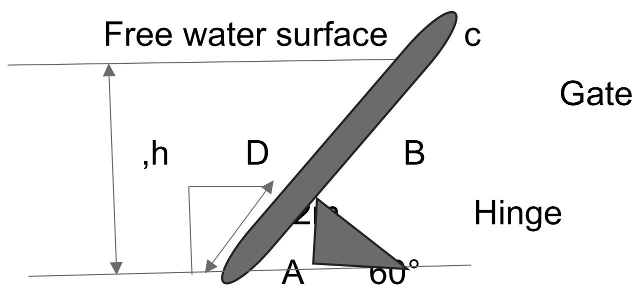
PX 1.5 = 181730.25 X .804

$$P = \frac{181730.25 \quad .804}{1.5} = 97407.41 \text{ N}$$

Force applied to open the gate.

Q 16

A gate supporting water is shown in fig. Find the height h of water so that gate begins to tip about the hinge take width of the gate as unity. [Ans ;3x $\sqrt{3}$ m]



If resultant force acts between B and C the gate will tip over the Hinge. , h is the depth of water.

$$AC=\frac{h}{sin60°} = \frac{2h}{\sqrt{3}} \quad , \quad AD=AB \sin60 =2x\sin60=1.732m$$

Resultant force passes through h∗ I, e center of pressure .

In limiting case when resultant force passes through B

$$h* =h\text{---} AD=h\text{—}1.732= h\text{--} \sqrt{3} \qquad \text{---------}$$

(i)

$$\text{but } h * =\frac{I_G}{A\bar{h}} + \bar{h}$$

$$M.O.I=\frac{bd^3}{12} = \frac{1xA^{\ 3}}{12} =\frac{(2h/\sqrt{3})^3}{12} = \frac{2h^3}{9\sqrt{3}} =I_G$$

Type equation here.

. $h* = \frac{\frac{2h^3}{9\sqrt{3}}}{\frac{2h}{\sqrt{3}}} x \frac{(\frac{\sqrt{3}}{2})^2}{\frac{h}{2}} + \frac{h}{2} = \frac{h}{6} + \frac{h}{2} = \frac{2h}{3}$ -------------(-ii)

Equating i) and ii) above

,h – AD $= \frac{2h}{3}$

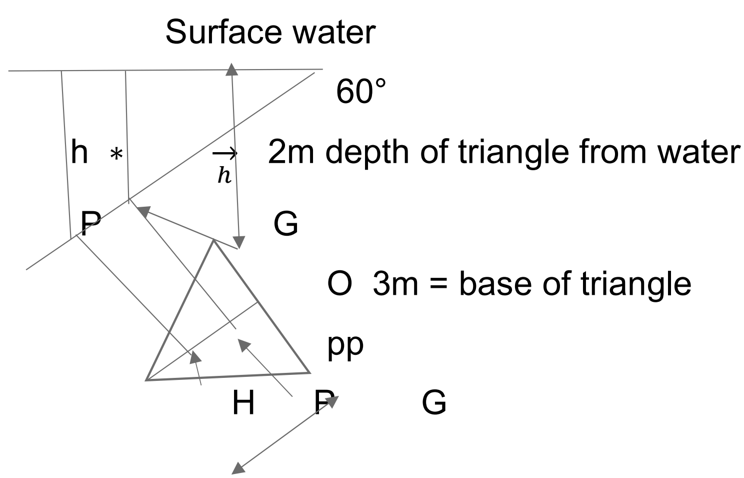
Or, ,h—2h/3 =AD=1.732 $=\sqrt{3}$

Or, $\frac{3h-2h}{3} = \sqrt{3}$

Or h $=3\sqrt{3}$ (height h of water)

Q 17

Find the total pressure and center of pressure on a triangular plate of base 3 m which is immersed in water in such a way plane of the plate makes an angle of 60°with the free surface The base of the plate is parallel to the water surface and at a depth of 2m from water surface. [Ans;126.52k N,2.996]



3m= height of triangle

Base of plate =3m, height =3m, A=area=1/2x3x3=4.5

Angle =60°

$\bar{h}$=2m+(1/3 x 3)sin 60 [GO=1/3 OH]

= 2m+.866=2.866m

(i) F =Total pressure= ρ g $\bar{h}$ A

= 1000 x9.81x2.866x4.5

= 126.5 19.57 N

= 126.52k N

(II) Center of pressure

$I_G = \frac{bh^3}{36} = \frac{3x3^3}{36}$ =2.25m4

,h∗ = $\frac{I_G}{A\bar{h}} x sin60^2 + \bar{h}$

= $\frac{2.25x(\frac{\sqrt{3}}{2})^2}{4.5x2.866}$+2.866

=.13+2.866= 2.996m =h∗

Q 18

Find the horizontal and vertical component of the total force *acting on a curved surface AB which is in the form of quadrant of* a circle of radius 2 m as shown in fig . take width of the gate 2m. [$F_x 117.72\ k\ N$ F_Y=140.114k N]

Solution

Given

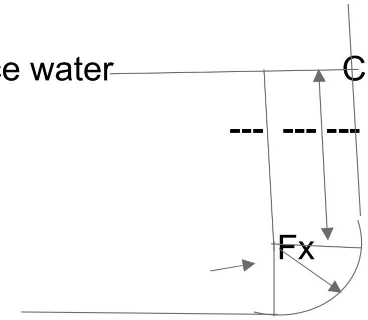
- surface water C D

--- --- --- ---- --

2m gate width

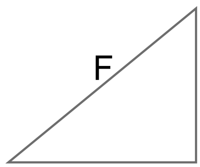
Fx O A

B 2m radius



quadrant

F



,dA sin θ

E G Angle FEG=θ

,dA cos θ

, d F=ρ g h x d A as per equation 3.11 ref book.

, d F_x =d F Sin θ = ρ g h d A sin θ

, d F_y =d F Cos θ = ρ g h d A cos θ

On integration

F_x = g ρ $\int h$ d A Sin θ d θ

F_y =g ρ $\int h$ d A cos θ d θ

Total pressure force on the projected curved surface on O B

Weight of liquid supported by curved quadrant

Width of gate =2m

Quadrant OA=OB =Radius =2m

Horizontal force Fx given by water on gate is given by (eq. 3.17)

A= projected area of curve surface on vertical plane

= OB x width of gate=2x2=4 m2

$\bar{h}$ = depth of C . G. of OB from free surf2mace =2m+2m/2

= 2+1=3m

1) So Fx= pressure on OB= g A ρ $\bar{\bar{h}}$ = 9.81 x 4 x 1000 x 3

= 117720 N

= 117.72 k N

2) Point of application Fx

,, h∗ = $\frac{I_G}{A\bar{\bar{h}}}$ + $\bar{h}$ [I_G =$\frac{d^3b}{12}$ =$\frac{2^3x2}{12}$ = 4/3]

= $\frac{4/3}{4x3}$+ 3 = 3$\frac{1}{9}$ m

3) Fy = water pressure on BOCDA area

= ρ g (volume of water in area COAD +

Volume of water in OBA area)

= ρ g {OAXCOX width +($\frac{\pi}{4}$ x$\frac{OB^2}{}$) x width}

= 1000x 9.81(OAXCOX Width+$\frac{\pi}{4}x2^2x2$)

=9810 (2x2x2 + 2π) = 9810x 14.28318

= 140117.9 N = 140.111 k N

Q 19

Fig shows a gate having a quadrant shape of radius of 3 m. Find the resultant force due to water per meter length of the gate . Find also angle at which the total force will act.[82.201k N Angle 57 degree.

Solution

Given

water surface R3m O Hinge

1) to find resultant force/m length of gate
2) angle at which total force will act

radius of gate =3 m

width of gate =1m

A =area =3x1=3 m2

$\bar{h}$ = 3m/2=1,5m

1) Horizontal force F_x =ρ g A $\bar{h}$ = 1000x9.81x 3x1.5

=44145 N= 44.145 k N

Fy= weight of water supported by AB quadrant

=ρ g $\frac{\pi}{4}$ x3^2 =1000x 9.81x 3.14x 2.25

= 69307.61 N

=69.307 k N

Resultant force =$\sqrt{Fx^2+Fy^2}$ =$\sqrt{44.14^2+69.3^2}$

=$\sqrt{\ 1948.33+4802.49\qquad}$

=$\sqrt{6750.82}$

= 82.16 k N

$$\text{Tan } \theta = \frac{Fy}{Fx} = \frac{69.307}{44.14} = 1.57$$

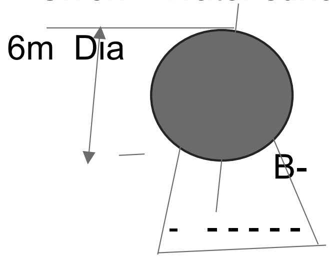
Θ =57°31min angle made by resultant

Q 20

A roller gate is shown in figure It is cylindrical form 6.0 m diameter. It is placed on the dam Find the magnitude and direction of resultant force due to water acting on the gate when water is just going to spill. The length of the gate is 10m.

Ans ;2.245MN, 38°.8 min

Solution Given Water surface



Roller gate length =10m

Dam

To find 1) magnitude and direction of resultant force.

when water is about to spill.

L = length of gate =10

Dia of roller gate =6m

A of gate = 6x10 =60m2

$\bar{h}$=6m/2=3m

Fx=ρ g A $\bar{h}$= 1000 x9.81x3x60

=1 765800 N=1.76 MN

Fy = force of water enclosed by the curved surface ACB

=ρ g x volume ACB enclosed water (half roller only)

= 1000x9.81x area of ACB x length of roller gate

= 9810 x $\frac{\pi}{4} x 6^2 x 10$ = 1386153N =1.386 MN

Resultant force R = $\sqrt{1.765^2 + 1.386^2}$

=$\sqrt{5.028}$ =2.242 MN (MN=mega neu ton

1000 k neu ton)

Direction of resultant =$\frac{F_x}{F_Y}$ =$\frac{1.386}{1.765}$ =.7852 =38 ° 2 min

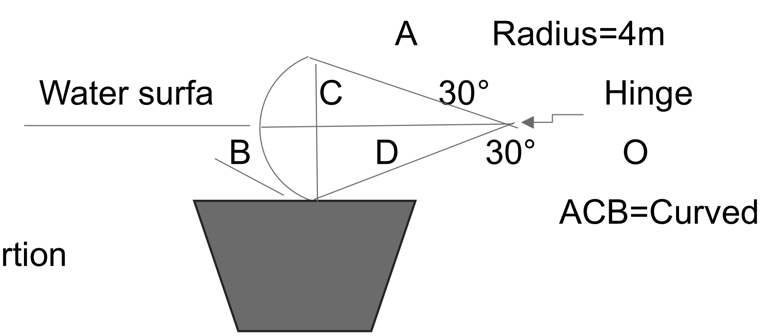
Q 21

Find the horizontal and vertical of the water pressure exerted on tainter gate of radius 4m as shown in fig . Consider width as unity.

[Ans Fx=19.62 Kn. Fy= 7102.44 N]

Solution

Given



portion

BD=2m

OA=OB=4m

Sin 30 =$\frac{BD}{OB}$,BD= OB Sin30=4mx ½=2m

Width of gate = 1m

A=area= BDX Width= 2mx1m=2 m2

$\bar{h}$ =$\frac{BD}{2}$=$\frac{4sin30}{2}$= 4x1/ 2x2 =1

F x=ρ$g\bar{h}A$ =1000x9.81x 1x 2 =19620 N

Fy = force due to water CB area

=ρ g {π/30/360 x $(4)^2$---triangle BOD}X1m

= 1000x9.81(π/12 x16---1/2 X BDXDO)X 1M

= 9810 (4.186---1/ x2x 4cos 30°)x 1

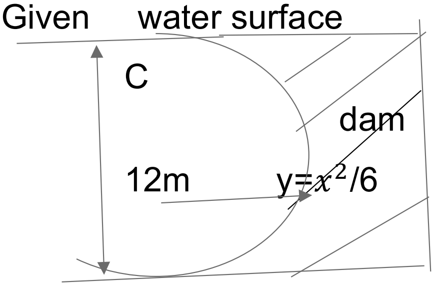
=7082..82 N

Q 22

Find the magnitude and direction of the resultant water pressure acting o a curved face of a dam which is shaped according to the relation y=$x^2/6$ as show in fig .The height of water retained by the dam is 12 m .Take width of dam as unity.

Solution

Given water surface



A

Area of

B

Width of dam= 1m

Y=$\frac{x^2}{6}$ or 6y=x^2 or x =$\sqrt{6y}$ = $\sqrt{6}\, y^{\frac{1}{2}}$

d A =d y .x , $\int A = \int x\, dY$

A = $\frac{x^2}{2}$ y .

For horizontal force, Area A = BC X Width = 12m x 1 m

= 12m2

$\bar{h}$ = $\frac{height}{2}$ = $\frac{12}{2}$ =6 m

Fx = ρ g $\bar{h}$ A =1000 x 9.81 x 6 x 12 =706320 N

Fy =weight of water on curved AB of the dam

= weight of water in the area ABC

= ρ g (area ABC x width of dam)

= 1000 x 9.81 x[$\int_0^{12} xdy$] x 1m(width)

= 9810 x [$\int_0^{12}\sqrt{6}$ ydy]x1

=9810 x$\sqrt{6}$ [$\frac{y^{\frac{1}{2}+1}}{\frac{3}{2}}]_0^{12}$x 1

= 9810 x 2.45 x 2/3 x (12)$^{\frac{3}{2}}$ x1

= 9810 x2.45 x .66 x 41.5692

= 659402.6 N

Resultant= $\sqrt{706320^2 + 659402.6^2}$ =

=$\sqrt{933699731286.76}$ = 966281.39 N

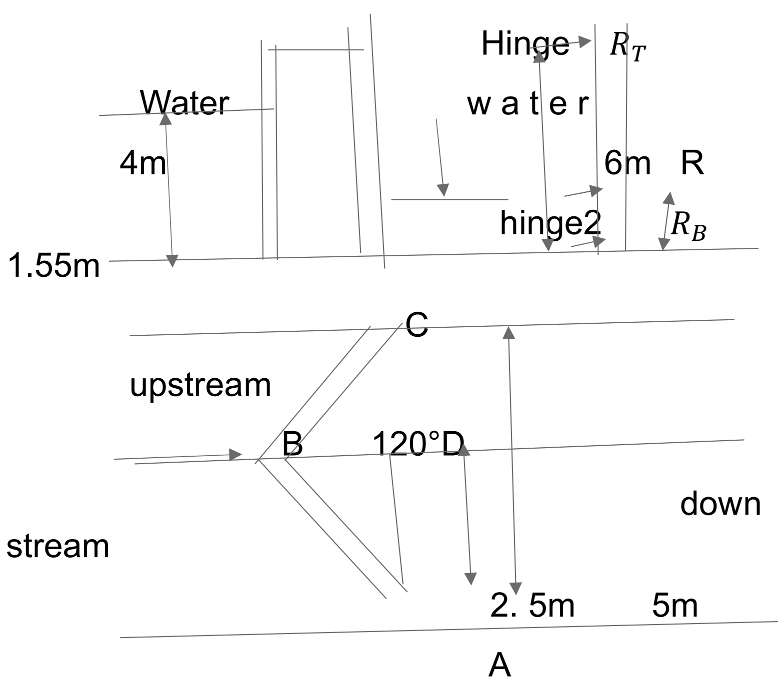
Type equation here.

Tan θ $=\frac{Fx}{Fy} = \frac{659}{706}$ =.9334 =43° 1 min =direction of resultant.

Q 23

Each gate of a lock is 5 m high and is supported by two hinges placed on top and bottom of the gate. When gates are closed they make a angle of 120 The width of the lock is 4m .If the depth of water o the two sides of the gates are 4m ad 3 m respectively, determine i) magnitude of resultant pressure of each gate ii) magnitude of hinge reaction.

Solution



Between R and

R_B=1.55M

To find resultant force on each gate

2. Magnitude of hinge reactions R_T, R_B ,

Top and bottom

Width of each lock =AD/Cos 30 =2m/$\frac{\sqrt{3}}{2}$ =$\frac{4}{\sqrt{3}}$ =L=2.31m

Angle between the gate =120°

Height of upstream water = H1=4m

Down stream=H2 =3m

$$\bar{h}_1 = \frac{H1}{2} = 4/2 = 2m$$

F1 = Total water pressure upstream =ρ g x $\overline{h_1}$x.A1

A1=H1X L =4 x 2.31=9.24 m2

F_1 =1000x 9.81x2 x 9.24 =181288 N

F_1 will act at a distance from bottom =$\frac{H_1}{3}$ =4/3= 1.33m

F_2=Total water pressure downstream =ρ g $\bar{h}_2$ A_2

$$A_2 = H2\ X\ L = 3X\ 2.31 = 6.93$$

$$\bar{h}_2 = \frac{H_2}{2} = 3/2 = 1.5m$$

F_2 =1000 X 9.81 X 1.5 X6.93 =101975 N

Resultant water pressure o each gate = $F_1 - F_2$

= 181288---10197

= 79313 N

= 79.313 k N

=79.313 k N

Resultant water pressure F acts at X distance from bottom. So taking moment of F_1 and F_2 and F we get

F x X= F1 x 1.33 --- F2 x 1

X = $\frac{181288x1.33--101975x1}{79313}$ =1.754 m

Let R is at hinges R_T and R_B

P is the reaction of common contact surface of two gates and acting perpendicular to the contact surface.

P= R =$\frac{F}{2Si\theta}$ =$\frac{79313}{2sin3}$ =79313/2x1/2=79313 N

R_T and R_B are two reactions = when added =R=79313 N

Takin moment of hinge reactions of all R s we get

R_Tx 5 +R_Bx 0=R x 1.754

R_T=$\frac{79313 \quad .754}{5}$ =27823N =27.823 k N

R_B = R--- RT = 79313--- 27823 =51490 N

=51.490 k N

Q 24

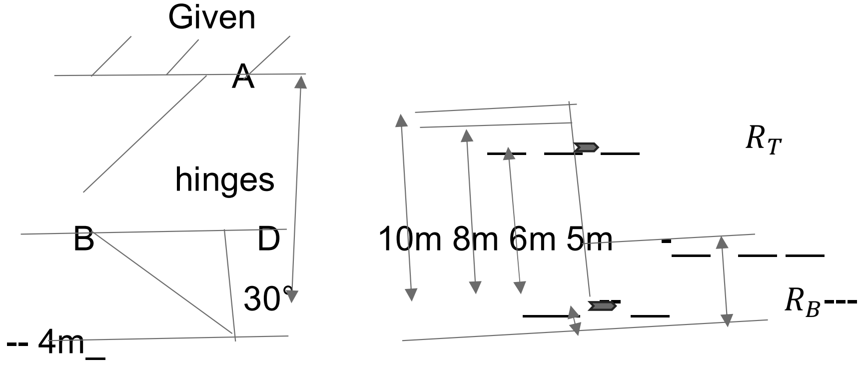
The end gates ABC of a lock are 8 m high ad when closed make a angle of 120°.The width of lock is 10 m. Each is supported by two hinges located at 1m ad 5 m above the bottom of the lock. The depth of water o the upstream ad down sides of locks are 6m ad 4m respectively.

Find i) resultant water force in each gate. Ii) reaction between gates AB and BC. Iii) Force on each hinge considering reaction at the gate acting in the same horizontal plane as resultant water pressure.

[Ans ; I 566.33 kN 2 566.33 k N3. 173.64k N ,4. 392.69k N]

Solution

Given



Plan C *elevation*

Width of lock =10m ∴ Each lock width=$\frac{\frac{10}{2}}{cos30°}$ =5.733m=L

End gates of lock =8 m

Upstream height =6m =H_1

Lower stream height =4m= H_2

Upper lock height =5m

Lower lock height =1m

$$Angle\ between\ two\ gates\ = 120°$$

Angle BCA=30°

1. Area of upper lock =A1 =L x H1 = 5.773 x 6=34.638m2
2. $\bar{h}$ =H1/2=6/2=3m
3. ∴ F1 = ρ g A1 $\bar{h}$=1000x9.81x34.638 x 3 =1019396.34 N

   = 1019.4 k N

H2=4m
A2=LxH2=5.773x 4= 23.092 m2
$\bar{h}$=4/2=2m
F2= ρ g $\bar{h}_2$ A2
1000x 9.81 x 2 x 23.092 =453.065 k N
Resultant water pressure = F1 – F2 =1019.4—453.065
= 566.335 k N

II)

Reaction between gate AB and BC i.e. P
= R = F/2Sin30
=566.335/2x1/2
= 566.335 k N

III)

Force on each hinge .Rt and Rb
Rt + Rb=R

Also R=P= 566.335 k N

F1 is acting at H1/3 =6/3 =2mfrom the bottom

F2 is acting at H2/3 =4/3 =1.33m from the bottom

Resultant F is acting at distance x m from the bottom hinge.

Taking moment of F1,F2, and F about bottom hinge will get F x X = F1 x 2--- F2 x 1.33

= 1019.4 x2 --- 453.06 x 1.33

$$X = \frac{1019.4x2 - -453.06x1.33}{566.336}$$

$$= \frac{1436.23}{566.335} = 2.536 \text{ m}$$

So R is acting at a distance of 2.536 m from bottom hinge.

Taking moment about bottom hinge we get

Rt x (5m – 1m) = R (2.563 – 1)

Rt = (566.335x1.536)/ 4m =217.472 k N

Rb= R – Rt = 566.335 – 217.472 =348.863 k N

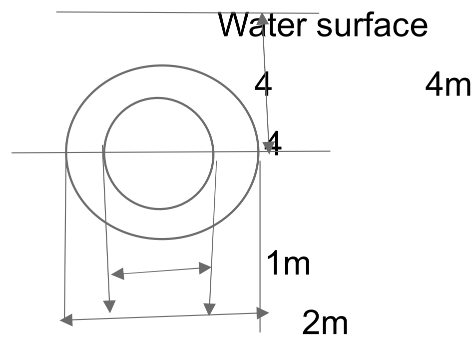
Q 25

A hollow circular plate of 2 m external and 1 m internal diameter is immersed vertically in water such that center of plate is 4 m deep from water surface. Find the total pressure and depth of center of pressure.

[Ans; 92.508 k N, 4.078 m]

Solution

Given



A =Area of hollow circular plot $=\frac{\pi}{4}(2^2-1^2)$ = 2.356m2

$\bar{h}$ = 4m(C.G of plate from water

I Total pressure = ρ g $\bar{h}$. A

= 1000 x 9.81 x 4 x 2.356

= 92.456 k N

II Depth of center of pressure = h∗

$= \frac{I_G}{\bar{h}A} + \bar{h}$

I_G For hollow circular plate$=\frac{\pi}{64}(2^4-1^4)$

= .7363 m4

h∗ = $\frac{.7363}{4x2.356}$ + 4 = 4.078 m

Q 26

A rectangular opening 2m wide ad 1m deep I a vertical side of a tank is closed by a sluice gate of the same size .The gate can turn about horizontal centroidal axis. Determine I total pressure on sluice gate ii torque on the sluice gate. The head of water above the upper edge of the gate is 1.5 m . [Ans 39.24 k N,16.35 Nm]

Solution

Given

ABCD is opening, AB= 2m ,AC=1m

From point E to mid of AB =1.5m

From that to half of side AC will give us C.G i. e $\bar{h}$

So $\bar{h}$ =1.5m + ½ x 1m=2m

Area of opening =2mx 1m=2m2

I Total pressure on sluice gate =ρ g .A . $\bar{h}$

=1000 x 9.81 x 2 x 2

= 39240 N=F force acting through C.G

= 39.24 k N

II Torque on sluice gate that is Moment of Force about ($h*$--$\bar{h}$).

For that Area A =2m2

$I_G = \frac{2X1^3}{12}$ =1/6

$h* = \frac{1/6}{2x2} + 2$ =2.0417 m

so F is acting at a distance of 2.0147m from water surface

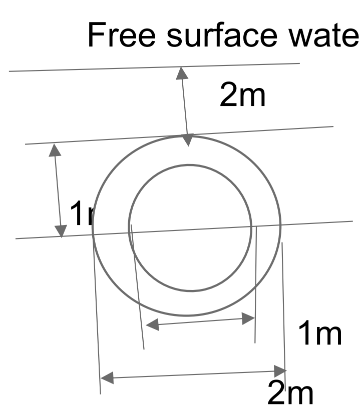
TORQUE = F x ($h*$ --$\bar{h}$) =39240 Nx(2.0417 –2)m

= 1636.3 Nm

Q 27

Determine the total force and location of center of pressure on the face of the plate shown in fig immersed in liquid of s p gr 0.9.[Ans;62.4Kn ,3.04m]

Solution



Area of plate = $\frac{\pi}{4}(2^2 - 1^2)$ = 2.355 m2

S p gr of liquid= .9

Density= .9 x1000 =900 kg/m2

$\bar{h}$ =distance from free water surface of C G.

= 2m +1m=3 m

Total Force = ρ g A $\bar{h}$ = 900x 9.81x 2.355 x 3

=62376.885 N

= 62.376 k N

II

Location of center of pressure h∗

$$I_{G} = \frac{\pi 2^4}{64} - \frac{\pi 1^4}{64} = .7363 \text{ m4}$$

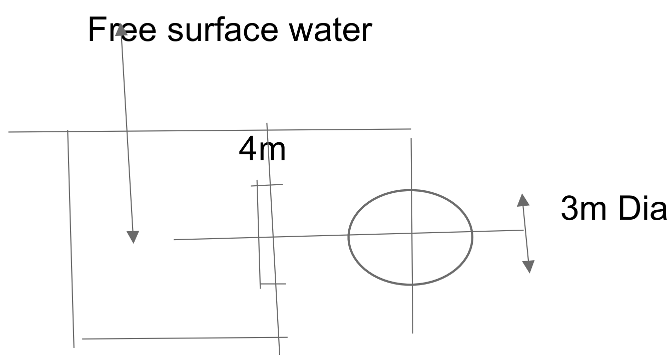
, h∗ = $\frac{.7363}{2..355x3}$ + 3 m = 3.1m from free. s. water

through which water pressure will work .

.

Q 28

A circular opening 3m diameter in vertical side of water tank is closed by a disc of 3m diameter which can rotate about horizontal diameter. Calculate 1) force on the disc ii)torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4 m.[Ans ; 270 k N , 38k Nm]



G center of disc.

1 Force on the disc =ρ g A $\bar{h}$

$\bar{h}$ =4m , A =Area of disc=π/4 x3^2

= 7.0685 m2

Force = 1000 x9.81 x 7.0685 x 4= 277367.94

= 277 kN

II Torque required for that we have to find out the center of pressure h∗ =$\frac{I_G}{A\bar{h}}$ + $\bar{h}$

I_G =π x$d^4 \frac{1}{64}$ =π x$\frac{3^4}{64}$ (moment of inertia of disc)

-=3.976 m4

h∗=$\frac{3.976}{7.06x4}$ + 4 =4.14m

Torque required to maintain the equilibrium

for disc to operate =F x (h∗-- $\bar{h}$) = 277kN x (4.14—4)

=277 x .14

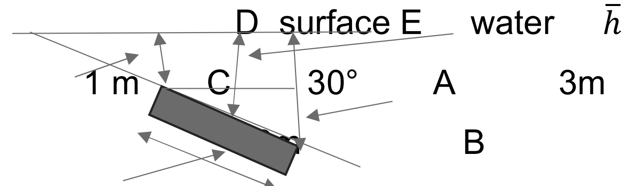
=38.78 k Nm.

Q 31

A circular plate of diameter 3 m is immersed in water in such a way that its least and greatest depth from the free surface of water are 1m ad 3 m respectively. For the front side of the plate Find I) total force exerted by water ii) position of center of pressure. [138684 N , 2.125 m]

Solution

Given



3m Diameter circular plate

A =area of circular plate =π/4 3^2 =7.068m2

DC=1m ,EB=3m , In triangle ABC

Sin ϑ =$\frac{AB}{BC}$ = $\frac{3-1}{3}$ = 2/3

= .666

C.G of plate is at the middle of BC ,1.5m from C

∴ $\bar{h}$ = CD + CG Sin θ =1 m +1.5 x 2/3 =2m

I Total pressure on front face of the plate

F =ρ .g A $\bar{h}$

= 1000 x 9.81x 7.0685 x 2

= 138683.97 N

II Position of center of pressure =$\frac{I_{Gsi\theta^2}}{A\bar{h}}$ +$\bar{h}$ =h∗

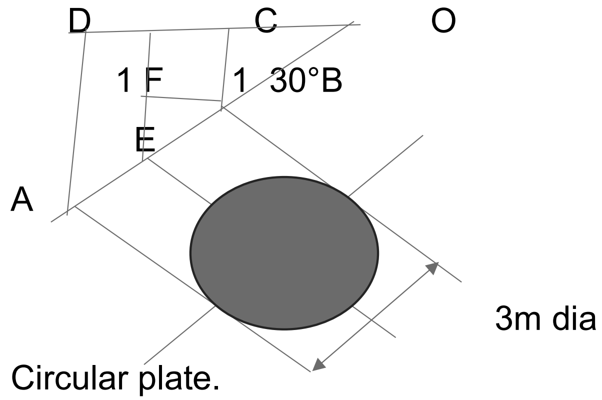
Where Ig =π/64 $x3^4.666^2$

h∗ =$\frac{3.97\quad .666x.666}{7.0685x}$+ 2m = .1248 + 2

=2.1248m

Q 33

A circular plate of 3m diameter is under water with its plane making an angle of 30°with the water surface .If the top edge of the plate 1m below the water surface ,find the force on one side of the plate and its location.



Angle = 30

Height of top edge = BC =1m

$\bar{h}$ =DE= EF+1m

=1.5 sin 30°+1m=1.75m

Area A $=\frac{\pi}{4}3^2 = \frac{9\pi}{4}$

Force on one side =ρ g A $\bar{h}$

= 1000x 9.81x

9π/4 x 1.75

= 121.35k N

$h* = \frac{I_{G sin\theta^2}}{A\bar{h}} + \bar{h}$

$= \frac{\frac{\pi}{64}3^4 si\,n3\ \ ^2}{\frac{\pi}{4}3^{2x1.75}}$ + 1.75=.08 +1.75 =1.83m

E N D OF CHAPTER 3

Fluid Mechanics

CHAPTER 4

Buoyancy And Floatation

Q 1

A wooden block of width 2m, depth 1.5m and length 4m floats horizontally in water. Find the volume of water displaced and position of center of buoyancy.

The s. gravity of wooden block is 0.7

[Ans; 8.4m3 , ,0.525 m from the base]

wooden block floating in water.

volume of block= 4 x2x1.5 =12 m3

density of wood = .7 x1000 = 700 kg/m3

weight of block =700 x 9.81x 12 =

for equilibrium

weight of wooden block =weight of water displaced by wooden block

700x9.81x12 = volume of water displaced x 1000x 9.81

∴ volume of water displaced =$\frac{700x9.81x1}{1000x9.81}$ = .7x12 =8.4 m3

II Position of center of buoyancy

Volume of wooden block in water = volume of water displaced

= 8.4m3

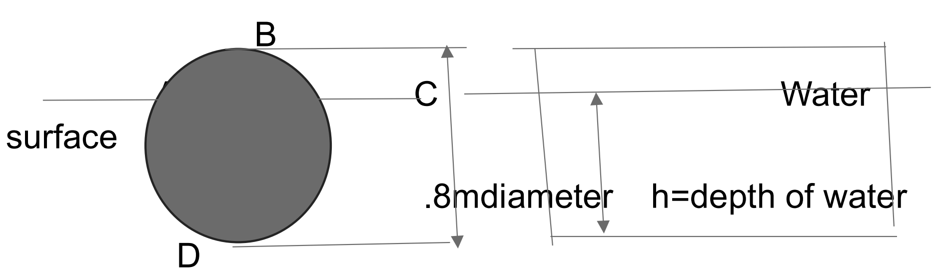
,h x 4 x2 = 8.4 where h m is submerged depth of wood block.

, h = 1.05 m

Center of buoyancy = h/2=1.05/2 =.525m

Q2

A wooden log .8m diameter and 6m long is floating in river water . Find depth of wooden log in water when s gr. Of wood is .7 [Ans; .54m]



Side view 6m long w .block

Diameter of log =.8 m

Length =6m

s. gr of log =.7

ρ = .7x1000 =7ookg/m3

=weight of water displaced of wooden log

let depth of log in water =h

Weight of log =weight density of log x volume

= 700 x9.81 x volume of log

Volume of log =π/4 x $.8^2 x6$

Weight of wooden log=weight density x vol .of log

= 700 x9.81x π/4 x$(.8)^2 x6$

= 2111.15 x 9.81

Weight of wooden log =weight of water displaced.

= weight density of water x vol . of water displaced

Vol. of water displaced =$\frac{weigh \quad 0f \; wooden \; log}{weigh \quad density \; of \; water}$

=$\frac{2111.15x9.81}{1000x9.81}$=2.111m3

Volume of log under water =area ADCA x length x L

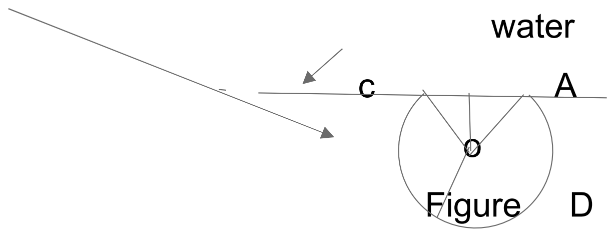
Type equation here.

=ADCA x 6 m3

Vol, of log inside water=volume of water displaced.

=2.111m3 =ADCA X 6

So ADCA = 2.111/6 = .3518 m2



Wooden log

from above figure r is radius ,AO=OC=r

Angle AOE=θ =EOC

So OE =r cos θ , so depth h=r + r cos θ

We know ADCA =.3518m2

From that we can calculate the value θ =71.5°

Diameter is .8m so r =.4m

, h = depth of water = ,4 +.4 cos 71.5

= .4+.4x.3173

= .54m

Q3

A stone weighs 490.5 N in air 196.2 in water. Determine the volume the volume of stone and its specific gravity. [.03m3 or 3 x$10^4 cm3$, 1.67]

Solution

Given

Weight of stone in air =490.5 N

Weight of stone in water =196.2 N

For equilibrium

W t in air –w t in water =w t of water displaced by stone

= 490.5 –196.2 =294.3 N

294.3N = Volume of water displaced x 1000 x9.81

1 . volume of water displaced = $\frac{294.3}{1000x\ \ .81}$=3 x 10^4cm3

II Specific gravity of stone

Mass of stone = $\frac{weigh\ \ in\ air}{g}$ =$\frac{490.5}{9.81}$ =50kg

Mass in air = density x volume

Density of stone =mass in air / volume

=50/.03m3 =1666.6 kg/m3

Specific gravity of stone=$\frac{density\ of\ stone}{density\ of\ water}$

= 1666.6/1000

=1.66

Q 4

A body of dimension 2.0 x1.0 x 3.0 weighs 3924N in Water. Find its weight in air. What will be its specific gravity. [62784 N 1.0667]

Solution

Given

Volume of body =2 x 1x 3 = 6m3

Weight of body in water =3924 N

Volume of water displaced =volume of body =6m3

Weight of water displaced by body =1000 x 9.81 x 6

= 58860 N

For equilibrium

W t of body in air --- w t of water displaced =W t in water

w.t of body in air =w t in water + w t of water displaced

= 3924 +58860 = 62784 N

II mass of body = w t in air /g = 62784/9.81=6400 kg.

Volume of body =6m3

Density of body =mass in air/volume of body

= 6400/6m3

= 1066.6 kg

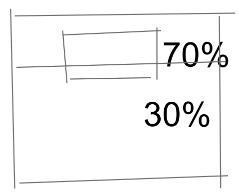
Specific gravity $=\frac{density\ of\ body}{density\ of\ water}$=1066.6/1000

= 1.066

Q 5

A metallic body floats at the interface of mercury of sp gr. 13.6 and water in such a way 30% of its volume is submerged in mercury and 70%in water. Find the density of metallic body.

[Ans 4780 kg/m3]



30 % submerged i mercury ad 70% iwater

Let volume of the body =Vm3

Volume of submerged in mercury =.3Vm3

Volume submerged in water =.7Vm3

For equilibrium

Total buoyant force (upward force) =weight of body

But here total buoyant force=buoyant force for water (a)+buoyant force for mercury. (b)

. a. force = 1000x9.81 x .7v

.b . force =1000 x13.6x 9.81 x .3v

(a + b) = total weight of body = ρ x g x v

1000x9.81x .7v + 13.6 x 1000x 9.81x .3v= ρ x 9.81 x v

700 + 1360 x 3 = ρ =4780 kg/m3

Q6

A body of dimension 0 .5 m x0.5m x 1.0 m and sp .gr 3.0 is immersed in water. Determine the least force required to lift the body. [Ans ;4905N]

Solution

Given

Volume of body =.5 x.5 x1 =.25m3

Weight of body =

Weight of water displaced =ρ g volume

= 1000 x 9.81 x .25

= 2452.5 N

Specific gravity = $\frac{density\ of\ body}{density\ of\ water}$

3 =density of body/1000

Density of body =3000 kg/m3

Density of body =$\frac{mass\ in\ air}{volume}$

Mass in air = 3000 x .25 =750 kg

Weight of body in air =750 x9.81 =7357.5 N

W t body in air ---w t of water displaced =w t in water

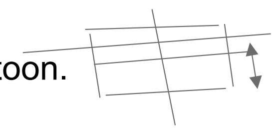
7357.5 --- 2452.5 = 4905 N =W t of body in water.

So to lift the body from water 4905 N minimum Force, is required.

Q 7

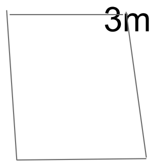
A rectangular pontoon 4m long 3m wide and 1.4m high. The depth of immersion of the pontoon is 1m in seawater. If the center of gravity is .7m above bottom of pontoon ,determine the metacentric height . Take density of seawater as 1030 kg/m3

[Ans;.45m]



Water level 1m from bottom of pontoon.

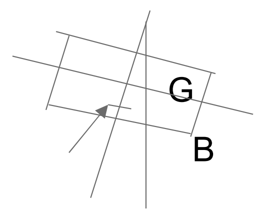
.7m =C.G, 1.4m =high



3m =wide

4m long

pontoon



Volume of pontoon = 4 x 3 x1.4 =16.8m3

Depth of pontoon = 1 m from bottom of pontoon.

AG = C.G of pontoon =.7m

AB=center of buoyancy=1m/2=.5m

Density of seawater =1030kg/m3

Metacentric height of GM= I (moment of Inertia)—BG

∀=volume of the submerged part of pontoon

=1m x3m x4m= 12m3

I =M.O.I=1/12 bd^3= 1/12 x 4 x 3^3=9m4

GM= $\frac{I}{\forall} - BG$ =$\frac{9}{12} - (AG - AB)$=.75—(.7--.5)

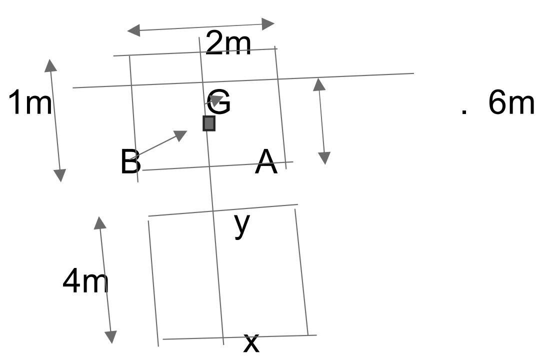
= .75---.2 =.55 m= Metacentric height.

Q 8

An uniform body of size 4m long ,2mwide ,1m deep floats in water. What is the weight of the body if depth of immersion is .6m? Determine the metacentric height also.[Ans47088N,0.355m]

Solution

Given



Volume of body= 4x2x1=8m3

Volume of immersed part= 4x2x.6=4.8m3

I W t body =4.8xρxg=4.8x1000x9.81=47088 N

II Metacentric height =$\frac{I}{\forall}$ ---BG

I=1/12 b d^3= 1/12 x 4 x2^3 =8/3 m3

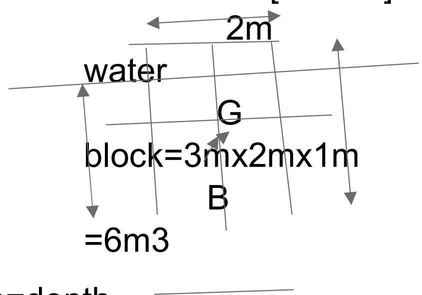
M.O.I =$\frac{\frac{8}{3}}{4.8} - (AG - AB)$= .555—(.5--.3)

=.555--.2 = .355 m

Q 9

A block of wood of specific gravity .8 floats in water .

Determine the metacentric height of the block size 3mx2mx1m.[.316m]

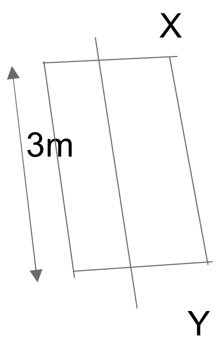


1m vol of block=3mx2mx1m

=6m3

,h=depth

A



Wooden block views

Specific gravity= $\frac{density\ of\ body}{density\ of\ water}$

$.8 = \frac{density\ of\ wooden\ block}{1000}$

Density of body=800 kg/m3

Mass in air =density of body x volume

=800x 6=4800 kg

I=Weight of body in air =4800 x9.81

= 47088N

Volume of water displaced=2mx3mxhm=6h m3

Weight of water displaced = 6hx1000 x 9.81=6000hx9.81

=47088 N

,h $=\frac{47088}{6000x\ .81}$=.8m

∀ =volume of water displaced = 6x .8=4.8 m3

II Metacentric height

BG=AG—AB= 1/1/2 ---h/2= .5--..4=.1m

Metacentric height = $\frac{I}{\forall}$ -- BG

I = 1/12 x b xd^3 =1/12 x 3 x2^3 = 2m4

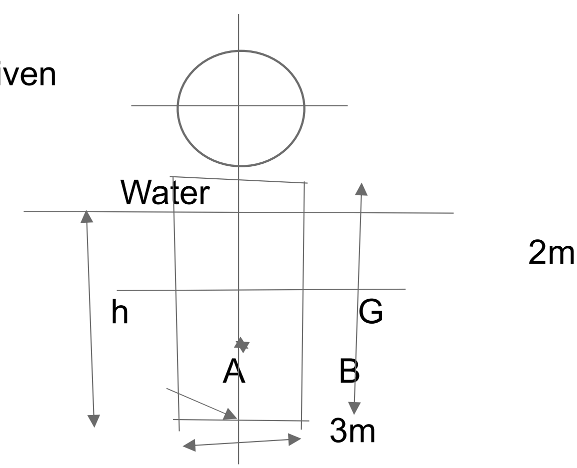
M. H = $\frac{2}{4.8}$ --- .1m=.416--.1m=.316 m

Q 10

A solid cylinder of diameter 3m. and height 2m . Find the metacentric height of cylinder when it is floating in water with its axis vertical. The specific gravity of cylinder is .7. [0.1017 m]

Solution

Given



, h= depth of water from bottom of cylinder.

Volume of cylinder under immersion=$\frac{\pi d^2}{4}$xh

=$\frac{\pi 3^2}{4}$xh=9π/4 x h

Volume of cylinder =(9π/4) x 2 =$\frac{9\pi}{2}$

Volume of water displaced =$\frac{9\pi}{4}$ h m3

I Weight of water displaced =weight in air

= $\frac{9\pi}{4}$x hx 1000x 9.81

= 69342.8 h

s.g of cylinder =.7= $\frac{density\ of\ body}{density\ of\ water}$

density of body= .7x 1000= 700 kg /m3

mass of cylinder in air = density of cylinder x vol. of c y l.

=700x 9π/2 = 9896.016 kg

II Weight of cylinder in air =mass of cylinder x g

= 9896.016 x 9.81

= 97079.9N

Now , I = II above'

69342.8 h= 97079.9 N

, h = 97079.9/69342.8 = 1.3999=1.4 m

Metacentric height

Moment of inertia I =π/64 X d^4= π/64 x 3^4

$= 3.976m4$

$\forall$ =volume of cylinder in water $=\frac{\pi}{4} 3^2 xh$

= 9x π/4 x 1.4 = 9.9m3

BG= AG—AB = 1--.7= .3

Metacentric height = 3.976/9.9 ---- .3

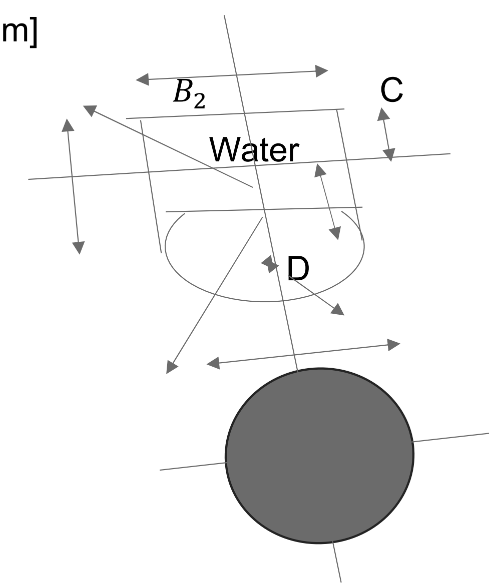
= .4016---.3

= .1016 m

Q 11

A body has a cylindrical upper portion of 4 m diameter and 2m deep, the lower potion is curved one which displaces a volume of .9m3 of water. The center of buoyancy of curved portion is at a distance of 2.10m below the top of cylinder. C.G of whole body 1.5 m below top of cylinder. The total displacement of water 4.5tonnes Find the metacentric height of body

[Ans ;2.387m]



Diameter of body= 4m

Depth of body =2m

Volume of water displaced by curved portion=.9m3

B_1 is center of buoyancy of curved portion.

B_2 is the center of buoyancy of rectangular portion.

G is center of gravity of whole body.

C is top portion of cylinder.

CG=1.5m, C B_1 =2.1m

Total displacement of water= 4.5tonne=4.5x 1000 kgf

= 4500 x 9.81 N =44145N

Metacentric height of body

Let x m be the distance between surface water and top of body. = x m.

Total weight of water displaced by body = weight density of water x volume of water displaced

= 1000x 9.81 x volume of body in water.

= 9810x [π/4 xd^2x depth of cylindrical part I water+ vol. of water displaced by curved portion].

44145N = 9810[π/4 X 4^2x (2--- x) + .9]

4.5m3 = π/4 x 4^2x (2—x) + .9

$$4x\pi/4(2-x) == 4.5--.9 = 3.6$$

2---x = $\frac{3.6x4}{\pi x4x4}$ =.2866

X =2.0--.2866 =1.71m

So depth of cylindrical portion in water

$= 2 — 1.71 = 0.29$

$\therefore CB_2 = X + .29/2 = 1.71 + .145 = 1.855$

$CB_1 = \frac{.9X2.1+3.6X1,855}{.9+3.6} = \frac{8.568}{4.5} = 1.904$

-

$B_1G = CB_1 — CG = 1.904 --- 1.5 = .404$

$M.O.I = \frac{\pi 4^4}{64}$

$\forall = 4.5m3$

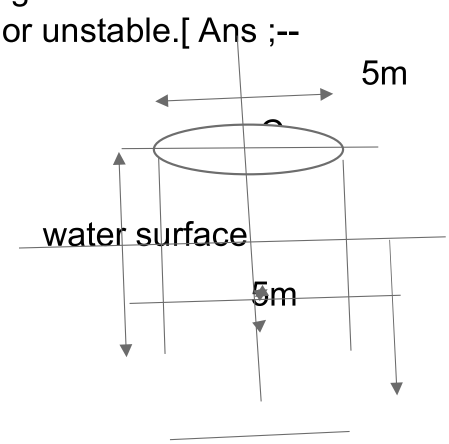
$G M. = \left(\frac{\pi}{64} X \frac{4^4}{4.5} \right) --- .404$

$= 2.787 --- .404$

$= 2.307$ m

Q 12

A solid cylinder of diameter 5m has a height of 5 m. Find the metacentric height of the cylinder if the specific gravity of material of cylinder .7 and it is flowing in water with axis vertical .State whether equilibrium is stable or unstable.[Ans ;-- 0.304,m,unstable]



D

Diameter of cylinder=5m

G hm

B

A

Height of cylinder=5m,   G =Center of gravity

Sp gr = .7   B =Center of Buoyancy.

AG= 5m/2=2.5m

AB=h/2 m , Immersed portion of cylinder =.7 x5m

=3.5m

So   ,h =3.5/2= 1.75m

BG= AG—AB=2.5—1.75=.75

Metacentric height=GM=$\frac{I}{\forall} - BG$

Volume of body portion in water

= $\frac{\pi}{4}5^2$x3.5   =$\frac{\pi x\ 25x3.5}{4}$

M.O.I =$\frac{\pi}{64}5^4$=$\frac{\pi x625}{64}$

GM =$\frac{\pi X625}{64}$X$\frac{4}{\pi X25X\ \ .5}$ --- .75

= .4464-- .75

= --- .3036 m

Since GM is negative so system is unstable.

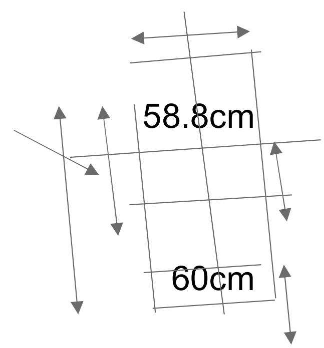
Type equation here.

Q13

A solid cylinder of 15 cm diameter and 60 cm long, consists of two parts made up of different material. The first part at the base

Is 1.20cm long and s p g r 5.The other part of cylinder is made of material having s p g r .6. State if it can float vertically. [Ans –5.26=GM, Unstable equilibrium.]

15cm



h_2

h1

Length of cylinder = 60cm=.6m

Base material = 1.2cm

Will be floating .6x5(s p g r)=3m below the free surface

Of water. = h1

The other part of body =.6x.6=.36 m=h2

So total =h1+h2=3m +.36m=3.36m

So buoyancy = 3.36/2 = 1.68m below

Surface

water.

And C.G. = h1/2= .6/2=.3m

Diameter of cylinder= 15cm=.15m

Length =60cm=.6m

Length of bottom part=1.2cm=.012 m

S p g r of bottom part=5

So density =5x1000=5000 kg/m3 =ρ1

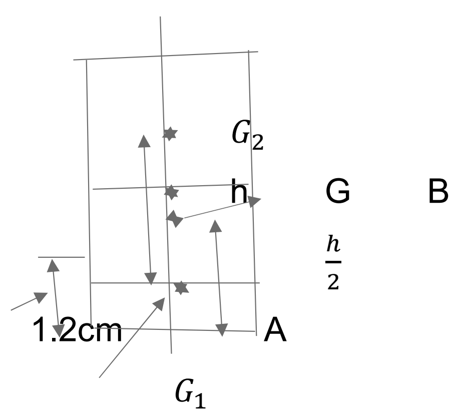
Length of 2^{nd} part=60—1.2=58.8cm=.588m

Density ρ2=.6(s p g r)x1000=600 kg/m3

The cylinder will float vertically if metacentric height +ve

to find metacentric height;

we have to have CG and center of Buoyancy of combined body;-



The distance of C G of solid cylinder from A is give as;-

Type equation here.

AG =W T of 1st part x CG of 1st part from A +WT of 2nd part x CG of 2nd from A

W t of 1st part +w t of 2nd part

$$= \frac{\frac{\pi}{4}xd^2x1.2x5x1.2/2\}+\{\frac{\pi}{4}xd^2x58.8x.6)x(1.2+\frac{58.8}{2})\}}{\frac{\pi}{4}d^2x1.2x5+\frac{\pi}{4}d^2x(58.8x.6)}$$

$$= \frac{\{(1.2x5x.6)+(58.8x.6)\}x30.6}{1.2x5+5\ \ .8x.6}$$

$=\frac{6x.6+1079.6}{6+34.8}$ $=\frac{1083.2}{40.8}$ =26.55m =AG

To center of buoyancy of combined body we to find depth immersion of cylinder , let it be h

Weight of cylinder = weight of water displaced by cylinder.

π/4 x(.15)^2 x58.8/100 x600x 9.81+π/4 (,15)$^2x\ 5000x1.2/100x9.81$ $=\frac{\pi}{4}$ x(.15)^2 x h/100x1000x 9.81

,h $=\frac{58.8x.6+50x1.2}{10}$ =41.28 cm

$B\ is\ the\ buoyancy\ point.$

AB= h/2= 41.28/2 = 20.64 cm

BG=AG---AB=26.55--- 20.64=5.91cm

I = M.O.I = π/4 x 15^4

∀ = volume = π/4 x 15^2x h=π/4 x 225 x 41.28

=7294.77cm3

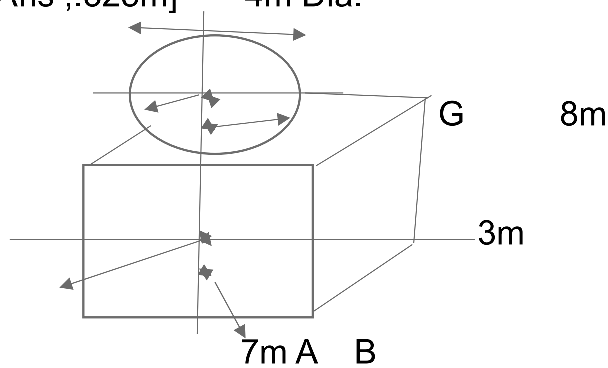
GM= $\frac{I}{\forall}$ -- BG = $\frac{\pi/64}{\pi/4}$ X $\frac{15^4}{15^2X41.28} - 5.91$ = .340--- 5.91

= --- 5.570m

Since GM is negative figure hence unstable equilibrium.

Q 14

A rectangular pontoon 8m long x 7m broad x 3m deep weighs 588.6 k N. It carries I its upper deck a empty boiler of 4m diameter weighs 392.4 k N. The center gravity of the boiler and the pontoon are at the respective center along the vertical line . Find the meta center height .Weight density of sea water is 10.104N/m3.[Ans ;.325m]   4m Dia.



Let G and B are common C G and buoyancy of whole.

G1 andG2 are C G for pontoon and boiler.

AG2=3+4/2=5m, AG1=3/2=1.5m

W1=pontoon w t=588.6k N, W2 =Boiler w t=392.4kN

Common center of gravity AG

$$= \frac{W1XAG1+W2XAG}{W1+W2} = \frac{588.6X1.5+392.4X5}{588.6+392.4} = 2.9\text{m}$$

Let h be depth of immersion of pontoon in water.

WT of pontoon with boiler =weight of seawater dis placed

981 = 7 x 8x h x 10.104 (w t density of seawater)

,h = 1.733m

AB=distance of common center of buoyancy=h/2

= 1.733/2=.866m

∴ BG=AG--- AB= 2.9--- .866=2.032m

Meta centric height GM

M.O.I =1/12 x bd^3 =1/12 x 8 x 7^3=228.66 m4

∀ = 8x7x h=56x 1.733=97.048 m3

GM=$\frac{228.6}{97.08}$ --- 2.034 =.322 m

Q 15

A wooden cylinder of s p g r .6 and circular in cross section is required to float in oil of s p gr .8 Find the L/D ratio of the cylinder to float with its longitudinal axis vertical I oil where L is height of cylinder and D its diameter [Ans L/D<0'8164].

sp.gr of cylinder =.6

L =Height of cylinder

D=Diameter of cylinder.

S p gr of oil = S2=.8

Let h be the depth of immersion body cylinder.

Weight of cylinder = weight of displaced.

$\frac{\pi}{4} x D^2 x L x$ .6 $x 1000$ $x 9.81$= h x$\frac{\pi}{4}$xD^4x .8 x 1000 x9.81

. h= $\frac{3L}{4}$

Distance of CG from A , AG=$\frac{L}{2}$

Distance of center of buoyancy from A

$$AB = \frac{h}{2} = \frac{3L}{8}$$

$$BG = AG — AB = \frac{L}{2} - \frac{3L}{8} = L/8$$

For equilibrium or stable GM> 0

M.O.I= $\frac{\pi}{64}D^4$

∀ =Vol. of cylinder in oil=$\frac{\pi}{4}D^2xh$

$$= \frac{\pi}{4}xD^2\frac{3L}{4}$$

$$= \frac{3\pi}{16}D^2xL$$

$$I/\forall = \frac{\frac{\pi}{64}}{\frac{\pi}{16}}\frac{D^4}{D^2L} = \frac{D^2}{12}$$

$$\therefore \quad GM = \frac{I}{\forall} \text{ ---BG } = \frac{D^2}{12L} \text{---} L/8$$

For equilibrium GM> 0

Or, $\frac{D^2}{12L}$ ---- L/8>O

Or, $\frac{D^2}{12}$ > L/8

Or, $\frac{D^2}{L^2}$>12/8> 3/2

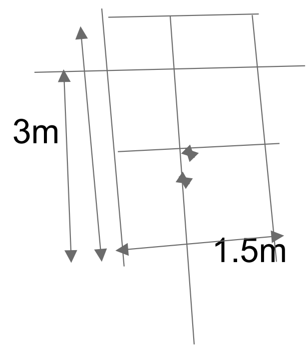
Or, $\frac{L^2}{D^2}$ < 8/12

Or ,$\frac{L}{D} < \sqrt{\frac{8}{12}}$ $<\frac{2.828}{3.464}$ = .81639

Q 16

Show that a cylindrical buoy of 1.5 diameter and 3m long weighing 2.5 tones will not float vertically in sea water of density 1o30 kg/m3. Find force necessary I a vertical chain attached at the center of the base of the buoy that will keep it vertical.

[Ans;10609.5N]



Diameter of cylinder =1.5mρ

Length=3m

Weight=2.5 tones = 2.5 x 1000 9.81 N

,ρ = Density of sea water=1030kg/m3

I Show that cylinder will not float vertically.

II Force required to keep the float vertically

I If GM is negative float will not vertical.

So we have to find out GM.

W t of cylinder = π/4 $(1.5)^2$x 3 x 9.81 x 1030 x h

= 2.5 ton x 1000 x 9.81

, h = .4578 m=from water to the bottom of cylinder

Center of buoyancy = h/2 =.4578/2=.2289m =AB

AG = CG Point from A= length of cylinder /2

=3m/2 = 1.5m

BG = AG--- AB= 1.5 -- .2289 = 1.2711m

Metacentric height =GM, we require

M.O.I =π/64 x $(D)^4$=π/64 X $(1.5)^4$=

∀ = π/4X1.5^2 X h m3

$$I/\forall = \frac{\frac{\pi}{64}X1.5^4}{\frac{\pi}{4}1.5^2X.4578} = .307$$

GM= $\frac{I}{\forall}$ --BG=   .307 --1.271m =---.9641m

Since is not positive so cylinder cannot stay vertical.

II Let h! is depth of immersion when force T is applied to keep buoy vertical.

Total downward force = w t of water displaced.

2.5x 1000 x 9.81 +T =Density of water x vol of c y l in water. x g

= 1030 x9.81 x π/4 x $(1.5)^2$h!

$$, h! = \frac{25 \quad 9.81+T}{1030X9.81 \quad \frac{\pi}{4}X2.25} = \frac{24525+T}{17885.77}$$

$$AB! = h!/2 = \frac{24525+T}{35711.54}$$

Combined CG, G! due to w t of cylinder and due Force T from A is AG! =(w t of cylinder x distance of CG of cylinder from A+TX distance of CG of T from A)/ (W.t of cylinder +T)

$$= \frac{\frac{24525X3}{2}+Tx0}{24525} = \frac{36787.5}{24525} = AG!$$

B!G! =AG! --AB!

$$= [\frac{36787.5}{24525+T} - \frac{24525}{35711.54}]$$

Type equation here.

Metacentric height

I $\frac{\pi}{64} X 1.5^4$=.2484

∀ = $\frac{\pi}{4} X 1.5^2$X h! =1.767 x($\frac{24525}{17855.77}$)

GM= $\frac{.2484}{1.767\ X(\frac{24525+T}{17855.77}}$ --- [$\frac{36787.5}{24525+T} - \frac{24525}{37711.4}$]

For stable equilibrium GM should be + ve

So, 2510/(24525 + T) --[36787.5/(24525+T)

--(24525 +T)/37711.4] > 0

OR, $\frac{-36787.5+251}{24525+T} + \frac{24525+T}{37711.4} > 0$

$(24525+T)^2$ > 34277.4 X37711.4

(24525+T) > √ 1292648742.36

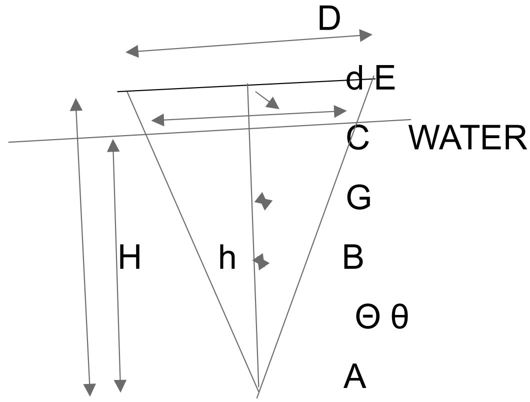
T > 35953.36 –24525

T > 11428.42 N

So force T must be at least 11428.42 N .

Q 17

A solid cone floats in water, its apex downwards. Determine least apex angle of cone for stable equilibrium. Sp gr of cone in water is .7.[Ans 39°7mi]



D= Dia of cone.

, d=dia. Of cone at water level ,ρ = s p gr x 1000 =.7 x1ooo

= 700 kg/m3

2θ = apex angle of cone.

AE = H = Height of cone

AC = h =height of cone up to water level.

B = Center of buoyancy

G= C.G.

AG =3/4 H from A

AB= ¾ h from apex A

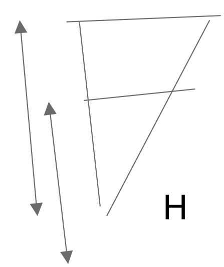
Volume of water displaced up to water

= 1/3 xπ$r^2 h$

Volume of cone = 1/3 π R^2 H∄θ xR^2 x H

F

R E



r

H h

θ

A

From ∆ AEF

Tan θ = R/H

R = H Tan θ

r= h tan θ

weight of cone = 700 g x1/3 π $R^2Tan\theta^2H$

=700 g 1/3 π $H^2Tan\theta^2H$

= 700 g 1/3 π $H^3Tan\theta^2$

(I)

weight of water displaced = 1000 g x1/3π $h^2tan\theta^2$ (II)

for equilibrium

I) weight of cone =(II)weight of displaced water.

We get finally

$\frac{H}{h} = (\frac{1000}{700})^{\frac{1}{3}}$ ------------(III)

For equilibrium GM be + ve

MOI=π/64 X $d^4$and ∀ =1/3 x π/4 x d^2xh

$\frac{I}{\forall}$ = 3/16 X $d^2x\frac{1}{h}$ = ¾ x $\frac{r^2}{h}$ =3/4 x $\frac{(htan\theta)^2}{h}$

=$\frac{3h}{4}xtan\theta^2$ (IV)

BG =AG –AB

=3/4 H---3/4 h

= ¾(H—h) --------(v)

So GM = ¾ h tan$\theta^2 - \frac{3}{4}(H-h)$ ------(-VI)

For equilibrium

GM > 0

From VI we have

, h $\tan\theta^2$> (H—h)

Or, h $\tan\theta^2 + h$ > H

Or , h(1+$\tan\theta^2$)>H

Or sec $\theta^2 > \frac{H}{h}$

[But (III) $\frac{H}{h}$ =1.126]

So sec θ^2 >1.12 .6

Or cos θ^2> 1/1.126=.888

Cos θ > √.888= .9423

θ = 19°57min

so 2θ = 39°14mins. Is the least apex angle

Q 18

A ship 60m long and 12 m broad has a displacement of 19620k N. A weight of 294.3 k N is moved across the deck

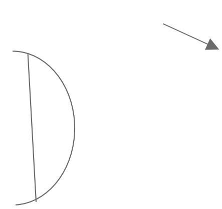
Through a distance of 6.5 m. The ship tilted through 5°. The moment of inertia of ship in water line about its force and aft axis is 75° of moment of inertia the circumscribing rectangle .

The center of buoyancy is 2.75m below water line. Find metacentric height and position of center of gravity of ship

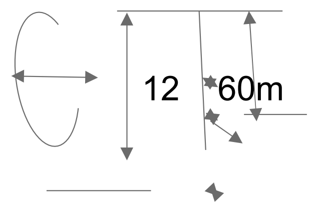
The specific weight of sea water is 10104 N/m3.

[1.1145 m, .53m below water surface]

Water lie



C



. G 2.75m to buoyancy

A B

point.

L =Length of ship=60m

,b=breadth of ship=12m

Displacement by ship=19600 k N

W1= Weight moved in deck= 294.3 k N

=294.3X 1000 N

=294300N

Distance moved across by W1 =6.5m =x

Θ = Tilt angle =5°

M.O.I of ship at water line about its fore aft axis

= 75% of M.O.I of circumscribing rectangle.

S p g r of sea water =10.104 k N/m3

To find ;

A--Metacentric height

B --Center of buoyancy

A GM $=\frac{W1X}{W.Tan\theta}= \frac{294.3x6.5}{19600xtan5°} =\frac{1912.95}{1715}$

= 1.115m

B Position of center of gravity

M.O.I of ship at water surface about =75% of M.O.I of circumscribing rectangle=1/12 x bxd^3x .75=1/12 x 60 x12^3x .75

= 6480m4

Volume ∀ ship in water= $\frac{weight\ of\ ship}{weigh\ \ density\ of\ sea\ water}$

=$\frac{displacement\ by\ ship}{weigh\ \ density\ of\ water}$

= $\frac{19620k\ N}{10.104kN}$ =1941.8m3

∴ $\frac{I}{\forall}$ = 6480/1941.8 = 3.337m

GM=$\frac{I}{\forall}$ ---BG

1.115=3.337--- BG

BG= 2.222m

CG = CB---GB = 2.75—2.222 = .528 m

Q 19

A pontoon of 1500 tones of displacement is floating in water. A weight of 20 tones is moved through a distance of 6m across the deck of pontoon when tilt of pontoon is 5°. Find the metacentric height of ship. [Ans; .9145m]

Solution

Given

W = Weight of displaced water

= 1500tons x 1000 kg

W1= weight moved on deck of potoo

=20 ton x1000 kg

Distance moved by w1 =6 m =x m

Tilt angle =5° =θ

Metacentric height $=\frac{w1\ x}{WTan\theta} \ = \frac{20x1000x6}{1500x1000 \quad tan5}$

$= \frac{20x6}{1500x\ tan5} = \frac{120}{1500x\ .0875}$

=120/131.25

=0.914 m

END OF CHAPTER FOUR

Fluid Mechanics

CHAPTER 5

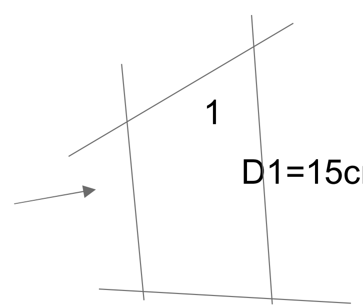
Kinematics of Flow and Ideal Flow

Q 1

The diameter of pipe at section 1 and 2 are 15 cm and 20 cm respectively. Find the discharge through the pipe if velocity of water at section 1 is 4 m/s .

Determine the velocity of section 2. [Ans ;0.07068m3/s, 2.25m/s]

Solution 2



1

D1=15cm D2 =20cm

1

2

V1 =4m/s

Pipe section 1 / 1 diameter 15 cm

2/2 diameter 20 cm

V1 =4m/s

A1= Area of section 1/1 = π/4 $(.15cm)^2$

=0.1767m2

A2=Area of section 2/2=π/4$(20cm)^2$=.031416m2

Discharge rate at section =Q1=A1XV1= .01767X4

= .07068m3/s

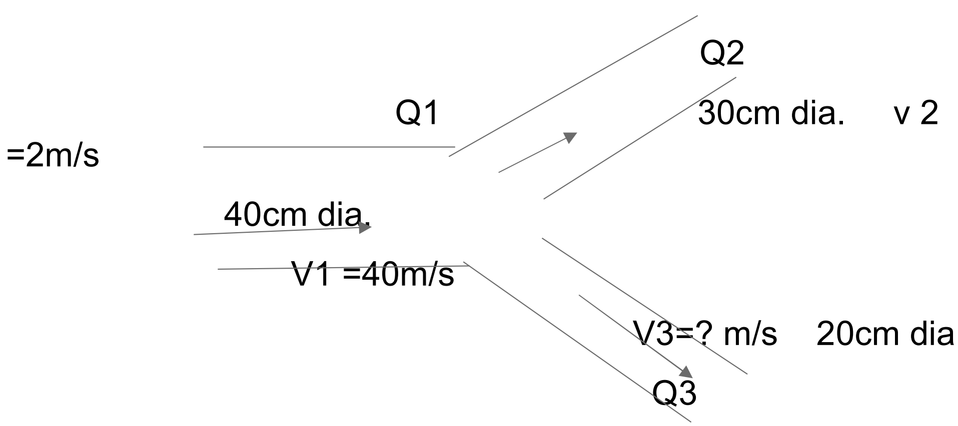
Since pipe is one continuity of discharge rate will be applied

Q = A1XV1 =A2XV2

$$V2 = \frac{A1XV1}{A2} = \frac{.0767}{.031416} = 2.25 \text{ m/s}$$

Q 2

A 40 cm diameter pipe conveying water branch into two pipes of diameter 30 cm and 20 cm respectively. If the average velocity in 40 cm dia. pipe is 3 m/s. Find the discharge in the pipe Also determine the velocity in 20cm pipe if the average velocity in 30cm diameter pipe is 2m/s [Ans;.3769m3/s,7.5 m/s]



$A1=\pi/4(.4m)^2$ =.12566m2

V1=40m/s

V2=2m/s $A2=\pi/4$ X$(.3m)^2$ =.0706m2

$A3=\pi/4$ $(.2m)^2$ = .031416m2   V3= ?

Taking above figures after calculation ,

Rate of discharge Q1=A1 XV1= .3769m3

Q2=A2 XV2 =.1412 m3

From the pipes configuration made above

Q1=Q2+Q3

∴ Q3=Q1—Q2

$V3=\frac{Q3}{A3}=\frac{.2357}{.031416}$ =7.5 m/s

Q3

A 30cm diameter pipe carries oil s p gr .8 at velocity 2m/s .At another section diameter 20cm. Find velocity at this section and mass rate of flow [Ans; 4.5m/s,113kg/s]

Solution

Given

Section 1 ,   D1=30cm=.3m

$A1=\pi/4$ X $(.3)^2$=.0706m2

V1=2m/s

Section 2

D2= .2m

So, A2 = .0314 m2

V2 = ?

Applying continuity of equation

Q =A1V1= A2V2

V2 $= \frac{A1V1}{A2} = \frac{.0706X2}{.0314}$

Mass rate of flow =mass density x Q

=ρ A1 V1

S P Gr of oil $= \frac{density\ of\ oil}{density\ of\ water}$ =

So, density of oil = .8 x 1000 =800 kg/m3

Mass rate of flow = density of oil x Q

=800 X .0706 X 2

= 112.96 Kg/s

Q 4

The velocity vector in fluid flow is given by V =$2x^2 i - 5x^2 yj$ +4tk. Find the velocity and acceleration of fluid particles (1,2,3)at time t =1.[Ans ;10.95,units ,16.12 units]

Solution

Given

V= $2x^3 i - 5x^2 yj + 4tk$

Velocity components u, v, w are

U=$2x^3, v = -5^2 y$,w=4t

For the point/particle (1,,2,3) x1,y=2, z=3

So velocity components are

U=2 $x1^2 = 2, v = -5x1^2x2 \quad w = 4x1 = 4$

U=2,units. v=10units, w= 4unit

So velocity vector V at (1,2,3)=2i—10j +4k.

Resultant velocity =$\sqrt{2^2 + 10^2 + 4^2}$ =√120

= 10.95 units.

$$a_x = u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} + \frac{du}{dt}$$

$$a_y = u\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz} + \frac{dv}{dt}$$

$$a_z = u\frac{dw}{dx} + v\frac{dw}{dy} + w\frac{dw}{dz} + \frac{dw}{dt}$$

U=2x^3 , du/dx= 6x^2 , du/d z=0, du/dt =0

V=--5x^2y , dv /dx=--10xy ,dv/d y =--5x^2 , dv/d z=0

W=4t, d w/dx =0, d w/d y=0. D w/dt=4t

Putting the values of differentiation --

, a_x = 2$x^3.(6x^2) - 5x^2y(0)$ +4t(0)=12$x^5(1)^5$ =12units

, a_y = 2$x^3.(-10xy) + (-5x^2y)(-5x^2) + 4t(0)$

= --20.$x^4y + 25x^4y$=5x^4y=5$1^4x2$=10 units

, a_z =2 $x^3(0)$+(--5x^2y)(0)+ (0) +4.1=4 units

A = acceleration =$a_xi + a_yj + a_wk$

= 12i+10j+4k

A = $\sqrt{12^2 + 10^2 + 4^2}$ =√260 = 16.10 units

Q 5

The following cases represents two velocity components determine the third component such that they satisfy the continuity equations;

I u=$4x^{2,}$ v =4xyz

$$II \quad U = 4\,X^2 + 3XY,, W = \; Z^3 - 4XY - 2YZ$$

Calculate the unknown velocity components, so that they satisfy the following equation;

I u =$2x^2$ v=2xyz, w=?

II u =$2x^2$+2xy, w=z^3—4xz +2yz

Solution

Continuity equation for incompressible fluid is given by equation (5.4)

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0 \text{ ---------- (1)}$$

Case 1 u =$4x^2$ v=4xyz

$\frac{du}{dx}$ =8x, $\frac{dv}{dy}$ = 4xz

Putting above value in (i)

8x + 4xz + $\frac{dw}{dz}$ =0

$\int dw$ =$\int(-8x - 4xz$)dz

W = ---8xz---4x $z^2$1/2 + c

=---8xz—2x z^2 + f (xy) $**$

$**$constant of integration will not be a function of z but can be a function x and y.

Case II Continuity equation (5.4)

$$\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}+\frac{\delta w}{\delta z}=0$$ ------- (II)

w = z^3—4xy—2yz

on differentiation

$\frac{dw}{dz}$ = 3z^2—0 –2y

U = 4$x^2 + 3xy$

$\frac{du}{dx}$ = 8x +3y

Putting the diff. value in (II)

8x +3y +3$z^2 - 2y$ +$\frac{dv}{dy}$ =0

$\int dv$ =$\int(-8x - y - 3z^2)$ dy

V = $\int -8xdy - \int ydy - 3\int z^2\, dy$

=--8xy --$\frac{y^2}{2}$ -- 3z^2y + constant f (x z)

2nd part of question 5

I U = 2x^2 v =2xyz. W =?

II U= 2x^2, w = z^3—4yz + x z v =?

Differentiation I and II

I $\frac{\mathbf{du}}{\mathbf{dy}} = \mathbf{4x}$ $\frac{\mathbf{dv}}{\mathbf{dy}}$ **=2xz**

So continuity equation becomes

4x + 2yz +d w/d z = 0

$\int dw$ = $\int -4x\, d\, z$ ---$\int 2xz\, dz$

W =--4xz--- x z^2

II U=2$x^2 + 2xy$

$\frac{dv}{dy}$= --$\frac{du}{dx} - \frac{dz}{dw}$

= --4x –2y --3$z^2 + 4y - x$

= --5x +2y –3 z^2

, $\int$(dv) =$\int$(-- 5x +2y--3z^2)d y

V = ---5xy + 2$y^2/2$ --3$z^2 y + f(xz)$

Q 6

A fluid flow is given by V= x$y^2(-2yz^2) - (zy^2 - 2z^3/3)$k

I Prove that i t is a case of possible steady incompressible fluid flow.

11 calculate the velocity and acceleration at the point

1,2,3 (x y z) [Ans 36.7 units.874.5 units]

Solution

I V = $xy^2 i - 2yz^2 j - (zy^2 - \frac{2z^3}{3})$ k

U =xy^2 , v =-- 2y z^2, w= --(z$y^2 - 2z^3/3$)

, du/dx=y^2, dv/d y=---2z^2, d w/d z= --y^2+6$z^2/3$

=--

$y^2 + 2z^2$

$\frac{du}{dx} + \frac{dv}{dy}$+$\frac{dw}{dz}$ should be =0 for continuity.

So $y^2 - 2z^2 - y^2 + 2z^2$ = 0 , proved fluid incompressible.

II Velocity and acceleration

V= x$y^2 i - 2yz^2 j - \left(zy^2 - \frac{2z^3}{3}\right) k$

Velocity at x=1,y=2, z=3

V =1,$2^2 i -\ \ 2.2.3^2 j - (3.2^2 - 2.3^3/3)k$

= 4i --- 36j + 6kb^2

Resultant velocity = $\sqrt{4^2 + (-36)^2 + (6)^2}$=√1348 =36.71

U= x y^2 , du/dx =y^2, du/d y =2xy, du/d z= 0

V=--2yz^2, d v/d x =0, dv/dy=--2z^2,du/dz=--4yz

W =--(z$y^2 - 2z^3/3$) , d w/dx=0 d w/d y=--2yz-0,

Dw/dz=--($y^2 - \frac{6}{3} z^2$)=-- y^2 −2z^2

Putting the values x=1,y=2, z=3 ,

a_x =u. d u/d x+ v du/d y+ w du/d z

= xy^4—4x$y^2 z^2$ -- 0 = 1.2^4 ---4x1.$2^2 3^2$

= 16—144= --128 units

Similar way we get

a_y = 504 units and a_z= 438 units

A= $a_x i + a_y j + a_w$k = (--128)i+ (548)j +(438)k

So acceleration = $\sqrt{(-128)^2 + (548)^2}$+(438)^2

=$\sqrt{508532}$ = 713.1 units ∗

∗ students may check as the last answer is not matching with

original answer from book.

Q 8

The velocity potential function Ø is given by Ø= $x^2 - y^2$

Potential derivative of Ø w.r.t x ad y we get

Ø = $x^2 - y^2$

, dØ/dx =2x ,

,dØ/d y=---2y

Velocity component

U =2x , v =-- 2y

II The given value Ø will represent a possible case of flow if it satisfies Laplace equation $\frac{d^2\emptyset}{d\,x^2} + \frac{d^2\emptyset}{dy^2}$ =0

dØ/dx= 2x , dØ/d y= ---2y

$\frac{d^2\emptyset}{dx^2}$ =d/dx(2x)=2

$\frac{d^2\emptyset}{dy^2}$ =d/d y(--2y) =-- 2

∴ adding both= 2 +(--2) = 2—2 =0

So is possible case of fluid flow.

Q 9

For the velocity potential function Ø=$x^2 - y^2$, find velocity component at the point 4,5. (Ans u=8, v=--10)

Solution

Given

Ø =$x^2 - y^2$

, dØ/dx $(x)^2$ =2x =u

,d Ø/dx $(--y)^2$ = --2y =v

Taking x=4,y 5

U= 2x =2x 4 =8 units.

V= --2y =--2x 5 = --10 units

Q 10

A stream function is given φ = 2x—5y. Calculate components in x and y direct and also magnitude and direction of velocity at ay point. [Ans; u=5, v=2 ,R=5.384, θ=21°48 min]

Solution

Given φ=2x—5y

, d φ/dx= 2 d φ/d y=--5

But as per (5.12)

U =--d φ/d y = ---(--5) =5 units

V =d φ/dx=2 units

R =$\sqrt{u^2 + v^2} = \sqrt{5^2 + 2^2}$ =√29 =5.385uits

Tan θ = $\frac{v}{u}$ =$\frac{2}{5}$ =..4

Θ =21°8min

Q 11

If for a 2 dimension potential flow the velocity is given by

By φ =4x(3y—1).

Determine the velocity at point 2.3) Determine also the value of stream function∵Ø at point (2,3)

[40 units , ∅=6x^2—4($\frac{3}{2}y^2$—44]

Solution

Given

Φ= 4x(3y—1)

U =-- $\frac{d\varphi}{dx}$=---d/dx [4x(3y—1)]

=--12y+4

U =--32 putting value (2,3)

V =--$\frac{d\varphi}{dy}$= --- d/d y [4x(3y—1)]=--12x

=---12x 2= --24.

R =$\sqrt{(-32)^2+(-24)^2}$ = 40 units.

= resultant velocity.

II Value of stream functioning at p (2,3)

$\frac{d\emptyset}{dy}$ =---u = -- (--12y+4) =4—12y ------(a)

d ∅/dx=v = --12x --------(b)

integrating (a)

∅ =$\int(-12y+4)dy$

C is not function of y but may be of x. let the value of

Of c is k

∅ = ---6y^2+4y ±k---------(c)

Diff.(c) w. r. t. x we get

$\frac{d\emptyset}{dx}$ = =-- dk/dx --------(-d)

But in a) $\frac{d\emptyset}{dx}=-u$ =-- 12x

Type equation here.

So --- dk /dx =---12x --------(--e)

Integrating (e)

-- K =--$\frac{--12x^2}{2}$= -- 6 x^2

Putting the value of k in(c)

Ø =---6$y^2 + 4y$ +6x^2

Putting the value of point p(x,y)φ

Ø =--- 6 $(3)^2$+ 4x 3 +6$x2^2$

= ---6x9 +12 +24=--- 54+36 =---18

Q12

The stream function for a 2dimensional flow is given by Ø=8xy. Calculate velocity at the point p (4,5). Find velocity potential φ[Ans u =---32, unit v=40unit φ=4$y^2 - 4x^2$]

Solution

Given

Ø =8xy

Components u and v in terms of Ø

U =-- $\frac{dØ}{dy} = -\frac{d}{dy}(8xy)$ $v = \frac{dØ}{dx}$ =$\frac{d}{dx}$(8xy)

= --8x = 8y

At the point (4,5) we get

U =--8x 4=--32

V =8x5 =40

II Velocity potential of function φ

$\frac{d\varphi}{dx}$ =--u =---(--8x)=8x

$\frac{d\varphi}{dy}$=--v =-- 8y ------- (i)

1. $\int d\varphi = \varphi$ = $\int 8x$ dx =8$x^2/2$ =4x^2+ k -----(-i i)
   K is constant and independent of x but can be a function of y.
   Differentiating (i i) w. r, t y
   We get
   $\frac{d\varphi}{dy}$ =dk/d y

But from (i)

∴ dk/d y =---8y iii)

Integrating iii) we get

K =--- $\frac{8\ y^2}{2}$ =--- 4 y^2

So φ = 4$x^2 + k$

= (4$x^2 - 4y^2$) is the velocity function.

Q 13

Sketch the stream lines represented by ∅ =x y Also find out velocity and its direction at point (2,3). [Ans ;3.6 units ,θ =56°18.6 min , 123° 42 min]

Solution

Sketch ∅ =x y

Velocity of component u and v

U = --- d∅/d y= ---d/d y (x y) =-- y

V = d ∅/ dx = d/dx (x y) = x

At the point (2,3) the velocity components are

U= --y =-- 3 units/s

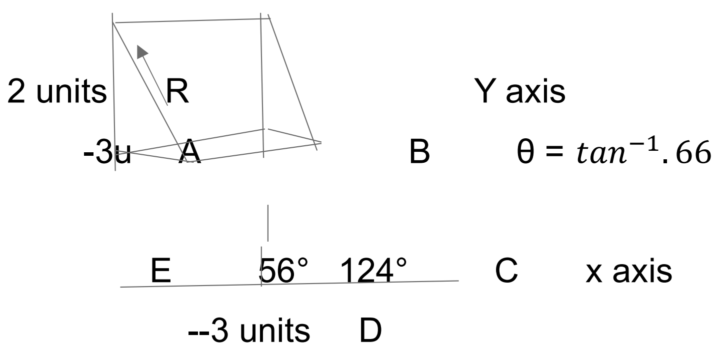
V = x = 2 units /s

R Velocity = $\sqrt{2^2 + (-3)^2}$ =$\sqrt{13}$

= 3.6 units/s

Tan θ = $\frac{2}{-3}$ =--.66

Θ=$tan^{-1}.66$



Q 15

A fluid flow is given by V=10 x^3i --- 8x^2yj . Find the shear strain rate is rotational or irrotational [Ans;- 8xy rotational]

Solution

Given

I V=10x^3i ----8 x^2yj

Let u = 10x^3 v =--8x^2y

. du/dx =30 x^2 and dv/dx= ---16xy

,du/d y =0 and dv/d y =--8x^2

We know shear strain rate =1/2 (d v/d x+ d u/d y)

Putting values = ½(---16xy +0)= ---8xy

II Rotational components are

w_z -= ½(dv/dx---du/d y)

=1/2 (--16x –0)= ---8xy

Since it is not 0 but has a value hence it is rotational.

Q 16

The velocity components in two dimensional flow are

U= $8x^2y - 8/3y^3$, V=--8x y^3+ 8/3x^3. Show that these velocity components represents a possible case of irrotational flow .[Ans; du/d x+ d v/d y=0 , $w_z = 0$]

Solution

U = $8x^2y - \frac{8}{3}y^3$

, du/dx = 16xy---0

=16xy

, du/d y=8x^2--8y^2

V= --8x y^2+ 8/3 x^3

, dv/d y=---8x2y+0 = --16xy

d v/d x = --8y^2 + 8/3 x 3x^2

=---8 y^2 +8x^2

I For two dimensional flow continuity equation

, d u/dx + dv/d y= 0

In this case 16xy ---16xy = 0

So it is a possible of continuous flow.

II w_z =1/2 (dv/dx--- du/d y)

= ½ (--8 y^2 + 8 x^2 ---8 x^2 + 8y^2)

=0

Since w_z is 0 so rotation is 0 which means case is irrotational flow

Q 17

An open circular cylinder of 20 cm diameter and 100 cm long contains water up to a height 80 cm. It is rotated about its vertical axis Find the speed of rotation when I) no water spill, II axial depth is 0. [267.5 r. p. m 422.98 r p m]

Solution

Given

Diameter =20 cm

R =20/2 =10cm

L =100 cm

Initial height of water = 80cm

I Let the cylinder is rotated

At an angular speed of ω radians /s

Then Rise of liquids at ends = fall of liquid at center.

But rise of liquids at ends = length of cylinder--- initial height.

= 100 –80= 20 cm

Height of parabola = 20cm +20cm=40cm

Z =40 cm

Now using the relation z = $\frac{\omega^2 R^2}{2xg}$

$$\omega^2 = \frac{40X2X981}{100} = 784.8$$

ω =28 rad/sπ but $\omega = \frac{2\pi N}{60}$

Where N is r. p. m=revolution/minutes

Putting the values

$28 = \frac{2\pi}{60}$ so $N = \frac{1680}{2\pi}$ =267.5 r p m

II when axial depth is = 0

then the depth of parabola

= total length of cylinder = 100 cm

Diameter =20cm R= 10 cm

Cylinder rotated at ω1 rad/s

Here z = 100 cm when axial depth =0

$$Z = \frac{{\omega_1}^2 R^2}{2g}$$

$${\omega_1}^2 = \frac{100x2x981}{100}$$

ω 1 = $\sqrt{1962}$ =44.29 rad/s

$= \frac{2\pi N}{60}$

$N = \frac{44.29X}{2X\pi}$ =423 .1 r p m

Q 18

A cylindrical vessel 15cm diameter 40cm long is completely filled with water, The vessel is open at the top Find he quantity of water left in vessel when it is rotated about its axis with a speed of 300 r p m [4566.]

Solution

Given

I =Quantity of water left in the vessel

Initial volume of water =Area x initial height of water

= π/4 x $(15)^2 x 40$ =7068 cm3

N =Speed=300 r. p. m , R= 7.5 cm

ω = 2πx300/ 60 =31.4 rad/s

height of parabola is given z =$\frac{\omega^2 R^2}{2g}$ =$\frac{31.4^2 7.5^2}{2X981}$=28.26 cm

as the vessel is full if is rotated it will spill over.

Volume of water spilled =volume of para bola

Volume of paraboloid = area of cross section x height of paraboloid.

Or, $\frac{\pi}{4} d^2 x \frac{z}{2}$ = $\frac{\pi}{4} 15^2 x \frac{28.26}{2}$ =2492.5cm3

Volume of water left in the vessel

= initial vol. ---vol. of water spilled from paraboloid

= 7068cm3---2492.5 cm3

= 4575.5 cm3

Q 19

An open circular cylinder of 20 cm diameter and 120 cm long contains water up to a height of 80 c m It is rotated about its vertical axis at 400 r. p .m Find the difference in total pressure force I. at the side of the cylinder and II. At the bottom of the cylinder. [Ans; 2465.45M ,14.52 N]

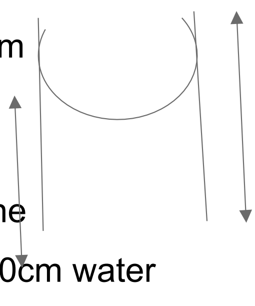
Solution

Given open cylinder vessel

I Force on the side of the cylinder $= \rho\, g\, \bar{h}\, A$

A = Area =surface area of the side of the cylinder

$= \pi$ D x height of water$= \pi$ x .2 x .8 = .5 m2



L = length of cylinder =120 cm

, h= height of water=80 cm

120cm , $\bar{h}$ = 80/2 = 40 cm= C G of the

80cm water wetted area of cylinder.

Open cylinder

ρ = 1000 kg/cm3

, g =9.81 kg/cm2

Wetted area $= \pi$ x D$= \pi$ X 20 cm

Height of water = 120 cm

, So the force on the side before rotation

= 1000 x9.81 x .5 x .4

= 1962 N

(b) After rotation

Water is up to top of cylinder and hence force on the

Side =1000 x9.81x wetted area of side x(1/2 x height of water.)

= 1000 x 9.8 1x(π x .2) x (½ 1.2)

= 4438 N

Difference in pressure = 4438 ---1962

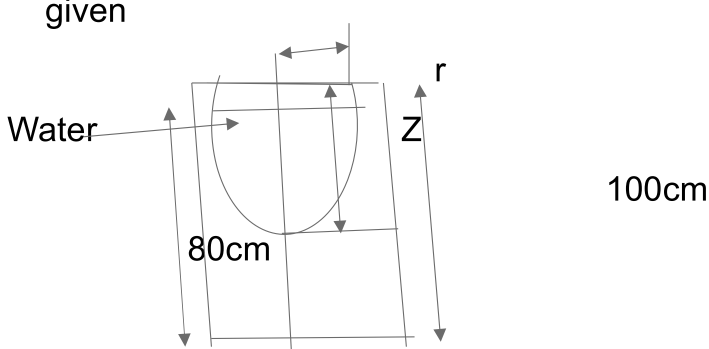
= 2476 N $*$ $*$

$**$ II Part is left for student to try as an exercise. Thanks.

Q 20

A closed cylinder vessel of diameter 15 cm and length 100cm contains water up to a height of 80 c m the vessel is rotated at a speed of 500 r p m about its vertical axis Find the height of paraboloid formed [Ans;56.06 cm]

Solution given



Closed cylinder with water.

Z= height of paraboloid ? r = radius of paraboloid

L= length of cylinder =100cm=1 m

80cm= height of water =.8 m

Diameter of vessel =15 cm =.15m

Speed of rotation =500 rpm

ω =2ΠN/60 = 2x π x 500/60 =52.35 rad/s

I When vessel is rotated

Volume of air before rotation=volume of air after rotation

π/4 x D^2(L – 80) =πr^2Z/2

π/4 x 15^2(100—80)= π x r^2x Z/2

Zr^2 = $\frac{225\ x20\ x2}{4}$ = 2250 ---------(i)

Z =$\frac{2250}{r^2}$ --------(i)

Now using the relation Z =$\frac{\omega^2 r^2}{2g}$ = $\frac{(52.35)^2 r^2}{2x981}$=1.396$r^2$

Z X Z= $\frac{2250}{r^2}$ x 1.396 r^2= 3141 cm2

Z =√3141 =56.04 cm

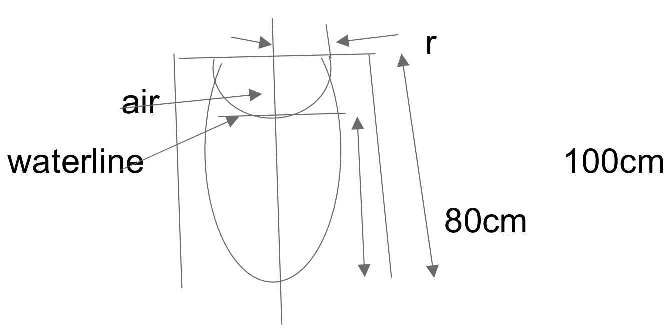
= height of paraboloid.

Q 21

For the data given Q 20. find the speed of rotation of the vessel when axial depth is zero.

Solution

Given



length 0f cylinder=100cm

r=radius of paraboloid.

Diameter of cylinder vessel=15 cm

$Initial\ height\ = 80\ cm$

Let ω be the angular speed.

When axial depth is o, paraboloid radius is r

And height is 100 cm.

Then using the following relation

Z = $\frac{\omega^2 r^2}{2g}$

$\omega^2 r^2$ = 100 x2 x 981= 196000 -----(i)

Vol. air before rotation = vol. of air after rotation

πR^2 (100---80) =π$r^2 x$ Z/2

$7.5^2\ X20$ = r^2x 100/2

So r^2 = $\frac{7.5^2 x20x2}{100}$ =22.5

, substituting in (i)

$\omega^2 r^2 =$ 196000

$\omega^2 = \frac{196000}{r^2} = \frac{196000}{22.5}$ =8711.11

, ω =$\sqrt{8711.11}$ =93.33 rad/s

=2ΠN/60

N = $\frac{\omega x60}{2\pi}$ =$\frac{93.33x60}{2x\pi}$ = 891.68 r p m

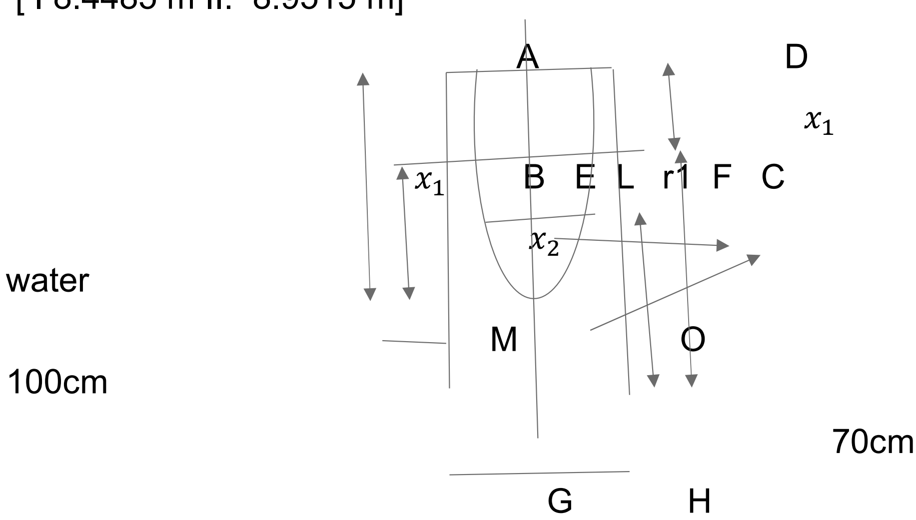
Type equation here.

Q 23

A closed cylindrical vessel of diameter 20 cm and height 100 cm contains water up to a height of 70 cm. The air above water surface is at a pressure of 78.48 Kn. The vessel is rotated at a speed of 300r p m about its vertical axis. Find the pressure heads

.at the bottom of the vessel I} at the center II) at the edge.

[I 8.4485 m II. 8.9515 m]



EL=LF=r_1 closed cylinder.

Diameter of vessel =20 cm

Radius = 10 cm

Initial height =70 cm

Length of cylinder= 100 cm

Pressure of air above water =78.48 kN/m2

= 78480 N/m2

Speed = 300 r .p .m

ω = 2Πn/60= 31.41 rad /s

pressure head = h = P/ρ g $\frac{78.48\ x\ 10^3}{1000\ x 9.81}$

pressure head = h = 8 m of water.

X 1 = height of paraboloid if the vessel assumed to be opened at top and very long

= $\frac{\omega^2 r^2}{2g}$ =$\frac{31.41^2 10^2}{2x\ 981}$ =50.285 cm ----(- i) x_1=.503 m

, r_1 is the radius of actual paraboloid of height x_2

So x_2 = $\frac{\omega^2 {r_1}^2}{2g}$ = $\frac{31.41^2 {r_1}^2}{2x\ 981}$ = .5 ${r_1}^2$------(-- ii)

Vol. of air before rotation = π$r^2(100-70)$

=30πx10^2

=9420cm3 ------(a)

Vol. of air after rotation =vol of paraboloid

= ½ x π${r_1}^2$x x_2

=1/2 π${r_1}^2$.5${r_1}^2$ -----(b)

Putting the value of x2 from ii)

Now we know that above mentioned (a) and b)

, are equal

$$\frac{1}{2} \pi\, r_1{}^4 x\ .5 = 9420$$

$$\therefore r_1{}^4 = \frac{9420x}{.5\pi} = 11994$$

$$r_1 = 10.46 \text{ cm}$$

$$x_2 = .5 r_1{}^2 = .5 \text{ x } 10.46 \text{ x} 10.46 = 54.70$$

=LO

I Pressure head at bottom of vessel

A. At the center = pressure head due to air+ OH
= 8m + (LH—LO)

= 8 m +(1m—52.57 cm)

= 8.475 m

B At the edge
= pressure head due to air +height of water above G.
= 8 m +AG
= 8m +(GM+MA)
=8m + (HO +x_1}
=8m +(1m+.503)
=9.503 m

Q 24

A closed cylinder30 cm and height 20 cm is completely filled with water. Calculate the total pressure force exerted on the top and bottom of cylinder, if it is rotated about its vertical axis

at 300 r p m .

Solution

Given ω H = Height of cylinder =.2 m

Speed = N = 300 r p m

. ω =2ΠN/60 = 2π X300/60 =31.41 rad/s

Total pressure force on the top of cylinder is given by equation 5.25.

F_T =ρ/4 Xω^2X π X R^4

= 1000/4 x $(31.41)^2$π x$(.15)^4$

= 392.46 N

II Total pressure force on the bottom of the cylinder is given by equation 5.26

F_B =ρ g x πR^2 x H +F_T

=1000x 9.81x π x $(.15)^2$x .2 +392.46N

=138.68 N +392.

= 5 3 0. 68 N

Q 27

State if the flow represented by u= 3x+4y and v =2x—3y is rotational or irrotational? [Ans Rotational]

Solution

U = 3x +4y

Differentiating , du/dx = 3

, du/d y =4

V=2x—3y

Differentiating

, dv /dx=2

, dv/d y=--- 3

For continuity

, du/dx +dv/d y should be =0

Putting values of differentiation

3+(--3) =0 hence continuity is there .and

Possible case of fluid flow.

Rotational w_z is give by equation

=1/2 (δ v/δ x ---δ u/δ y)

=1/2 (2 –4)

=1/2 x –2

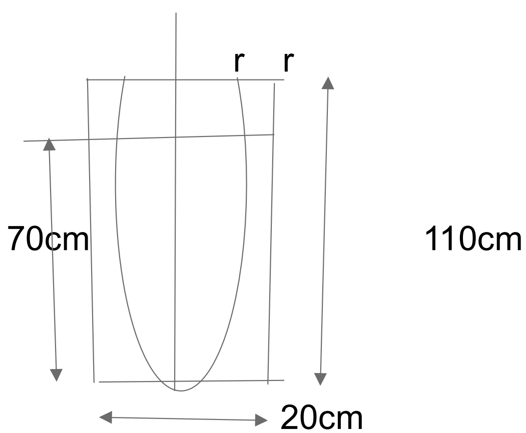
= ---1.

So there is a value of w_z and not 0, and therefore it is rotational.

Q 28

A vessel, cylindrical in shape and closed at the top and bottom contains water up to a height 700 mm The diameter o vessel is 200 mm and length of vessel is 1100 mm. Find speed of rotation if the axial depth of water is zero. [no answer is provided in original book] Note; I have considered prepared answer is correct.

Solution



, r =radius of paraboloid

Diameter of vessel =20 cm

Radius =10cm

$Initial\ height\ \ of water\ =\ 70mm = .7m$

Length of vessel =1.1 m

Let ω be the angular speed

When axial depth is 0 then depth of paraboloid is equal to = length of vessel during rotation =1.1 m =Z =11Ocm

Using the following relation in this case--

Z= $\frac{\omega^2 R^2}{2g}$

$\frac{\omega^2 r^2}{}$ =110x 2x 981 =215820 ------ I

. r=radius of paraboloid.

In such case

Vol of air before rotation= vol. of air after rotation

$\Pi R^2 X(110-70) = \frac{\pi r^2 110}{2}$

Or πx40x $(10)^2$ =π$r^2 x55$

r^2 =$\frac{4000}{55}$ =72.72

Putting the value in equation I

$\omega^2 = 215820/72.72$ = 2967,8

ω = $\sqrt{2967.8}$=54.47 rad/s=speed of rotation

or, N=60xω/2π =60x54.47/2π =520.15 r p m

Q 29

Define rotational and irrotational flow.

The stream function and velocity potential flow are given by

∅ =2xy , φ = $x^2 - y^2$

Show that condition of continuity and irrational flow are satisfied

∅ =2xy i.e for stream function

δ∅/δ y =δ/δ y(2xy) =2x =--u so u =---2x

δ∅/δ x = δ/δ x (2x y)=2y = v so v =2y

I for continuity

,d u/dx +dv/d y = δ/δ x (--2x) +δ/δy (2y)

= --2 + 2

= 0 satisfies continuity.

II For irrotational

w_z =1/2(d v/d x ---du/dy)

=1/2 ($\frac{d}{dx}(2y)$ --- $\frac{d}{dy}(2x)$

= ½ (0---0) = 0

Hence function is irrotational

B For velocity potential flow

Φ = x^2 --y^2

$\frac{\delta\varphi}{\delta x}$ =--- u =---2x

$\frac{\delta\varphi}{\delta y}$ =-- v =---(--2y)=2y

For continuity

d/dx(2x) + d/d y(--2y) = 2 –2 =0 hence continuity satisfied.

For irrotational

w_z =1/2 $(\frac{\delta v}{\delta x} - \frac{\delta u}{\delta y})$ should be 0

= ½ $\{\frac{\delta}{\delta x}(-2y) - \frac{\delta}{\delta}(2x)\}$

=1/2 (0--- 0)

= 0 so irrotational condition satisfied.

Q30

For steady incompressible flow are the following values of u and v possible ?

I U=4xy +y^2 , v =6xy+3x

II U=2$x^2 + y^2$ v =--4xy

Solution

I a) $\frac{\delta u}{\delta x}$ = 4y+0 , b) $\frac{\delta v}{\delta y}$ =6x+ 0

For continuity a) + b) =0

But it I is not 0 but it is 4y +6x , so these values are not possible.

II a) $\frac{\delta u}{\delta x}$ = 4x +0 b) $\frac{dv}{dy}$ =---4

As a) +b) =0 hence it satisfies continuity condition so values are possible in this case.

Q 31

Define two dimensional stream function and velocity potential.

Show that following stream function Ø=6x—4y+7xy+9

Flow. Find the velocity potential φ [Ans φ=4x+6y—3.5$x^2 + 3.5y$^2 + c$

Solution

I Ø= 6x—4y+7xy+9 -----i

If $w_z = \frac{1}{2}\left(\frac{\delta v}{\delta x} - \frac{\delta u}{\delta y}\right) = 0$ then it is irrotational.

Since it is *stream* function diff. equation i

$\frac{d\emptyset}{dy}$ =--- u = 0—4 +7x so u= 7x-- 4

d Ø/δ x = v = 6—0+7y+0 =7y +6

∴ u=7x—4 v= 7y+6

, du/d y =0 dv/dx=0

w_z=1/2(dv/dx---du/d y= ½ (0---0) =0

So it represents irrotational flow.

II To find velocity potential φ

$\frac{d\varphi}{dx}$=--u=---(7x—4)=4—7x ----(ii)

$\frac{d\varphi}{dy}$ =--v= --(7y+6) =--6—7y ----(iii)

Integrating(ii) we get φ=4x—7/2 x^2=4x—3.5x^2± C

C is constant and independent of x but can be function of y ---- iv)

Diff iv w.r.t y we get $\frac{d\varphi}{dy} = \frac{dc}{dy}$

And from iii we get $\frac{d\varphi}{dy}$ =--6—7y

,d c = (--6 –7y) d y

Integrating we get

C =± 6y + 7$\frac{y^2}{2}$ = ±(6y+3.5y^2)

Putting the value of C we get

φ =4x—3,5 ±C

=4x –3.5x^2 ±(6y +3.5 y^2) =value of potential.

= 4x +6y –3.5 x^2+3.5y^2

Q 32

Check if ∅ =$x^2 - y^2 + y$ represents velocity potential for dimensional irrotational flow. If it does then determine stream function φ [yes, φ=---2xy+x]

Solution

∅=denotes velocity potential

φ= represents stream function

I Velocity potential

U= --d∅/dx(f)=---d/d x ($x^2 - y^2 + y$)

=--2x

V =---d φ/d y(f)=---d/d y ($x^2 - y^2 + y$

= ---(--2y+1) =2y---1

(i) whether irrotational =?

w_z =1/2 (dv/dx---du/d y)

= ½ {d/d x(--2y+1)—d/d y(--2x)}

= ½ (0 –0)

= 0

Hence irrotational.

(ii) stream function

. d φ/d y= --- u =(--2x) =--2x ------- a.

. d φ / d x=v = (---2y+1) ---------b.

Integrating b.w.r.t. x

φ =-- 2xy + x ± k (constant)-------- c.

differentiating w. r .t y

. d φ/d y =d/d y(2xy+x+k)

. d φ/d y = 2x +0+dk/d y --------d.

As in a. above d φ/dx=2x

S o d. is

2x=2x +0+d k/d y

. d k /d y = 0 -------e.

Putting value of e. in c above , we get

.φ = --2xy +x + 0

=---2xy + x is the stream function.

Q 33

If stream function for steady flow is given by Ø=$y^2 - x^2$, determine whether flow is rotational or irrotational then determine velocity potential φ. [irrotational, φ =--2xy+c]

Solution

For stream function

Velocity components u and v in terms of Ø are

Ø =$y^2 - x^2$

U =---d Ø/d y=--d/d y $(y^2 - x^2)$ = --2y

V= dØ/dx = d/dx $(y^2 - x^2)$ =--2x

(i) To find rotational or otherwise

w_z=1/2 (d v/d x --- d u /d y

=1/2 {d/dx (--2x) ---d/d y (--2y)}

=½ { --2 – (--2)} = ½ (--2+2) = 0.

Hence the function is irrotational.

ii

Velocity potential φ to find out --

We know d φ/d x= u, =(--2y) =---2y ----a.

, d φ/d y= v =(--2x) =---2x ---- b.

Integrating equation a.

$\int d\varphi.$ =$\int -2y$ dx

Velocity potential φ = ---2xy + c

END OF CHAPTER FIVE

Fluid Mechanics

CHAPTER 6

Dynamics of Fluid Flow

Q 1

Water is flowing through a pipe of 100mm diameter under a pressure of 19.62 N/cm2(gage)and with mean velocity of 3m/s. Find the total head of the water at a cross section, which is 8m above the datum line. [Ans 28.458 m]

$Solution$

Given

Diameter of pipe =100mm= .1m

Pressure (gage) =19.67 N/cm2

= 19.67 x 10^4 N/m2

Velocity =v =3m/s

Datum head = 8m

Total head = pr head+ kinetic head +datum head

Pressure head = P/ρ g= $\frac{19.67x1\ \ ^4}{1000x\ \ .81}$ =20m

Kinetic head = $\frac{v^2}{2g}$ =$\frac{3x3}{2x9.81}$ =.4567m

Datum head = Z= 8m

Total head = Adding all three heads as mentioned above

=20m +.4567m+8m

= 28.4567m

Q 2

A pipe through which water is flowing is having diameters 40 cm and 20cm at the cross section at 1 and 2 respectively. The velocity of water is given 5m/s . Find the velocity head at I and 2 and also rate of discharge [Ans 1.274m, 20.387m.0.628m3/s]

Solution

Given

D1 = 40cm=.4m

A1 =π/4 X $(.4)^2$ =.12566 m2

D2=20cm =.2m

A2 =π/4 X$(.2.)^2$=.0314

I) Velocity head at section 1=$\frac{v_1^2}{2g}$ =$\frac{5^2}{2x9.81}$=1.274m

II) Velocity head at section 2=$\frac{{v_2}^2}{2g}$

[$v_2 = \frac{A_1V_1}{A_2}$ =$\frac{.125x5}{.0314}$ =19.9m/s]

= $\frac{19.9x19.9}{2x9.81}$

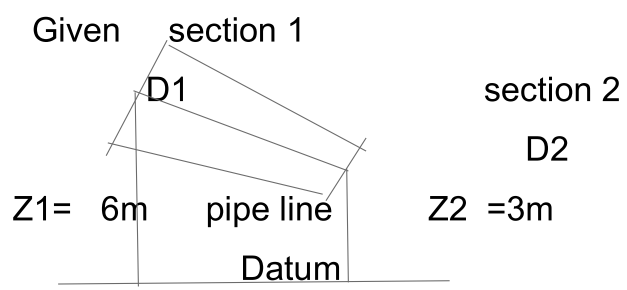
= 20.183 m

III)Rate of discharge=A1XV1 =.125X5 =.628 m3/s

Q 3

The water is flowing through a pipe having diameter 20cm and 15cm at section 1 and 2 respectively. Rate of flow through pipe is 40 lit/s . Section 1 is 6m above datum line and section 2 is 3 m . If the pressure in section 1 is 29.43 N/cm2, find intensity of pressure in section 2. [Ans 32.19N/cm2]

Solution



D1=.2m, D2=.15m,A1= π/4X,2X.2 =.0314 m2

A2=π/4 X .15 X.15 =.07m2

Pressure head at section 1 =29.43N/cm2

Z1=6m Z2=3m we have to find Pressure head at sec,2

Q =Rate of discharge= 40 lit /s=4o/1000 m3/s

Q=A1XV1=A2XV2=.04

V1=Q/A1 =.04/.0314=1.27 m/s

V2 =Q/A2=.04/.O7=.571m/s

Applying Bernoulli's equation at sec 1 & 2

$$\frac{P1}{\rho g}+\frac{V1^2}{2g}+\frac{Z1}{1}=\frac{P2}{\rho g}+\frac{V2}{2g}+\frac{z2}{1}$$

$$\frac{29.43}{1000x\ \ .81}+\frac{1.27x1.27}{2x9.81}+6=\frac{P2}{1000X\ \ .81}+\frac{.571X.571}{2X9.81}+\frac{3}{1}$$

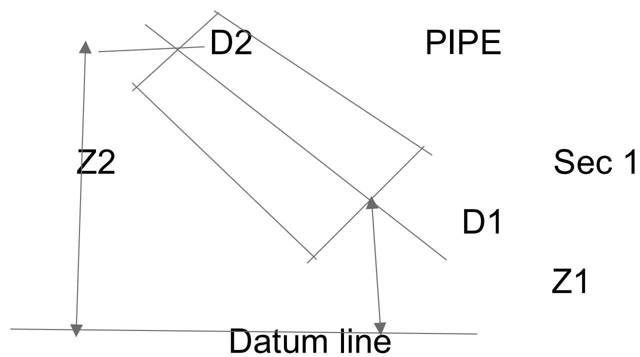
P2/9810 =33+.0822 ---.0166 =33.0656

P2= 33.0656X9819 =32.437N/cm2

Q 4

Water is flowing through a pipe having diameter 30cm and 15cm at the bottom and upper end respectively. The intensity of pressure at the bottom end 29.43 N/cm2 and pressure at upper end is14.715N/cm2. Determine the difference in datum head if the rate of flow through pipe is 50lit/s [Ans 14.618m]

Solution Given Sec--2



D2=15cm A2= .225π/4 m2

D1=30cm A1= .09π/4 m2

P2=14.715 X 10^4 N/m2

P1 =29.43X 10^4N/m2

Q -=A1XV1=A2XV2 = 50/1000 m3/s

V1= 50/1000 X 4/Пx.09 =.7 m/s

V2= 50/1000 X4/Пx.225=.283 m/s

Applying Bernoulli's equation

$$\frac{P1}{\rho g}+\frac{V1^2}{2g}+\frac{Z1}{1} = \frac{P2}{\rho g}+\frac{v2^2}{2g}+\frac{z2}{1}$$

Z2—z1 =(P1—P2)/ρ g $+\frac{V^2{}_1-V^2{}_2}{\rho g}$

$$Z2---Z1=\frac{(29.43--14.715)10^4}{1000X\ .81} + \frac{.7^2--.283^2}{2X9.81}$$

= 147.15/9.81+.20/9.81

=15+.02

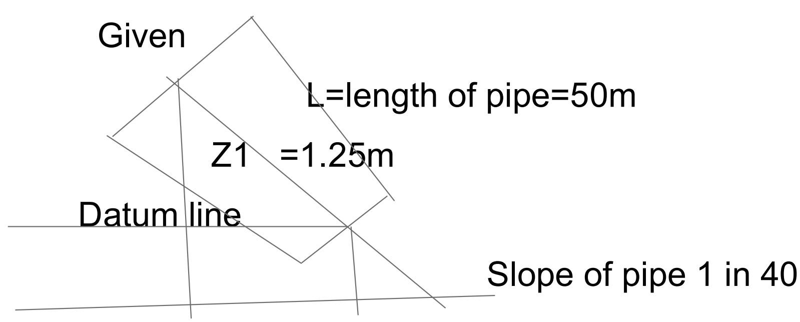
=15.02m = difference in datum head.

Q 5

Water is flowing through a taper pipe 0f length 50 m having diameter 40 cm at the upper end 20cm at the lower end ,at the rate of 60 liters /s. Pipe has a slope of I in 40. Find the pressure in lower end if the pressure in higher level is 24.525 N/cm2.

-[25.58 N/cm2]

Solution



Z1 = 50x 1/40=5/4=1.25m

Z2= 0= 0

D1=40cm,D2=20cm

A1=π/4X .4X.4= .1256m2

A2=π/4X.2X.2 =.0314m2

Q= A1XV1=A2XV2= 60/1000 m3

V1=60/1000 x 1/.1256=.477m/s

V2 =60/1000 x1/.0314= 1.91m/s

P1= 24.525 N/cm2

Applying Bernoulli's equation

$$\frac{P2}{\rho g} + \frac{V2^2}{2g} + Z2 = \frac{P1}{\rho g} + \frac{v1^2}{2g} + Z1$$

Or $\frac{p2x10^4}{109.8100} + \frac{1.91x1.91}{2x9.81} + 0 = \frac{24.525x10^4}{1000x9.81} + \frac{.477x.477}{2x9.81} +$ 1.25

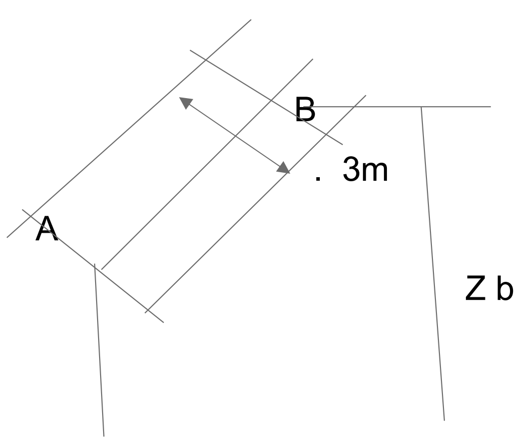
$$P_2 = 26.075X\ \frac{9.81}{10} = 25.579\ N/cm2$$

Q 6

A pipe of diameter 30cm carries water at a velocity 20 m/sec. The presser at the points A and B are given as 34.335N/cm2 and 29.43 N/cm2 respectively, while the datum head at A and B are 25m and 28m, Find the loss of head between A and B.[Ans 2m]

Solution

Given



Za

Datum line

Dia. of pipe=.3 m

At point A Pa=34.335N/cm2= 34.335x 10^4N/m2

Za=25 m

At point B Pb= .43x 10^4N/m2

Z b=28.0 m

Total energy at A (E

$$= Pa/\rho\ g + \frac{v^2}{2g} + Za$$

$$= \frac{34.335x\ 10^4}{1000x9.81} + \frac{20x2}{2x9.81} + 25$$

=35+20.38+25

=80.38 m

Total energy at B (E b)

$$= \frac{29.43x10^4}{1000x9.81} + \frac{20x2}{2x9.81} + 28$$

=30+20.38+28

=78.38 m

Loss of energy = E a ---E b

= 80.38---78.38

= 2 m

Q7

A conical tube of length 3.0 m is fixed vertical with its small end upwards. The velocity of flow at smaller end is 4 m/s while at lower end is 2m/s. The pressure head at the smaller end 2m of liquid. The loss of head in the tube is .95(V1—V2$)^2/2g$,V1 and

V2 at smaller end lower end resp. Determine pressure head at lower end .Flow takes place downward in direction.

[Ans 5.56 m of fluids]

3m=L=length of conical tube.

Datum line Z2=0 section 2

V1= velocity of small end = 4m/s

V2= velocity of lower end =2m/s

P1/ρ g= 2m =pr head of smaller end

To find pr head of lower end ? -=P2/2g

. h = loss of head = ..95 (V1—V2$)^2$/2g

= .95(4-- 2$)^2$/2x9.81

= .193 m

Applying Bernoulli's equation

$$P1/2g+ \frac{v1^2}{2g} +Z1= P2/\rho\ g +\frac{v2^2}{2g} + Z2$$

$$2 + \frac{4X4}{2X9.81} + 3 = P2/\rho\ g +\frac{2x2}{2x9.81} + 0$$

P2/2g=presser head at lower end

= 5+6/9.81=5.61 m

Q8

A pipe line carrying oil of specific gravity .8 changes in diameter from 300mm at a position A to 500 mm diameter to position B which is 5 m at a higher level. If the pressure

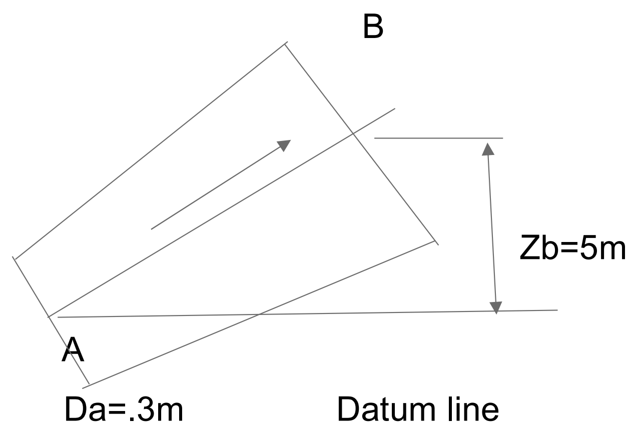
A and B 19.62N/cm2 and 14.91N/cm2 resp. and discharge is

150lit/s, determine the loss of head and direction of flow ?

[Ans 1.48m , flow from A to B]

Solution

Given



Db=500mm=.5m

Zb=5m

Aa=π/4$(.3)^2$=.07m2, Ab=π/4$(.5)^2$=.1963m2

Oil s p ,g r =.8 , ρ=density=.8x1000=800 kg/m3

Pa=19.62x10^4 N/m2, Pb=14.91x 10^4N/m2

Za=0 Z b=5 m

Discharge rate =150 lit/s= 150/1000 m3/s

Va=Q/area=150/1000x.07=2.1 m/s

V b=Q/area=150/1000x .196 =.7641

m3/s

Total energy at A

$$=Fa=\frac{P_a}{\rho g}+\frac{{V_A}^2}{2g}+Za$$

$$=\frac{19.62x1\quad ^4}{800x9.81}+\frac{2.14x2.14}{2x9.81}+0$$

= 24.99+.234

=25.224 m

Total energy at B= $Fb = \frac{P_b}{800X9.81}+\frac{Vb^2}{2g}$ + Z b

$$= \quad \frac{14.91x10^4}{800x9.81}+\frac{.7641\quad .7641}{2x9.81}+5$$

=18.9984+.0297+5

=24.028 m

So loss of head= Fa --Fb= 25.224—24.028

= 1.1959 = 1.196 (say)m

So flow will be from A to B.

Q 9

A horizontal venturi meter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential connected to inlet and throat is 10 cm of mercury. Determine the flow. Take Cd=0.98. [Ans 88.92lit/s]

Solution

Given

Diameter of inlet = .3m ,Area= π/4 x (.3x.3)m2=706.85 cm2

Diameter of throat= .15 m Area= π/4 x(15x15)=176.7cm2

Cd=.98 Manometer reading=10cm of mercury.

Difference of pr. head= x (s g of mer./s g of water-----1)

So h = x (13.6/1----1) =10 (13.6—1)=12 6 cm of water

The discharge through Venturi meter is given by

$$Q = Cd\left(\frac{a1xa2}{\sqrt{a1^2-a2^2}} x\sqrt{2gh}\right)$$

$$= .98\left(\frac{706.85x176.7}{\sqrt{.706.85^2-176.7^2}} x\sqrt{2x981x126}\right)$$

= .98x 182.5x497.2

=88924.22 cm3/s

=88.924 lit/sec.

Q 10

An oil of s p g r .9 is flowing through a venturi mete having inlet diameter 20 cm and out let diameter 10 cm .The oil mercury differential manometer shows a reading of 20 cm .Calculate the discharge of oil through the horizontal venturi meter.

[Ans 59.15lit/s] take Cd=.98.,

Solution

So = .9   Sg=13.6

Oil-mercury differential in manometer =x=20 cm

, h= difference of pr head = x(So/Sg---1)

= 20(13.6/.9--1)=20x14.1=282cm

Dia of inlet = 20cm Dia of outlet=10cm

A1= area = π/4 $(d1)^2$ = π/4x 20x20 =100π=314.16 cm2

A2= π/4 x (10x10) =78.54cm2

The discharge Q is given by equation (6.8)

$$=Cd\ x\frac{A1XA2}{\sqrt{A1^2-A2^2}}\quad X\sqrt{2gh}$$

$$= .98\frac{314.16x7\ \ .54}{\sqrt{314.16^2-78.54^2}}\ x\sqrt{2x981x282}$$

= 60336.34 cm3/s

= 60.336 lit/sec.

Q 11

A horizontal venturi meter with inlet diameter 30cm and throat diameter15 cm is used to measure the flow of oil of s p g r.8.The discharge of oil through venturi meter is 50 lit/s, find the reading of the oil mercury differential measurement. Cd=.98

[Ans ; 2.489cm]

Solution

$$Given$$

Inlet dia.=d1= 30cm. throat d i a . =15cm

A1= π/4 (30x30) =706.82cm2

A2= π/4(15X15) = 176.7 cm2

Specific gr of oil=.8 = So

“ of mercury=13.6= Sg

Discharge of oil =Q= 50 Lit/s= 50000 cm3/sec

$$Q= Cd\ x\frac{a1xa2}{\sqrt{a1^2-a2^2}}\sqrt{2gh}$$

$$50000\quad = .98x\ \frac{706.8x176.7}{\sqrt{706^2-176.7^2}}x\ \sqrt{2x981xh}$$

$$=.98x182.5x\ \sqrt{2x981xh}$$

$\sqrt{2x981xh}$ =$\frac{50000}{182.5x.98}$ =279.56 x279.56

.h=78120.25/1962 =39.81 cm

Differential h =x [Sg/So – 1]

39.81=x [13.6/.8 ---1]

X = 39.81/16=2.488cm

=reading of oil mercury

differential

Manometer.

Q 12

A horizontal venturi meter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow water. The pressure at inlet 14.715N/cm2 and vacuum pressure at the throat is 40 cm of mercury. Find the discharge of water through venturi meter. [Ans 162.539 lit/sec]

Solution

Given

Horizontal venturi meter

Inlet dia. =d1=20 cm

A1 = π/4(20x20) =314.16 cm2

Throat d i a= d2 =10cm

A2 =π/4 (10x10) = 78.54cm2

P1=Inlet pressure =14.715 N/cm2=14.715x 10^4

,ρ=1000kg/m3,

P1/ρ g = 14.715x 10^4/1000x9.81 =15 m

Vacuum pr=40cm of mercury

P2/ρ g = ---40 cm x Sg=--(40/100)m x 13.6

= ---.4x13.6= ---5.44 m of water

So differential head (h) in water

=P1/ρ g---P2/ρ g=15m---(---5.44m)

= 20.44m

Q =Discharge of water through venturi meter

$$= \quad Cd \frac{A1XA2}{\sqrt{A1^2 - A2^2}} x\sqrt{2gh}$$

$$=.98 \frac{\sqrt{2x981x204} \quad X314.16x7 \quad .54}{\sqrt{314.16^2+78.54^2}}$$

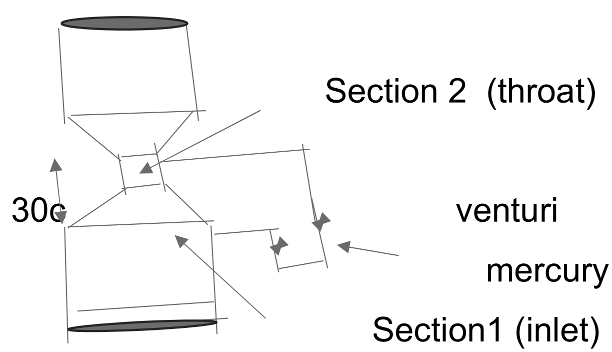
$$=\frac{49411899.2}{304} x2002.58$$

= 162539.14 cm3/3

= 162.539 lit/sec.

Q13

A 30cmx15cm venturi meter inserted in a vertical pipe carrying water flowing in upward direction A differential mercury manometer connected to the inlet and throat gives a reading of 30cm. take Cd=.98 [Ans 154.02lit/s]



Inlet dia. -=30cm= d1

Area A1= π/4 (30x30)=706.85cm2

Throat dia.= 15cm =d2

A2= π/4(15x15) =176.7cm2

Sg=13.6, S w= 1

X= reading of mercury of inlet and throat=30cm

.h differential= x [Sg/S w ---1]

=30[13.6/1 --1] =30x12.6

= 378cm

Q = Discharge rate= Cd$\frac{A1xA2}{\sqrt{A1^2-A2^2}}x\sqrt{2gh}$

=.98$\frac{706.85x176.7}{\sqrt{706.85^2-176.7^2}}x\ \sqrt{2x981x378}$

=.98 x 182.5x $\sqrt{741636}$

=154022.56 cm3/s

=154022.56/1000 lit/sec

=154.022 lit/sec.

Q14 In problem 13 instead of water, oil of specific gravity is flowing through a venturi meter , determine the rate of oil in lit/sec. [Ans 173.56 lit/s]

Solution

.d1 =inlet dia.=30 cm

A1= π/4 X(30x30)=706.85 cm2

. d2=π/4 x(15x15) =176.7cm2

Specific gravity of mercury =13.6, s p. g r of oil=.8

Section 1=inlet, section 2= throat.

Differential h =x (Sg/S o---1) = 30(13.6/.8 ---1)

= 480 cm

$$Q = Cd\frac{A1xA2}{\sqrt{A1^2-A2}}\sqrt{2gh}$$

$$= .98x\ \frac{706.85x176.7}{\sqrt{706.85^2-176.7^2}}\ x\sqrt{2x981x480}$$

= 173563 cm3/s

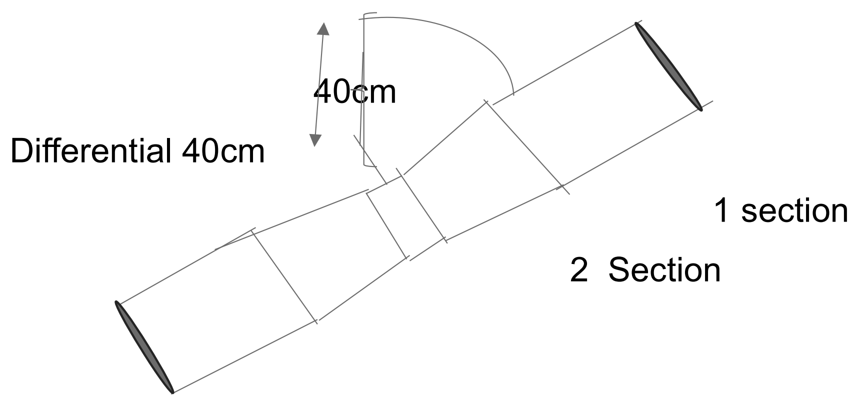
=173.563 lit/sec.

Q 15

The water is floating through a pipe of dia. 30 cm. The pipe is inclined and a venturi meter is inserted in the pipes. The d i a of venturi meter of throat is 15 cm. The difference of pressure between inlet and throat of the venturi meter is measured by a liquid of s p g r .8 in inverted u -tube which is giving a reading of 40 cm. The loss of head between inlet and throat is .3 times the kinetic head of the pipe. Find the discharge in lit/sec,

[Ans 22.64 lit/sec]

Inclined pipe with venturi meter



Oil s p g r= .8 s p g r of water =1

Dia. Of pipe =d1=30cm

Area = 706.85 m2

Dia of throat = 176.7 m2

Differential manometer reading =40 cm=x

Differential pressure head =h = x (1—S l/S w)

=40(1--- .8/1)

= 40x .2) =8 cm

of water

Also h =(P1/ρg+Z1) ---(P2/ρ g +Z2) =8 cm

As per question

Loss of head h_l =. 3x kinetic energy of pipe

$= .3x \frac{v^2}{2g}$ ---(i)

Applying Bernoulli's equation

P1/ρ g +$\frac{v_1{}^2}{2g}$ +Z1 = P2/ρg +$\frac{v^2{}_2}{2g}$ +Z2

∴ head loss h_l= (P1/ρ g +Z1)---(P2/ρ g +Z2) +($\frac{v_1{}^2}{2g}$-- $\frac{v_2{}^2}{2g}$)

= 8 + 1/2g{ $v_1{}^2(1-.3)-v^2{}_2$}

=8 +1/2g (.7 $v_1{}^2-v_2{}^2$) ------ (ii)

As per Bernoulli's equation

A1V1=A2V2 =Q

V1= A2X V2 /A1 =176.7/706.85 X V2

= .25V2 =V1 (iii)

Solving (ii) and (iii)

8 +1/2g(.7$v_1{}^2-v_2{}^2$) =0

Replacing v1 = .25v2 above we get

Type equation here.

$8+.7x\ (.25\ V2)^2/2g\ \ --(V2)^2\ /2g\ \ =0$

From this we get

v_2 =128.13 cm

Discharge rate Q = A_2V_2

= 176.7X 128.13

= 22640.57cm3/s

= 22.64 lit/sec.

Q 16

A 20X10 cm venturi meter is provided in a vertical line carrying oil of s p g r .8, the flow being upward The difference in elevation of the throat section and entrance section of the venturi meter is 50 cm. The differential U-tube mercury manometer 132shows gage deflection of 40 cm. Calculate i) discharge of oil ii) pressure difference between the entrance section and throat section. Cd=.98, s p g r of mercury=13.6[89.132 lit/s,5.415 N/cm2]

Solution

D1 = Inlet dia.=20cm A1 =Area =π/4 x 20x20

= 314.16cm2

D2=throat d i a =10cm A2=Area=π/4 x10x10

=78.83 cm2

Elevation of throat –entrance of venturi meter=50cm

Differential of mercury gage deflection

= 40 cm = X

1 Difference in pressure head=(entrance sec. & throat sec.)

. h =(P1/ρ g +Z1)--- (P2/ρ g +z2)

= X (Sg/So--1)=40(13.6/.8----1)

= 640 cm =6.40 m

6.40= 1/ρ g [(P1--P2)--(Z1 --Z2)]

=1/ρ g[(P1—P2)—50]

1/ρ g(P1—P2) = 640+50=690

P1—P2=690X .8X1000X981

Pressure difference =6.9x.8 x1000 x9.81N/m

=6.9x800x9.81/10^4 N/cm2

=5.411 N/cm2

2. Q=Cd x $\frac{\sqrt{2gh}}{\sqrt{a^2+b^2}}$ .A1Xa2

=.98 . $\frac{\sqrt{2.981.640}}{\sqrt{314.16^2-78.53^2}}$ x 314.16x78.53

= 89120.33 cm3/sec

=89.120 lit/sec. = rate of discharge of oil.

Q 17

In a 200mm diameter horizontal pipe a venturi meter of .5 contraction ratio has been fixed The head of water on the venturi meter when there is no flow is 4 on gage. Fid the rate of flow for which throat pressure will be 4 meter of water absolute Take Cd .97 atmospheric pressure head 10.3 m of water. [as 111.92 lit/s]

D1=Dia .of pipe =200mm =20cm

A1=π/4 x20x20 =314.16 cm2

D2=.5D1=.5X20=10m

A2=π/4 x 10x10=78.53 cm2

Head of water for no flow =4m (gage)

P1/ ρ g =4+10.3(absl) =14.3 (abs) ----- a,

Throat pressure head =P2/ρ g =4 m (abs) water--- b.

(i) difference of pressure head

.h= a. ---- b. =14.3---4 =10.3 cm2

(ii) rate of flow Q=Cd $\frac{\sqrt{2gh}}{\sqrt{A1^2-A2^2}}$ x A1Xa2

= .97$\frac{\sqrt{2x981x1030}}{\sqrt{314.16^2-78.53^2}}$ x 314.16x78.53

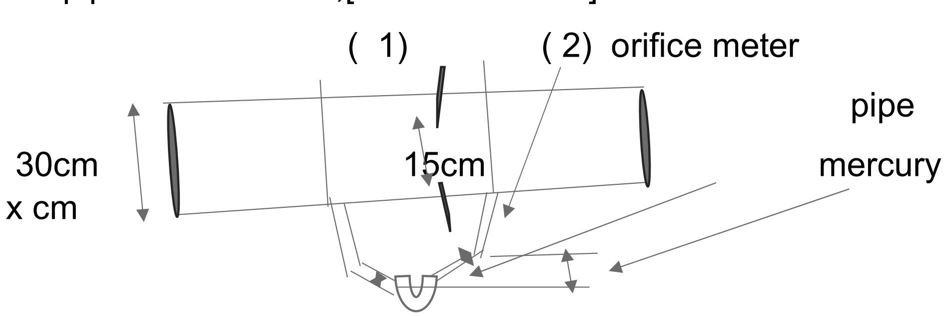
= $\frac{23930}{304}$x$\sqrt{2020860}$

= 111905.06 cm3/sec

=111.90 lit/sec

Q 18

An orifice with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure gage fitted up stream and down stream of the orifice give readings of 14.715 N /cm2 and 9.81 n/cm2 respectively. Find the rate of flow of water through the pipe in lit/s Cd=.6,[Ans 108.43 lit/s]



X cm is manometer reading.

Orifice dia. = 15cm = d1

A1= Orifice area =π/4 x15x15 176.7 cm2

Pipe d i a =30cm

A2= Pipe area = π/4 x 30x30 = 706.85 cm2

Up orifice pressure P1=14.7X10^4 N/m2

Down orifice pressure P2=9.81X 10^4N/M2

∴ Difference in pressure head =1/ρ g (P1—P2)

= 10^4/1000x9.81(14.715—9.81)

= 15—10= 5m =500cm

Cd= 0.6

Rate of discharge of water =Q= Cd$\frac{\sqrt{2gh}}{\sqrt{a^2-b^2}}$x A1XA2

= .6X $\frac{\sqrt{2x981x500}}{\sqrt{706.85^2-176.7^2}}$x 706.85x176.7

=$\frac{74940.23x990.45}{\sqrt{468414}}$ =108452.00cm3/s

=108.45 lit/sec.

Q19

If in problem 18instead of water oil of specific gravity .8 flowing through orifice meter in which pressure difference is measured a mercury oil differential manometer on the two sides of the orifice meter, find the rate of flow of oil when the reading of manometer is 40 cm. [Ans 122.68 lit/s]

Solution

D1=orifice d I a =15cm

A1 =Area=176.7cm2

D2=pipe d I a =30 cm

A2= area =706.85cm2

Pressure difference of two sides =reading of manometer

=x = 40 cm

Pressure head =h=x [13.6/.8—1]=40[17—1]

= 640cm=6.4 m of oil

Discharge of oil = Cd $\frac{\sqrt{2gh}}{\sqrt{a^2-b^2}}$ A1 x A2

= .6$\frac{\sqrt{2x981x640}}{\sqrt{706.85^2-176.7^2}}$ x 176.7x706.85

After calculation

= 122699.84cm3/s

=122.69/lit/sec

Q20

The pressure difference by the two tapings of a pilot static tubes, one taping pointing up stream and perpendicular to the flow, placed in the center of pipe of d i a 40 cm is 10 cm of water.

The mean velocity in the pipe is .75 times the central velocity. Find the discharge through pipe Take coefficient of discharge pilot tube as .98 [Ans .1293m3/s]

Solution

D1=40 cm

P1—P2=10cm =.1m =.1 m of water.

Area of pipe = π/4 x 40x40 =.12566m2

Cd =-98

Mean velocity=$\bar{v}$ = .75x central velocity.

Central velocity is given by the equation (6.14)

=Cd x $\sqrt{2gh}$

= .98 $\sqrt{2.9.81x.1}$ =1.372m/s

Mean velocity =$\bar{v}$ = .75x central velocity

= .75x 1.372 =1.029

m/s

So discharge through pipe

= A$\bar{V}$

=.12566x1.029

=0.12930m3/sec.

Q 21

Find the velocity of flow of an oil through a pipe When the difference of mercury level in a differential U-tube manometer connected to the two tapings of the pilot tube is 15 cm. Take spec. gravity o oil .8 and Cd coefficient of pilot tube as .98. Difference between mercury level in differential U tube=x=15cm [Ans 6.72m/s]

Solution

Specific gravity of oil= .8

-- do –mercury=13.6

Difference of pr head =h = x [Sg/So ---1]

= 15/100 [13.6/.8---1]m =.15x 16 m=2.40 m of oil.

Central velocity of flow= Cd $\sqrt{2gh}$

=.98 $\sqrt{2x9.81x2.4}$

=.98 x6.86

=6.72 m/sec

Q 22

A submarine moves horizontally in sea and has its axis 20m below the surface of water A pilot static tube placed in front of submarine and along its axis, is connected to two limbs of a U tube containing mercury The difference in mercury level is found to be 20 cm. Find the speed of submarine take s p g of mercury as 13.6 sea water 1.026.

Solution

Difference in mercury level=x=20 cm=.2m

Sg=13.6, So= 1.026

Difference in pressure head h=x (Sg/So ----1)

=.2(13.6/1.026 ---1)

= 2.451 m

Speed of submarine = $\sqrt{2gh}$ =$\sqrt{2x9.81x2.451}$

=6.934 m/s

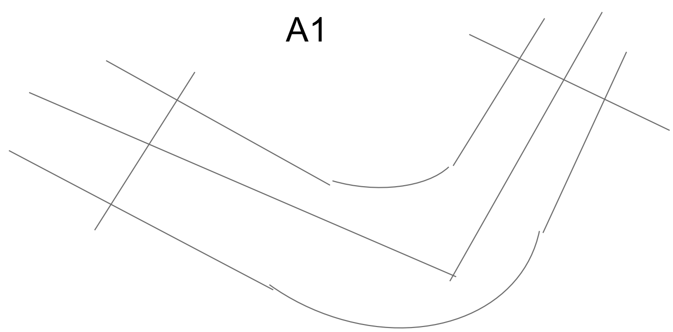
=$\frac{6.934x60x60}{1000}$

=24.962 km/hr.

Q 23

A 45° reducing bend s connected n a pipe line, the diameters at inlet and outlet of this bend being 40cm and 20 cm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet of the bend is 21.58 N/cm2. The rate of flow of water is 500 /sec. [22696.5n , 20°35 min]

Solution



45° Connected reducing bend.

D1=40cm, A1 =π/4 x .4x.4 =.1256m2

D2=20cm , A2 =π/4 x .2x . 2 =.0314m2

Pressure at inlet =21.58N/cm2=21.58x10^4N/m2

Discharge rate =Q=500 lit/s=.5m3/s

Velocity at section (1) V1=$\frac{Q}{A1}$ =$\frac{.5}{.1256}$

=3.98 m/s

Velocity at section (2)V2 =$\frac{Q}{A2}$

=.5/.0314=15.92

Applying Bernoulli's equation=

P1/ρ g +$\frac{{V_1}^2}{2g}$ +Z1=$\frac{P_2}{\rho g}$ +$\frac{{V^2}_2}{2g}$ +Z2

Here Z1=Z2

Type equation here.

$$P1/\rho g + \frac{v^2{}_1}{2g} = P2/\rho g + \frac{{v_2}^2}{2g}$$

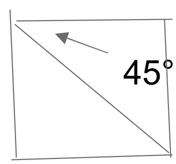
$$\frac{10^4 x\ 21.58}{1000x\ \ .81} + \frac{3.98x\ \ .98}{2x9.81} - \frac{15.92x1\ \ .92}{2x9.81} = P2/\rho g$$

P2/ρ g = 21.997+.807—12.91

= 9.894

P2= 9.894X1000X9.81= 9.7x 10^4 N/m2

Cos45=.707,



Sin45=..707

Forces on the bend in x and y direction are given by equation

(6.18) and (6.20).

$F_{X=}$ρ Q [V1—V2 cos θ]+P1 A1—P2A2.cos θ

= 1000 x.5 [3.98 --15.92 cos45°]+(21.58 x 10^4x.1256—9.7x10^4 x.0314 cos45)

=500(3.98—11.25)+21.58x1256—9.7 x314 x .707

=--- 3635---2153.68 +27104.48=21315.8 N

F_Y =ρ Q [--V2 sin θ] ---P2A2 sin θ

=1000x .5[---15.92 sin 45°] ---[9.7 x10^4x .0314 sin 45°

= ---5x 1592x .7071---(9.7x314x.7071)

=---5627---2153.68

= -- - 7780.68 N

$$F_R = \sqrt{{F_x}^2 + {F_Y}^2}$$

$=\sqrt{(21315.8)^2 + (-7780.66)^2}$

$=\sqrt{514897019.8}$

=22691.34 N

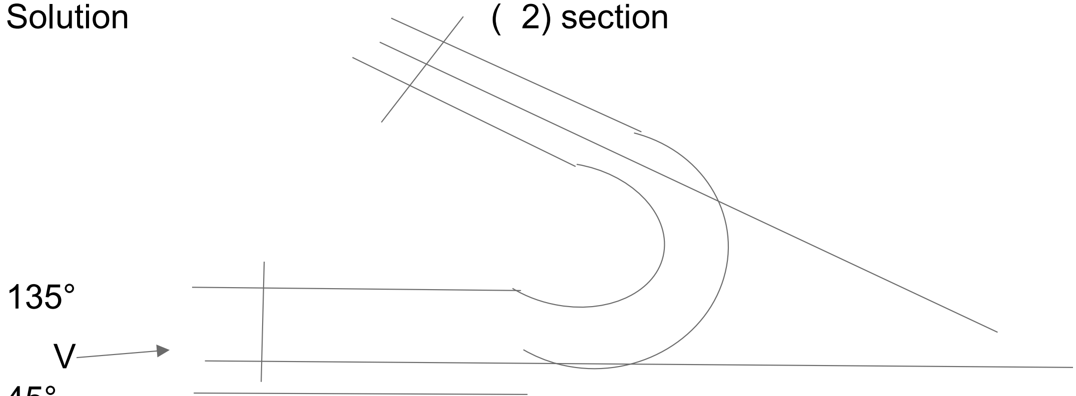
Tan θ $= \frac{F_Y}{F_X} = \frac{7780.68}{21315.8}$ =.3650193

Θ =20° 6’

Q 24

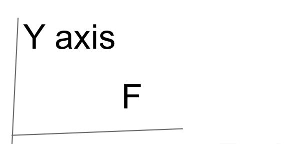
The discharge of water through a pipe of diameter 40 cm is 400 lit/s. If the pipe bend is 135 °,find the magnitude and direction of the resultant force on the bend The pressure of the floating water is 29.43N/cm2. [Ans;7063.2 N θ=22°29.9’]

Solution



(1) section

135° Bend pipe



axis

P1=P2 =29.43 N/cm2 =29.43 x10^4 N/m2

Q=400 lit/s, D1=D2=40cm=.4m, A1=A2=.1256m2

∀=volume of water =V1=V2=Q/A1=.4/.1256=3.18 m/s

F_X =ρ Q[V1x –v2x] +(P1xA1+P2xA2)

V1x= initial vel. In direction =3.184 m/s

V2x=final vel.=--- V2 cos45= ---3.184 x .707 = ---2.25m/s

P1x= 29.43x 10^4N/m2, P2x=29.43x10^4cos45

=29.43 x10^4 x .707N/m2

F_X = Putting above values we get

=1000x .4[3.184—(--2.25)]+(29.43x$10^4 x.1256)$

+(29.43x

$10^4 x.707x.1256)$

= 2174+36964+26133.6= 65271.6 N

F_Y =Force along y axis =ρ Q [V1y---V2y]+(P1yA1+P2yA2)

V1y=0,V2y=V2sin 45= 3.184x .707=2.25

P1 component in y direction will be 0 as cos 90 is 0.

P1A1=0, P2 A2 = 29.43x 10^4x.1256x,707=26133

F_y = putting the values we get

= 1000x . 4[0---2.25]+(0—26133)

=---400 2.25---26133

= --- 27033 N

F_R =$\sqrt{65271^2 + -27033^2}$

=$\sqrt{4991086530R}$

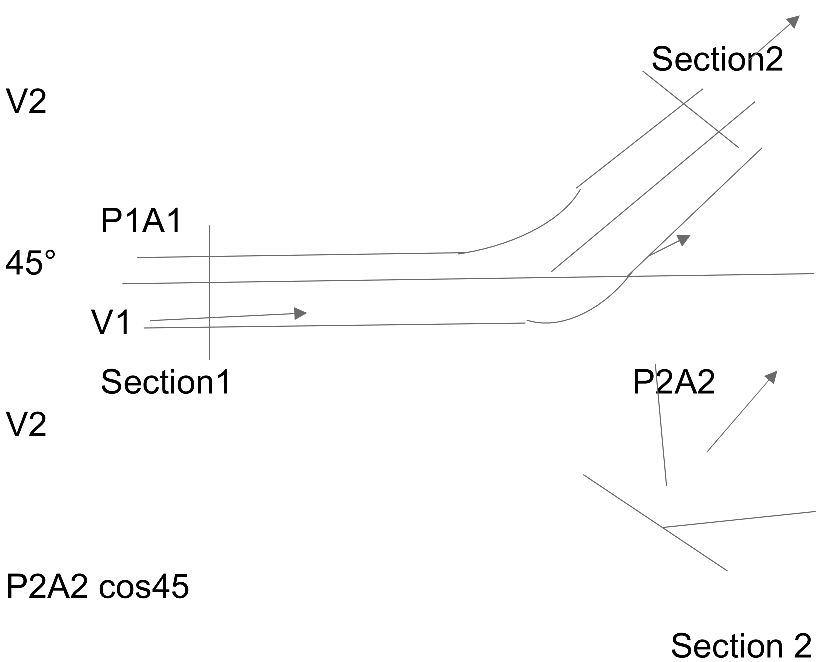
=70647.6 N =Magnitude of resultant force.

Tan θ $=\frac{F_Y}{F_X}=\frac{27033}{67271.6}$ =.41416

Θ =22° 5’ = direction of resultant force.

Q 25

A 30 cm diameter pipe water under a head 0f 15 m with velocity of 4 m/s. If axis of pipe turns through 45°, find the resultant and direction force at the bend [8717.5N θ=67° 30.’]



Dia of pipe=D1=D2=30 cm =.3

Area=A1 =A2 =π/4 (.3x .3)=.07068 m2

V=V2=V1=4Mp1

Θ = 45°

Discharge=Q= AV= .07068 X 4= .28272 m3/s

Pressure head = 15 m of water

P/ρ g=15/ρ g

P=15ρ g=15 x1000x 9.81

=pressure intensity

= 147150 N/m2

=P1=P2

V1x=4m/s2

V2x=V2 cos45 =4x .707=2.828 m

V1y=0, V2y=V2 sin45=2. 828 m

(P1A1) x= P1A1=147150x .07068 =10400.56

(P1A1)y=0

(P2A2) x = --P2A2 cos45 =---147150x.07068x.707=.7353.19

(P2A2)y=--- P2A2 sin45 =147150x.07068x.707 =.7353.19

F_x =ρ Q [V1x—V2x] +(P1A1) x+(P2A2) x

=1000x.28272(4---2.828) +10400.56+(7353.19)

=331.34+3047.37

= 3378.71 N

F_y = ρ Q [V1y---V2y] + (P1A1) y+(P2A2) Y

=1000x .28272[0---2.828] + [0+(--7353.19)]

=--- 799.53—7353.19

=---8152.72N

F_R =$\sqrt{3378.71^2 + (-8152.72)^2}$

$= \sqrt{77882524.65}$

$=8825.1$ N = Magnitude of resultant force.

Tan θ $= \frac{F_Y}{F_X} = \frac{8152.72}{3378.71} = 2.412$

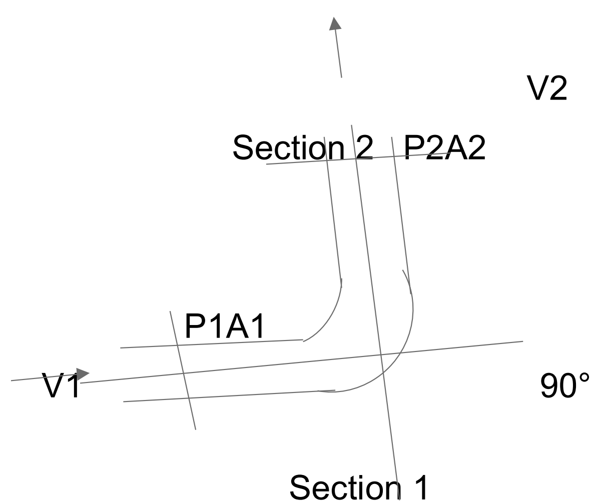
Θ =67° 48' Direction of resultant force

Q 26

A pipe of 20 cm diameter conveying .20m3/s of water has a right angled bend in a horizontal plane Find the resultant force exerted on the bend if the pressure at inlet and out let of the bend are 22.563 N/cm2 and 21.582N/cm2 respectively.

[Ans 11604.7N, θ =43° 54.2']

Solution



bend

D=D1=D2=20cm=.2m

$A=A1=A2=\pi/4$ x .2x.2 =.0314 m2

P1=22.563 N/cm2= 22.563 x 10^4 =225630 N/m2

P2= 215820 N/m2

Q= .20m3/s = A1V1 So V1= .2/.0314=6.37m/s =V2

(Same d i a pipe)

Angle of bend =90°

Force on bend along x axis

F_X= ρ Q[V1x---V2x] + (P1AI) x +$(P2A2)$x

=1000x .2 [6.37—0]+[7084.78+0]

= 8358.78 N

F_Y= ρ Q [V1y---V2y] + (P1A1)y+(P2A2)y

= 1000x.2[0 ---6.37] + (0---6776.74)

=--- 1274 ---6776.74

= ----8050.74 N

$F_R = \sqrt{F_X{}^2 + F_Y{}^2} = \sqrt{8358.78^2 + 8050.74^2}$

$= \sqrt{134683617.62}$

= 11605.32 N = Magnitude of resultant force.

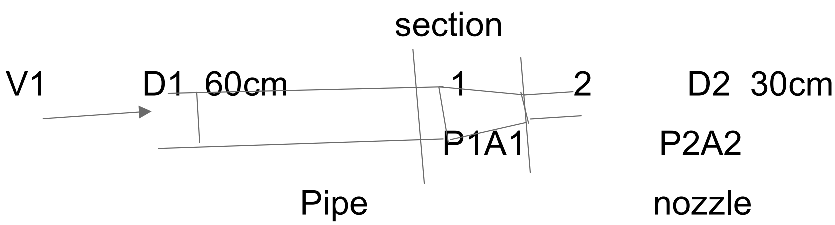
Tan θ $=\frac{F_Y}{F_X} = \frac{8050.74}{8358.78}$ = .96314

Θ = 43° 50’ =Direction of resultant force.

Q 27

A nozzle of diameter 30cm is fitted to a pipe of 60cm diameter. Find the force exerted by the nozzle on the water which is flowing through the pipe at a rate of 4m3/min.

[Ans 7057.7 N]



Nozzle

D1=60cm=.6m ,A1=π/4 x (.06x.06)=.002827m2

pipe D2=30cm=.3m. A2=π/4 x (.03x.030=..00070m2

Q = 4m3/min. =.066m3/s = discharge .

Applying continuity equation at sec1 ad 2

A1V1=A2V2 , V1= A2V2/A1 = .0666/.0028274

= 23.34m/s

A2V2= Q So V2 = Q/A2= .0666/.00070

= 93.48 m/s

Since pipe is horizontal Z1=Z2 also P2/ρ g=0

as atmospheric pr=0

so Bernoulli's equation stands as

$$P1/\rho\ g + {V_1}^2/2g = \frac{{V_2}^2}{2g}$$

$$P1/\rho = \frac{94.33^2 - 23.55^2}{2}$$

$$P1 = \frac{117.88X7\ \ .78}{2}\ X1000$$

$$= 117.88X35.39X1000$$

F_X = ρ Q[V2—V1] + (P1A1—P2A2)

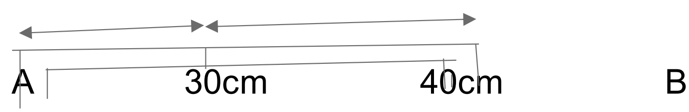
= 1000X .066[93.48--23.34]--4171773.2X .0028274---0

=4629.24---11795.27

= ---7166.03 N

Force exerted by nozzle. =7166.03 N

Q 28 A lawn sprinkler with 2 nozzles of diameters 3mm each is connected across a tap of water. The nozzles are at a distance of ip40cm ad 30 cm from the center of tip the rate of water through tap is 100 cm3/s .The nozzle discharges water in downward direction. Determine the angular speed at which sprinkler wii rotate free. [Ans 2.83 rad/s]



Nozzle

Nozzle

Nozzle dia. =D1=D2=3mm=.003 m

Area of nozzle=π/4 x(.003x.003) = .00000706m2

Q= Discharge = 100cm3/s

Assuming that the discharge is equally divided between two direction

QA=QB=Q/2=100/2=50m3/s

∀a=∀b=velocity of water at outlet of each nozzle

$Q/A=\frac{10^{-6}}{.00000706}$ x50 =50/7.06 =7.08 m/s =Va=V b

Force exerted by jib will be upward direction and torque also will be opposite direction. Hence Torque at B will be

Anticlockwise and torque A will be clockwise direction, But torque at B is clockwise direction, so sprinkle will be free.

Let ω be the angular speed of sprinkler

Then absolute velocity of water at A

= V1=VA+ ω x r a (r a=30cm=distance of nozzle from center, ω r a=tangential velocity)

= 7.08+ωx.3

Similarly V2= VB---Tangential vel. (ω x rb)

, r b=.4=dis. of nozzle from B

=(7.08 –ω x.4)

Applying equation (6.23) T= Resultant torque

= ρ Q (V2x r b---V1.ra)

=1000 x 50 x 10^{-6}[(7.08--.4 x ω) x .4 –{(7.08+.3ω).3}]

=0 Since no external force is applied T=0.

So 0 = 7.08x .4—7.08 x.3 -- .09 ω --.16 ω

Therefore . 25 ω = .708

, ω = 70.8/25

= 2.832 radians / sec.

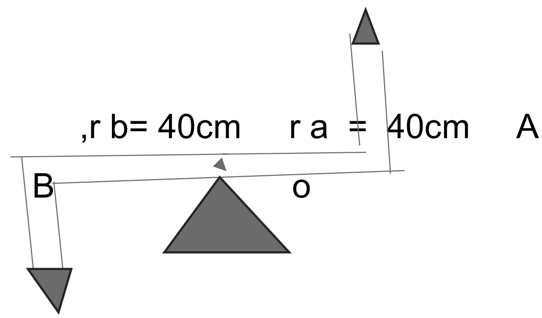
=angular speed at which sprinkler

, will rotate free .

Q 29

A lawn sprinkler has 2 nozzles of diameter 8mm each at the end of rotating arms and velocity of flow of water from each nozzle is 12m/s. The nozzles are at a distance of 40cm from the center of the rotary arm. Determine the torque to hold the rotatory arm stationary. Also determine constant speed of rotating arms if it is free to rotate.[Ans 5.78Nm, 30 rad/s]

Solution



Dia. Of nozzle=8mm both A and B both arm of nozzle

A=Area $=\pi/4 \times (.008 \times .008) = .0000502\ m2$

ρ

OA=OB=r a=r b=40 cm= both side nozzle arm.

Discharge from each nozzle= Q= A X V

$= .0000502 \times 12 = .0006024 m3/s$

Torque exerted by water through nozzle

= moment of momentum through A

$= r_a \times \rho \times Q \times V_a$

$= .40 \times 1000 \times .0006024 \times 12$

= 2.89 Nm clockwise. (1)

Torque exerted by nozzle B

$= r_b \times \rho \times Q \times V_b$

$= .4 \times 1000 \times .00006024 \times 12$

= 2.89 Nm (2)

Total Torque required to hold the Arms stationary

= (1) + (2) = 2.89+2.89

= 5.78 Nm

Constant speed of rotating Arms : ----

Let ω speed of rotation of sprinklers.

Absolute velocity of flow at nozzles

$V1a = 12$ m/s ---$\omega \times r\, a = (12 ---.4\, \omega)$

$V2b = 12$m/s --- $\omega \times r\, b = (12 --- .4\, \omega)$

Torque exerted by Water coming out at A sprinkler

$= r\, a \times \rho \times Q \times V1a = .4 \times 1000 \times .00006024 \times (12--.4\omega)$

$= .024096(12-- .4\omega)$ (3)

Torque exerted at B sprinkler

$= r\, b \times \rho \times Q \times V2\, b = .4 \times 1000 \times .00006024 \times (12-- .4\omega)$

$= .024096 (12--.4\omega$ (4)

Total torque exerted by both sprinklers = 3) +4)

$= .024096 (12--.4\, \omega) + .024096(12--.4\omega)$

$= 2 \times .024096 (12--.4\omega)$ -- 5)

Since moment of momentary flow entering is 0 and no external torque is applied on sprinklers so resultant torque must be zero.

So 5) above will be

, $2\times .024096 (12--.4\omega) = 0$

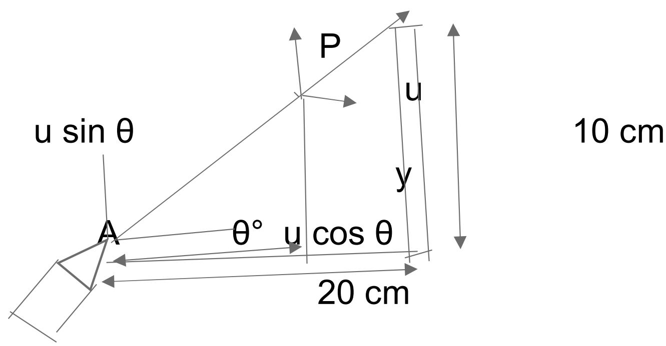
$.4\, \omega = 12$

$.\omega = 12/.4 = 120/4 = 30$ rad/sec

= Speed of rotation of sprinkler

Q 30

A vertical wall is of 10 m in height. A jet of water is issuing from nozzle with a velocity of 25m/s. The nozzle is situated at horizontal distance of 20 from vertical wall. Find the angle of projection of nozzle to the horizontal so that jet of water just clear the top of the wall. [79° 55’ , 36° 41’]



Height of wall=10m

Velocity of jet =25 m/s = U m/s

Distance of from wall =20m

Let the projected angle =θ°

As per equation (6.24) we have

$$Y = x \tan \theta -- \frac{gx^2}{2u^2} \quad sec\theta^2 \quad (1)$$

U =velocity of jet of water =25m/s

Coordinate of P (x,y) at point A

X=20cm , ,y=10 cm

Putting the values in equation (1) we get

$$10=20 \tan \theta --- \frac{9.8x20^2}{2x25^2} \quad sec\theta^2$$

$$= 20\tan \theta \; ---3924/1250 \; (1\_+\tan \theta^2)$$

$$=20\tan \theta \; -- 3.14 –3.14 \tan\theta^2$$

$$3.14 \tan\theta^2 — 20 \tan \theta + 13.14 = 0$$

(a) (b) (c)

$$\text{Tan } \theta = \frac{20 \pm \sqrt{20^2 - 4x3.14x13.14}}{2x3.14}$$

$$= \frac{20 \pm \sqrt{234.96}}{6.28}$$

$$= \frac{20 \pm 15.328}{6.28}$$

$$= 5.62, \quad 0.7439$$

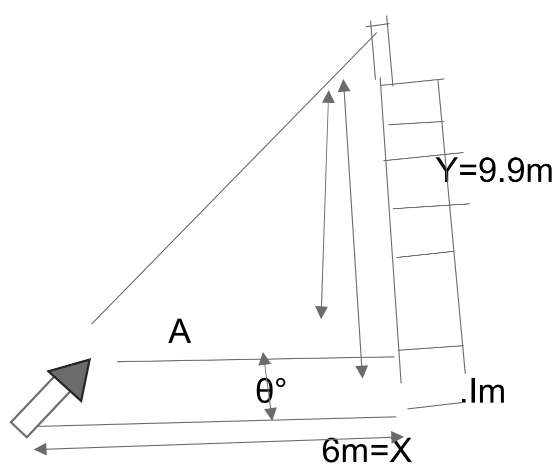
Θ =79° 40' ,36° 40' =angles of projections

Q 31

A fire brigade man is holding fire stream nozzle of 50 mm diameter at a distance of .1 m above the ground and 6m from a vertical wall. The jet is coming out with a velocity of 15 m/s. The jet is to strike a window situated at a distance of 10 m above ground in the vertical wall . Find the angles of inclination with horizontal ,made the jet, coming out of nozzle .What will be amount of water falling on the window ? [79° 16.7',67 3.7',.0294m3/s]

Solution

10cm



Dia. Of nozzle =50mm= .05m

Area of nozzle = π/4 x .05x.05 =0.00196 m2

(1) U=velocity of jet =15 m/s

Θ =angle of inclination of jet of nozzle by which the will reach window.

The coordinates at A , x=6m y= 10 -- .1 = 9.9m

Applying equation (6.24)

$$Y = X \tan\theta - \frac{g}{2}\frac{x^2}{u^2} sec\theta^2$$

$$9.9 = 6 \tan\theta - \frac{9.81}{2}\frac{6^2}{15^2} sec\theta^2$$

$$=6 \tan\theta - .7848\ (1+ \tan\theta^2)$$

$$9.9= 6\tan\theta - .7848 - .7848 \tan\theta^2$$

$$.7848 \tan\theta^2 - 6\tan\theta + 10.6848 = 0$$

(a) (b) (c)

$$\tan\theta = \frac{6 \pm \sqrt{6^2-4.7848x10.6848}}{2x.7848}$$

$$= \frac{6 \pm \sqrt{36-33.54}}{1.5696}$$

= 4.8165 2.8226

, θ = 78° 20’, 70° 26’

3 amount of water falling on window

= Q = AX V

= .00196 X 15

= .0294 m3/s

Q 32

A window in a vertical wall is at a distance 12 m above the ground level. A jet of water issuing from a nozzle of dia.50mm,is to strike the window. The rate of flow of water through nozzle is 40 lit/s. The nozzle is situated at a distance 1 m above the ground level. Find the greater horizontal distance from the wall to the nozzle so that jet of water strike the window [29.38m]

Solution

Given

Dia. Of nozzle =50 mm =.05 m

Area of nozzle = π/4 x(.05x.05) =.00196 m2φ

U =velocity of jet =Q/A = .04/.00196= 20.408 m/s

Let greatest horizontal distance of nozzle from wall=x m.

As per equation (6.24)

$$Y = x \tan\theta --- g/2 \times \frac{x^2}{20.4^2} sec\theta^2$$

$$11 = x \tan --- \frac{9.81}{832.}\frac{x^2}{32} \sec\theta^2$$

$$X \tan\theta ---.0117 \frac{x^2}{cos\theta^2} -- 11=0 \quad -------- 1$$

Differentiating we get max value of x w.r.t θ dx/d θ =0

$$X \sec\theta^2 + \tan\theta \times dx/d\theta -- .0117[\ x^2 \frac{-2}{cos\theta^3} (--\sin\theta\ )$$

$$--- \frac{1}{cos\theta^2} x2x\ .dx/d\theta\ ] =0$$

$$X \sec\theta^2 + \tan\theta\ xd\ x/d\theta --- .0117[\ \frac{2x^2 sin\theta}{\cos\theta^3} + \frac{2x}{cos\theta^2} dx/d\theta$$

For max value of x , dx/d θ=0

$$So\ x \sec\theta^2 = .0117 \times \frac{x^2 2sin\theta}{cos\theta^3}$$

$$X = \frac{.0117}{cos\theta} x\ 2x^2 \sin\theta$$

$$X = .0234\ x^2 \tan\theta$$

$$X \tan\theta = 42.73$$

$X = 42.73/\tan\theta$

Putting the value x tan θ =42.73

$$42.73 --- \frac{.0117x^2}{cos\theta^2} -- 11=0$$

Putting the value x = 42’73/tan θ, we arrive at

$$42.73 — 21.36/\sin\theta^2 — 11=0$$

We get sin θ=.8204

Cos θ =.5716

Tan θ =1.43

$$\text{So } x = \frac{42.73}{\tan\theta}$$

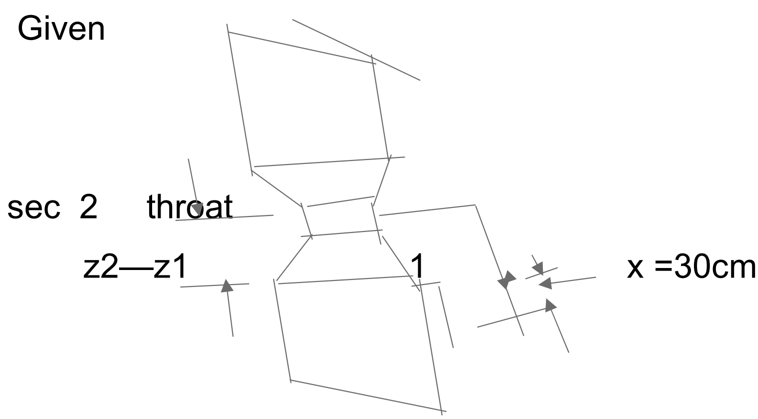
$$= \frac{42.37}{1.43} = 29.88 \text{ m}$$

Q 35

A 30 x 15 cm venturi meter is inserted in a vertical pipe carrying an oil of s p g r 0.8 flowing in up ward direction A differential mercury manometer connected to inlet and throat gives a reading of 30 cm. The difference in the elevation of the throat section and inlet section 50 cm Find the rate of flow of oil.

Solution

Given



manometer

vertical pipe with venturi meter

and manometer,

difference between elevation of throat section

and inlet section =50 cm

manometer reading of throat and inlet =x=30cm

pipe dia. =30cm

throat d I a =15 cm

oil s p g r=.8

pipe area = π/4 x 900 =706.85 cm2

throat area= π/4 x 225=176.71 cm2

difference in pressure head= x [Sg/So—1]

= manometer reading 30x [13.6/.8 ---1] =480 cm of oil.

Q= Rate of oil flow $=\frac{a1xa2\sqrt{2gh}}{\sqrt{a1^2-a2^2}}=\frac{706.85x1 \quad .71\sqrt{2x981x480}}{\sqrt{706.85^2-176.71^2}}$

$=\frac{124907.46x9 \quad .40}{684.4}$ =176119.5 cm3/s

= .176 m3/s

ii)

pressure difference between inlet section and throat sect of pipe (P1—P2)

, h =($\frac{p1}{\rho\ g}$ + Z1) –(P2/ ρ g + z2)

= x (Sg/So ---1)

480 =1/ρ g(P1—P2)—(Z2—Z1)

P1—P2 =(480+50)X ρ g= 530 x 1000x .8x 981

= 5.30 x1000x.8x9.81

=5.30 x 800 x9.81/10000

=5.30x 8 x9.81/100

=.530x,8x 9.81=4.16N/cm2

Q 36

A venturi meter is used for measurement of discharge of water in horizontal pipe line. If the ratio of upstream pipe diameter to hat of throat is 2: 1, upstream diameter is 300 mm and difference in pressure between throat and upstream is 3m head of water and loss of head through meter is 1/8 th of trot velocity head . Calculate discharge in the pipe

Solution

Given

Upstream pipe diameter =300 mm =30 cm = D1

Throat diameter = 150 mm=15 cm =D2

A1= 706.85 m2   A2 =176.71 m2

Difference of pr head between throat and upstream pipe

$\frac{P_1 - P_2}{\rho \ g}$ =3m =h =300 cm -- 1)

Velocity head -=$\frac{{V_2}^2}{2g}$-- $\frac{{V_1}^2}{2g}$ = h ---2)

v_1 =$\frac{a_2 v_2}{a_1}$ ----3)

From above 1),2),3) by solving we get

v_2 =$\frac{a_1}{\sqrt{a^2 - b^2}}x\sqrt{2gh}$

$$=\frac{706.85}{\sqrt{706.85^2-176.71^2}} \text{ x}\sqrt{2x981x300}$$

$$=\frac{706.85x767.2}{\sqrt{468410.5}}$$

$$= \frac{542295-32}{684.4} = 792.36 \text{ cm/s}$$

Q=Discharge in the pipe

= a_2v_2 =176.71 x 792.36 cm3/s

=140017 cm3/s

=$\frac{140017}{1000000}$ m3/s

=.140m3/s

Note : Balance few sums hints are given in the original book so I have not proceed in those sums.

END OF 6TH CHAPTER

Fluid Mechanics and Hydraulic Machine

Orifices and Mouthpieces

Chapter 7

Q 1

The head of water over an orifice of dimeter 50mm is 12 m

Find i) actual discharge ii) actual velocity of jet vena contract a Cd=.6, C v=.98

Solution

Given

Head of water over orifice =12m =h

Orifice dia.= 50mm=.05 m

Orifice area =π/4 x .05 x.05 = .001963m2

Theoretical velocity V2 = $\sqrt{2gh}$ =$\sqrt{2x9.81x12}$ =15.34 m/s

Theoretical discharge = V2x area of orifice

= 15.34 x .001963

=.03012 m3/s

Therefor actual discharge = .6 x .03012

=.018 m3/s

Now actual velocity =C v x theoretical velocity

=.98 x 15.34

= 15.033 m/s

Q 2

The head of water over the center of orifice of diameter 30mmis 1,5 meter .The actual discharge through the orifice is 2.35 lit/s .Find the coefficient discharge .

Solution Cd = $\frac{actual\ discharg}{theoretica\ \ dischage}$

Dia. Of orifice = 30mm =.03 m

Area of orifice = π/4 x .030 x.030 =.00070m2

H =water head =1.5 m

Theoretical Velocity = $\sqrt{2gH}$

$=\sqrt{2x\ 9.81x1.5}$

= 5.425 m/s

actual discharge =2.35 lit/s=.00235 m3/s

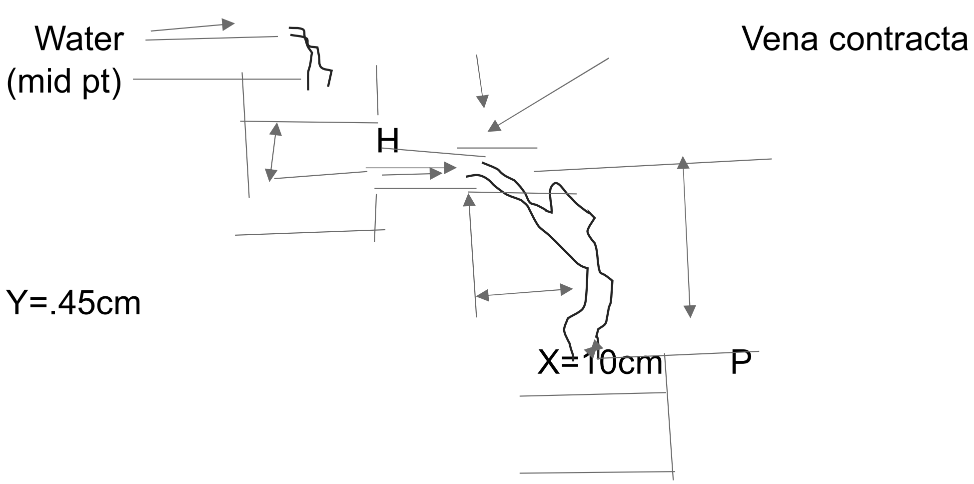
Theoretical discharge = theoretical velocity x area of orifice

= 5.425 x .00070=.0038 m3/s

Cd $=\frac{.00235}{.0038}$ = .62

Q 3

A jet of water issuing from a sharp edged vertical orifice under a constant head 60 cm has the horizontal vertical coordinates measured from vena contracta at a certain point As 10cm and .45 cm respectively. Find the value of Cc and Cv if Cd is .6. [A ns .962, .623]



Measuring Tank.

P =End of water falling on measuring tank.

H = Water head = 60 cm =.6 m

Mid point of vena-contrcta to P = coordinates (x,y}

X = 10cm =,1m ,y= .45 cm =.0045m

Theoretical Velocity =$\sqrt{2gH}$ =.$\sqrt{2x9.81x.6}$

=3.43 m/s

Now, x = actual velocity x time

= v x t so t = X/V

Y=1/2 g t^2 =1/2 g $(\frac{x}{V})^2$

$V^2 = g\frac{X^2}{2Y}$ =$\frac{9.81 \ .1x.1}{2x.0045}$ =$\frac{.0981}{.009}$ =10.9

V=3.3 m/s= actual velocity

Cv=$\frac{A.Velocity}{th\ .velocity}$ = $\frac{3.3}{3.43}$ = .962 m/s

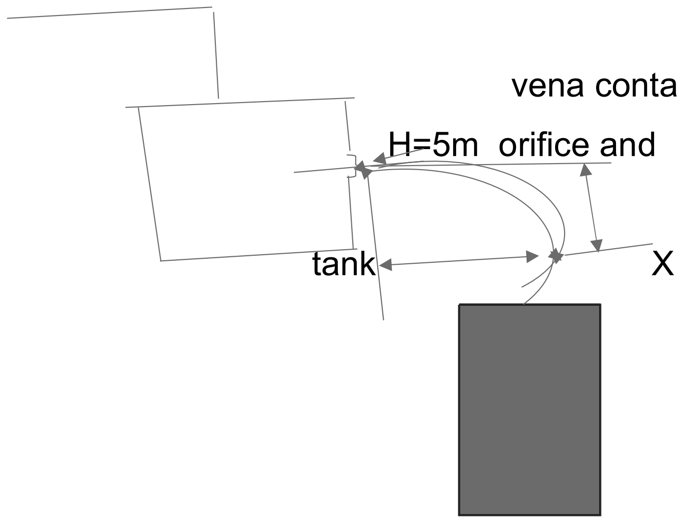
Cc =co. of contraction t vena contracta

=$\frac{coefficient\ of\ dis\ charg}{coefficient\ of\ velocity}$ =$\frac{.6}{.962}$ = .623.

Q 4

The head of water over an orifice of diameter 100 mm is 5 m. The water coming out of orifice is collected in circular tank of diameter of 2m. The rise of water in circular tank is .45 min 30 seconds Also coordinates of certain point on the jet measures from vena-contracta 100cm horizontal and 5.2cm vertical Find the hydraulic coefficient Ca Cv Cc [Ans .605 ,.98, .617]

Solution



Circular water tank

Dia. Of orifice=100mm= .1m rise of water =.45m/30s

=3/200 m/s

Area of orifice =π/4 x .1x.1 =.00785 m2

Water head= H=5m

P(X,Y)=(.1,.052) in m

Total water accumulated =π/4 x d^2 x .45 /30 m3/s

In measuring tank =.04712 m3/s

Theoretical velocity =$\sqrt{2gH}$ = $\sqrt{2x9.81x5}$ =9.9 m/s actual velocity , $v^2 = \frac{gx^2}{2y} = \frac{9{,}81x1^2}{2x.052}$ =94.3

so v= 9.711m/s

C v=$\frac{actual\ velocity}{theoretical\ velocity}$ =$\frac{9.711}{9.9}$ = .98

Cd=$\frac{ctual\ dischaer}{theoretica\ \ dischrg}$ =$\frac{.04712}{theoretica\ \ velocityxarea\ of\ orifice}$

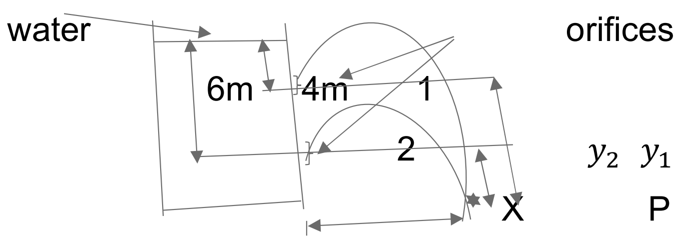
=$\frac{.04712}{9.9x.00785}$=.606

Cc =$\frac{Cd}{Cv}$ = $\frac{.606}{.98}$ = .618.

Q 5

A tank has 2 identical orifices in one of its vertical sides. The upper orifice is 4 m below the water surface and lower one 6m below the water surface. If value of C v for each orifice .98 find the intersection of two jets. [Ans 9.6 cm]

Solution



P =Point of intersection of two jets, P(x,y)

X is to be found out.

As mentioned in above sketch position of two orifices are

1. upper one =y_1
2. lower one= Y_2

H_1 = 4 m, below water surface

H_2 = 6m, below water surface

C v for both orifice is .98

As per sketch y_1 = y_2 +6 –4) = y_2 +2

We know here Cv=Cv1 =Cv2

Cv1=$\frac{x}{\sqrt{4y_1H_1}}$, Cv2=$\frac{x}{\sqrt{4y_2H_2}}$

$Y_2H_2 = Y_1H_1$

=$(Y_2 + 2)\ H_1$

6$Y_2 = (Y_2 + 2)4$

SO, Y_2=4

Y_1= 6

Cv=$\frac{x}{\sqrt{4y_2H_2}}$ =$\frac{X}{\sqrt{4X4X6}}$ =.98

SO x= .98 X √96 =.98x 9.798 =9.601 m

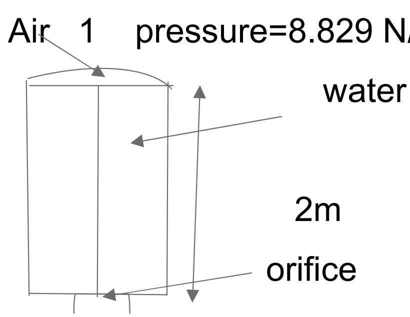
Point x is at a horizontal distance of 9.601 cm

Q 6

A closed vessel contains water up to a height of 2m and over the water surface there is air having pressure 8.829N/cm2 above atmospheric pressure. At the bottom of the vessel there is an orifice of diameter 15 cm. Find the rate of flow of water from orifice. Take Cd=0.6 [Ans 15575m3/s.

Solution

Given



dia.=15cm=.15 m

area of orifice =π/4(.15x.15)=.0176m2

water H=2m=height of water,

air pressure= 8.829 x 10^4 N/m2

applying Bernoulli's equation

$$\frac{P_1}{\rho g}+\frac{V^2{}_1}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2{}^2}{2g}+z_2$$

Z1=2m Z2=0,P2/ρ g=0 because of atmospheric Pr.

$$.\frac{p_1}{\rho g}+z_1=0+v_2^2/2g+0$$

$$\frac{10^4x8.829}{1000x\ .81}+2=\frac{v_2^2}{2x9.81}$$

$$v_2=\sqrt{11x2x9.81}=14.69\text{m/s}$$

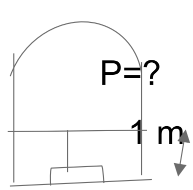
Q=Rare of flow from orifice= Cd x Ax V

= .6 x .0176 x 14.69

=.15512 m3/s.

Q 7

A closed tank partially filled with water up to a height of 1m having an orifice of diameter 20mm at the bottom of tank. Determine the pressure required for discharge of 3 lit/s through the orifice. Take Cd=.62



diameter of orifice =20mm=.02m

Area=π/4 x (.02x.02)=.0003144m2

H =1m , H=1m

Section 1 =surface water , vel. Here =0, V1=0 , Z=1m

Section 2= orifice, here Z 2=0

So, Bernoulli's equation

$\frac{p_1}{\rho g}$+0+1= 0 + v_2^2 /2g + 0 --------- .1

Now Q =Discharge = Cd A.v

.003 =0.62x .000314x v_2

So, v_2 =15.416

Putting the value of velocity of section 2

$$\frac{P_1}{1000X9.81} + 1 = 15.416^2\ /2g$$

From above equation we get

P_1/1000X9.81 =(237.65/19.62) ---1

P_1 =$\frac{109.01X10}{10^4}$ = 10.9 N/cm2

Alternatively,

Pressure head = $\frac{p}{\rho g}$ =$\frac{px10^4}{1000x\ \ .81}$ = $\frac{10p}{9.81}$ on the water,

V_2=At orifice = $\sqrt{2g(H+\frac{P}{\rho g}})$ = $\sqrt{2x9.81(1+\frac{10}{9.81}})$

Discharge Q = C d A V_2

.003 =.62 x.00314 $\sqrt{9.81x2(1+\frac{10\ )}{9.81)}})$

From above value of p comes out as P=10.89 N/cm2

Q 8

Find the discharge through a rectangular orifice 3m wide and 2m deep fitted to a water tank. The water level in the tank is 4 m above the top edge of the orifice. Take Cd =.62 [Ans; 36.77m3/s]

Solution

Distances from surface water are Hi and H 2

H is the midpoint of orifice

Width of orifice= b =3m   d=depth of orifice = 2m

(As per original book, chapter 7   equation no 7.8)

The discharge through the orifice

= 2/3 Cd x b x√2g x $[H_2^{3/2} - H_1^{3/2}]$

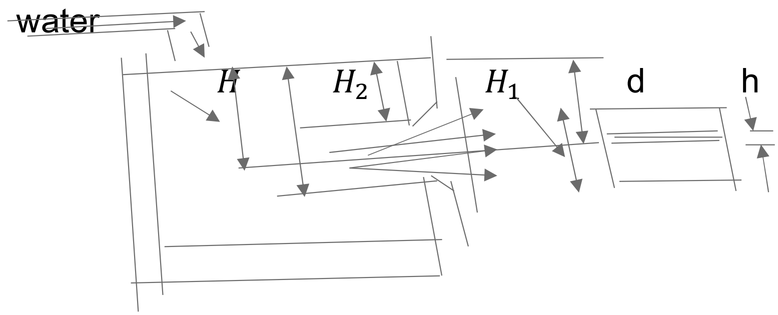
=2/3 x.62 x3x$\sqrt{2x9.81}$ x $[6^{3/2}$---$4^{3/2}]$

=2 x .62 x 4.429 x [14.694—8]

=1.24 x4.429 x6.694= 36.763 m3/s Ans.

Q 9

A rectangular orifice 2.0 m wide 1.5 m deep is discharging water from a tank If the water level in the tank 3m above of the top edge of the orifice, find the discharge through the orifice. Take Cd =.6 [Ans 15.40m3/s]



width of orifice=b=2m

d=depth of orifice =1.5 m ,H_1=height of water surface from the top edge p\of orifice =3m, H_2= 3m+ depth of orifice=3m +1.5=4.5m ,Cd=.6

Discharge through orifice =Q =Equation 7.8 of Chapter 7

= 2/3 x Cd x b x $\sqrt{2g}$ x $[H^{2/3} - H^{\frac{2}{3}}$]

=2/3 x .6 x 2 x $\sqrt{2x9.81}$ $[4.5^{1.5} - 3^{2/3}$]

=2 x. 2 x2 x 4.429 x [4.5x2.12—3 x1.73]

= .8 x4.429 x [9.54-- 5.19]

= .8 x 4.429 x 4.35

= 15.41 m3/s =discharge rate through orifice.

Q 10

A rectangular orifice ,1m wide and 1.5m deep is discharging water from a vessel. The top edge of the orifice is .8m below the water surface in the vessel. Calculate the discharge through orifice if Cd=.6. Also calculate % age error if orifice is treated as a small orifice.

Type equation here.

d=1.5 m=depth

b=1.m

HI=height of water from top edge of orifice

= .8 m

Cd =.6

H2 =H1 +1.5 m =.8m +1,5m=2.3m

Q= Discharge through orifice

= 2/3 x Cd x b x $\sqrt{2g}$ x [$H_2{}^{3/2}$--$H_1{}^{3/2}$]

=2/3 x .6 x $\sqrt{2\ x 9.81}$ $[2.3^{1.5} - .8.^{1.5}]$

=.4 x 4.429 x 2.773

= 4.912 m3/s

Discharge when considered as small orifice = Q =Cd x A x $\sqrt{2gh}$

A=b x d = 1.5 x1=1.5m2

,h =H_1+d/2=.8 +1.5/2=1.55m

So Q = .6 x 1.5 x $\sqrt{2x9.81x\ 1.55}$

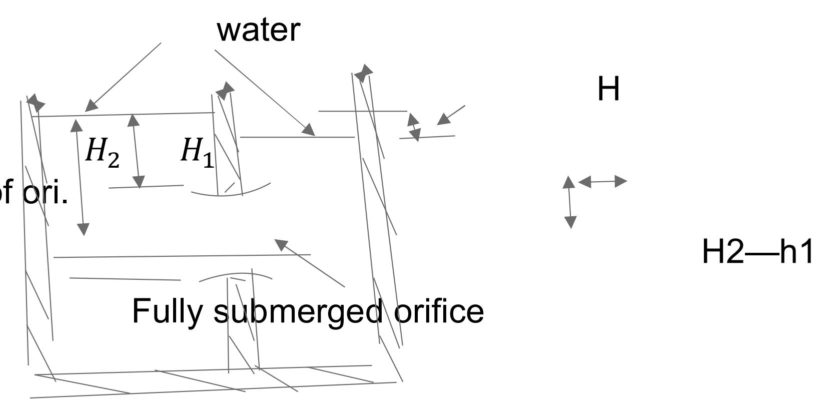
=.6 x 1.5 x 5.5146 =4.963 m3/s

Type equation here.

% error $=\frac{dischage\ as\ small\ orifice--dischrge\ as\ big\ orifice}{discharge\ as\ big\ orifice}$

$=\frac{4.963-4.912}{4.912} = \frac{.051}{4.912}$ x 100 =.01038 = 1.038%

Q11 Find the discharge through a fully submerged orifice of width 2 m if the difference of water level on both sides of the orifice is 800 mm. The height of water from top and bottom of the orifice are 2.5 m and 3 m respectively. Take Cd=.6 [Ans 2.377m3/s]



$H_1 = 2.5m \qquad H_2 = 3m$

Cd=.6 H=Difference of water level=.8m

Width of orifice =b = 2m, Area=$(H_{2-}—H_1)$ x b

Q =Discharge from fully submerged orifice

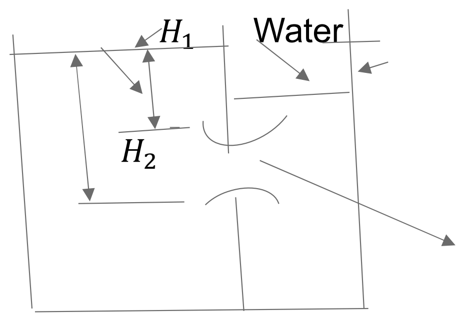
= Cd x b x $(H_2 - H_1)$ b x $\sqrt{2x9.81xH}$

$= .6 \times 2 \times (3—2.5) \times \sqrt{2\ 9.81\ x.8}$

$= .6 \times 3.96 = 2.377$ m3/sec.

Q 12

Find the discharge through a totally drowned orifice 1.5 m wide and 1 m deep. If the difference of water level on both sides of orifice be 2.5m. Take Cd=.62



H =2.5m

Orifice (b x d=1.5m x 1m)

Discharge through orifice

$= Cd \times b \times d \times \sqrt{2gH}$

$= .62 \times 1.5 \times 1 \times \sqrt{2x9.81x\ 2.5}$

$= 6.513$ m3/s

Q 13

A rectangular orifice of 1.5 m wide and 1.2m deep is fitted in one side of the large tank. The water level on one side of the orifice is 2 m above edge of orifice ,while on the other side of the orifice the water level is .4 m below the top edge. Calculate the discharge through the orifice if Cd=0.62[Ans 7.549 m3/s] since this question is partially submerged orifice ,hence the discharge will follow the following formulae

Q= Q1 +Q2

Q1 =Cd x b x(H2—H1) $\sqrt{2gH}$

Q2= 2/3 x Cd x b x $\sqrt{2g}$ x $(H^{3/2} - H_1^{3/2})$

$H_1 = 2m$ H_2= 2m+1.2m =3.2m

H =2 m +.4m=difference of water level

Applying above formula

Q1 =.62 x 1.5 x (3.2—2.4) x $\sqrt{2\ x\ 9.81\ x 2.4}$

=.62 x 1.5 x .8 x 6.86

= 5.10384 m3/s

Q2 =2/3 x .62 x (1.5 x $\sqrt{2\ x 9.81}$ x $(2.4^{3/2} - 2^{\frac{3}{2}})$

=.66 x .62 x 1.5 x 4.429 x (3.71—2.82)

= .66 x .62 x1.5 x 4.429 x .89

= 2.41948 m3/s

So total discharge Q = Q1+Q2

=5.10384 +2.41948

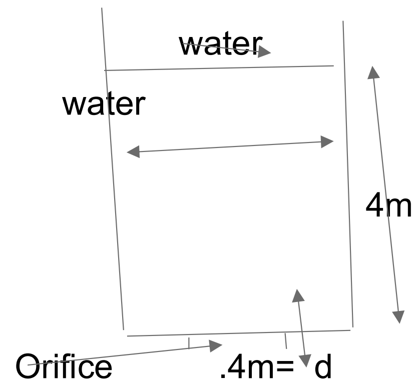
= 7.523 m3/sec

Q 14

A circular tank of diameter 3 m contains water up to 4 m The tank is provided with an orifice of diameter.4m at the bottom Find the time taken by water i) to fall from 4 m to 2m ii) for complete emptying the tank. Cd =.6 [Ans I.24.85S,II.84.75S]

Solution

H_1 =4m H_2= 2m



3m=tank diameter

4m = water height

2m = height after fall

Area of tank = π/4 x (3 x 3)=7.06 m2

Area of orifice = π/4 (.4x.4) =.1256 m2

i)

time to fall of level from 4 m to 2m

$=(\sqrt{H_1} --- \sqrt{H_2})$ X $\frac{2A}{Cd.xa\,x\sqrt{2g}}$

$=\frac{2x7.06}{.6x.1256x\sqrt{2x9.81}}$ x [$\sqrt{4}$ -- $\sqrt{2}$]

$=\frac{8.3308}{.3337}$ = 24.96 seconds.

ii)

for emptying the tank

$$= \frac{2A\sqrt{H_1}}{Cd.a\sqrt{2g}.} = \frac{2x\ 7.06x\sqrt{4}}{.6x.125 \qquad .429}$$

$$= \frac{4x7.06}{.3337}$$ =84.62 seconds.

Q 15

A circular tank of diameter 1.5 m contains water up to a height of 4 m . An orifice of 40mm diameter is provided at its bottom . If Cd=.62 find the height of water above the orifice after 10 min. [Ans 2m]

Solution

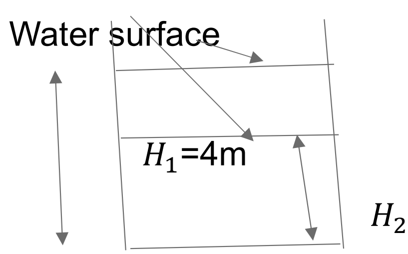
Given

Area of tank = π/4 x 1.5 x 1.5 =1.767 m2

Area of orifice = π/4 x (.04 x.04) =.001256m2

$$Time\ = 10min\ = 600\ seconds. H_1 = 4m$$

H_2 =? after 10 Minutes .



Circular tank

T=Time = $\frac{2A}{Cd\ xa\ x\ \sqrt{2g}}$ x $(\sqrt{H_1} - \sqrt{H_2})$

600 S= $\frac{2x1.767}{.62\ x.001256\ x\sqrt{2x9.81}}$ $(\sqrt{4} - \sqrt{H_2})$

600 =1039 (2--$\sqrt{H_2}$)

$$\sqrt{H_2} = 2 - \frac{600}{1039} = 2 - .5774 = 1.422$$

$$H_2 = (1.422)^2 = 2.02 \text{ m}$$

Height of water after 10min =2.02 meter

Q 16

A hemispherical tank of diameter 4m contains water up to a height of 2.0 m . An orifice of diameter 50mm is provided at the bottom . Find the time required by water i)to fall from 2m to 1m

Ii) to completely emptying the tank. Take Cd .6. [Ans 30min 14.3s

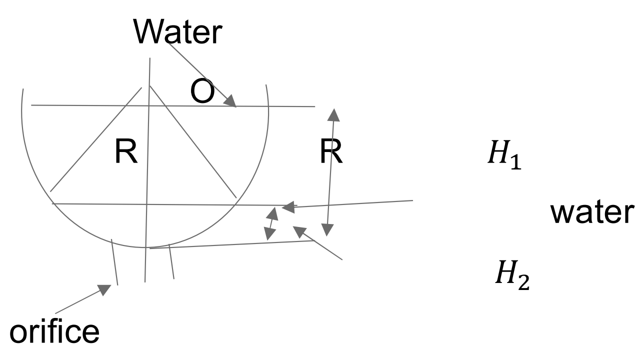
(ii)52min 59s]

Solution

D=4m, R=2m, orifice d i a =d= 50 mm

Area of orifice = a= .001963m2, H_1 = 2m, H_2 = 1m

Cd=.6



case i Time to bring layer of water H1 to H2 level through orifice

$$T = \frac{\pi}{Cd \times a \times \sqrt{2g}} [4/3 \times R \times (H_1^{3/2} - H_2^{\frac{3}{2}}) - 2/5(H_1^{\frac{5}{2}} - H_2^{\frac{5}{2}})]$$

$= \frac{\pi}{.6\ x\ .001963\ x\sqrt{2g}}$ [4/3 x 2 $(2^{3/2} - 1^{\frac{3}{2}}) - 2/5(2^{\frac{5}{2}} - 1^{\frac{5}{2}})]$

= 601.95(2.66x 1.828 -- .4 x 4.656)

= 601.95 x 3 = 1805.85 seconds=30.0975 min

= 30 min 5.85seconds.

Case ii

Emptying the tank i. e H_2=0

T = $\frac{\pi}{Cd\ xa\ x\ \sqrt{2g}}$ [4/3 x R x $H_1^{3/2} - \frac{2}{5} x\ H_1^{\frac{5}{2}}$]

= $\frac{\pi}{.6x.001963\ x\sqrt{2\ x9.81}}$[4/3 x 2 x $2^{3/2} - \frac{2}{5} x2^{\frac{5}{2}}$]

=601.95 [7.522---2.262]

= 601.95 x 5.26

=3166.257 seconds

=52.77 min =52 min 46.2 seconds.

Q 17

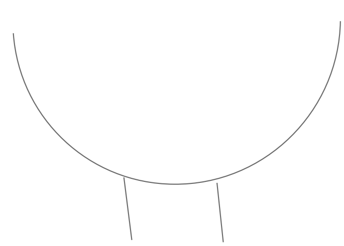
A hemispherical cistern of 4m radius is full of water . It is filled with 60 mm diameter sharp edged orifice at the bottom . Calculate the time required to lower the level in the cistern by 2 m. Cd=.6

[Ans 58m 45.9 s]

Solution

R=4m , diameter of orifice= 60 mm , Cd=.6

H_1 =4m H_2 = 2m



orifice →

Hemisphere

Area of orifice= π/4 (.06x06)=.002827 m2

To find time to fall of water for 2m through orifice

$$= \frac{\pi}{Cd\ x\ a\ x\ \sqrt{2g}} \text{[4/3 R } (H_1^{3/2} - H_2^{\frac{3}{2}}) - 2/5(H_1^{\frac{5}{2}} - H_2^{\frac{5}{2}}]\text{)]}$$

$$= \frac{\pi}{.6\ x\ .002827\ x\ \sqrt{2x9.81}} \text{[4/3x 4 x } (4^{1.5} - 2^{1.5}) - (4^{2.5} - 2^{2.5})]$$

= 418.20 [5.333x (8—2.828)— .4(32—5.656)]

=418.20[5.333 x 5.172 --.4 x 26.344]

=418.20[27.58 –10.5376]

=418.20 x 17.04 =7126.12 seconds

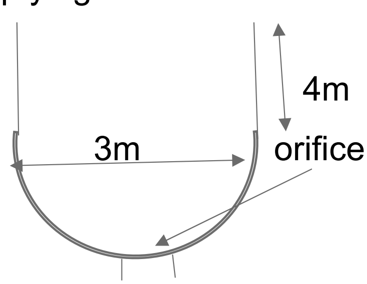
=118.768 min =1.979 hour

= 1hr 58 min 44.4 sec.

Q 18

A cylindrical tank is having a hemispherical base. The height of cylindrical portion is 4 m and diameter is 3 m . At bottom of this tank an orifice of diameter 300mm is fitted. Find the time required completely emptying the tank. Take Cd=.6 [Ans 2m7.37s]

Solution



Height of tank=4m =H

Diameter of tank =3m R=1.5m

Diameter orifice =300mm=.3m

Tank is split into two .

Time for cylindrical potion = T2

Time for hemispherical portion =T1

H_1 =1.5.0m ,H_2=0

$$T_1 = \frac{\pi}{Cd\ x\ a\ x\sqrt{2g}} [4/3 \text{ x R x } H_1^{3/2} - \frac{2}{5} H_1^{5/2}]$$

$$= \frac{\pi}{.6\ x\ .0706\ x\sqrt{2x9.81}} \text{x } [4/3 \text{ x } 1.5 \text{ x } 1.5^{1.5} - \frac{2}{5} x 1.5^{2.5}]$$

= 16.746[3.724—1.102] =16.746 x2.622=43.90 s

=43.9seconds.

Time to emptying the cylinder =T2

$$= \frac{2A[\sqrt{H_1} - \sqrt{H_2}}{Cd\ x\ a\ x\sqrt{2g}} = \frac{2x7.06\sqrt{5.5}\ -\sqrt{1.5}}{.6\ x\ \ .0706\ x4.429} -$$

$$= \frac{17.2264}{.1876} \text{ =91.82 seconds}$$

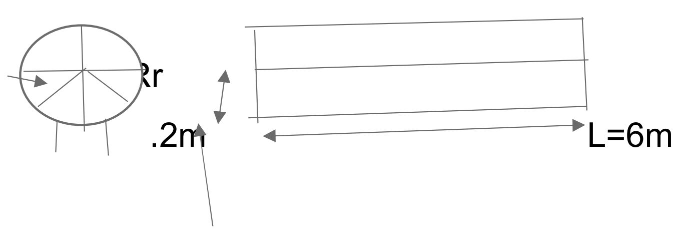
Total time = $T_1 + T_2$ =43.9 s + 91.82s =135.72 s

= 2.262 m=2m15.72s

Q 19

An orifice of diameter 200mm is fitted at the bottom of a boiler drum of length 6m and of diameter 2 m. The drum is horizontal and half full of water. Find the time required to empty the boiler given the value of Cd=.6 [Ans 2min 55.20 s]

Solution



Boiler drum

Orifice diameter =.2 m   R = Drum radius=H_1=1m

Area of orifice =a= π/4 x .04 =.0314 m2

Cd=.6

T =time required for emptying the drum (half full)

$$= \frac{4L}{3\ Cd\ x\ a\ x\sqrt{2g}}\ [\ (2R)^{\frac{3}{2}} - (2R - H_1)^{1.5}]$$

(7.16Eqan)

$$=\frac{4x6}{.0314\ x3\ x.6\ x\sqrt{2x9.81}}\ [\ (2x\ 1)^{1.5} - (2x1 - 1)^{1.5}]$$

$$= \frac{24}{.05652x4.429}[\ 2.828---1]$$

$$=\frac{24}{0.2503}x1.828=95.88\ x1.828$$

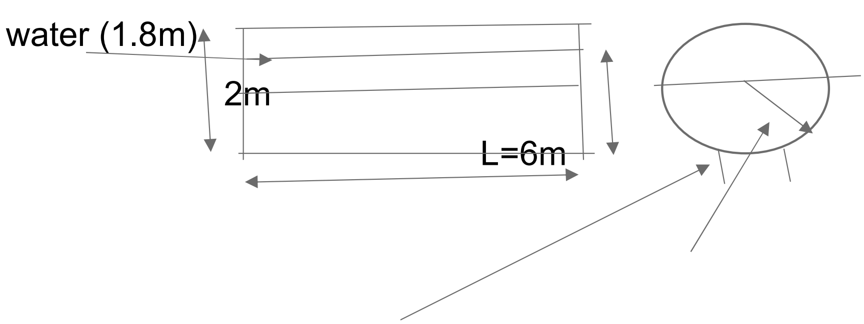
= 175.268 sec.

=2.92 min

=2min 55.2 sec.

Q 20

An orifice of diameter150mm is fitted at the bottom of a boiler drum of length 6m and of diameter 2m. The drum is horizontal and contains water up to a height 1.8m. Find the time required to empty the boiler. Cd=.6 [Ans 7min 46.64s]



Boiler drum

R=Radius

=1m

orifice (dia.=.15m)

D=Diameter =2m

Cd=.6

Area of orifice = π/4(.15 x .15)= .0176m2

1.8m =H_1

To find time T to empty drum from 1.8m (equation7.16)

$$= \frac{4L}{3Cd\ x\ a\ x\sqrt{2g}}\ \text{x}\ [(2\text{R})^{3/2} - (2R - H_1)^{1.5}]$$

$$= \frac{4\ x\ 6}{3\ x.6\ x.0176\ x\sqrt{2x9.81}}\ [(2\ \text{x}1)^{1.5} - (2\ x\ 1 - 1.8)^{1.5})]$$

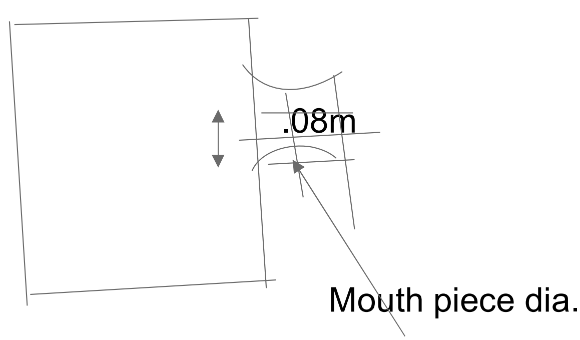
$$= \frac{24}{1.8\ x.0176\ x4.429}\ (2.828\text{--}.089\ )$$

$$= \frac{65.736}{.140}\ = 469.54\text{sec}$$

= 7min 49.5 sec.

Q21

Find the discharge from 80mm diameter external mouthpiece ,fitted to a side of a large vessel if head of over the piece is 6m .



Large vessel

Area of mouth piece =π/4 x (.08 x .08)

=π x .0016 m2

Cd = Cc x C v=1 x .855 =.855

H =HEAD =6 m

Discharge through the mouthpiece

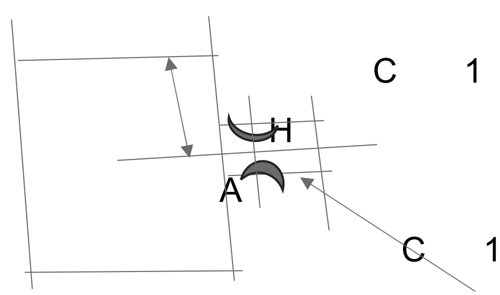
=Cd x Area of mouth piece x $\sqrt{2gH}$

= .855 x (π x .0016)x $\sqrt{2 \; x \; 6 \; x \; 9.81}$

= .00429 x $\sqrt{117.72}$ = .04654 m3 /sec

Q22

An external cylindrical mouthpiece of diameter 100mm is discharging water under constant head of 8 m . Determine the discharge and absolute pressure head of water at vena contract. Take absolute head 10.3m of water [Ans .084m3/s,3.18m]



External cylindrical mouthpiece

H = Water head at A =8 m

Mouth piece diameter = 100mm=.1m

Do area =π/4 x (. 1 x. 1) = .00785m2 =a

Cd=.855, Cc=.62, Ha=10.3 m

i) discharge at vena-contract=Cd x a x $\sqrt{2gH}$

= .855 x .00785 x$\sqrt{2\ x 9.81 x 8}$

=.855 x.00785 x12.528

=.084m3/s

ii) Absolute pressure head at vena-contract

As per Bernoulli's equation ;

$$\frac{P_A}{\rho g}+\frac{v_a^2}{2g}+\ z_A\ =\frac{p_c}{\rho g}+\frac{v\ _c^2}{2g}+z_c$$

But $p_A/2g$ =Ha+ H.$Va = 0$

Also H=pressure head at A

Va= velocity at out let

Za=Z c

$\therefore$ Ha+H+0= ($\frac{Pc}{\rho g}+\frac{v_c^2}{2g}$=H c+$v_c^2/2g$

$\therefore$ H c=Ha +H--$-\frac{V_C^2}{2g}$

But V c= $\frac{V1}{.62}$

$\therefore$ H c =Ha+ H --($\frac{V_1}{.62})^2 X1/2g$

But H=1.375$\frac{V_1^2}{2g}$

$\frac{V_1^2}{2G}=\frac{H}{1.375}$ =.7272H

Therefore H c=Ha+ H--.7272H x 1/.62 x.62

= Ha---.89 H

=10.3---.89 x 8

=10.3—7.12

=3.18 (absolute)

Q23.

A convergent -divergent mouth piece having throat diameter 60mm is discharging water under a constant head of 3 meter. Determine the maximum outer diameter for maximum discharge Find maximum discharge also. Take atmospheric pressure head 10.3m of water and separate pressure head of 2.5m of water absolute [6.88 cm, Q max =0.01596m3/s]

Dc =diameter of throat = 60mm =.06 m

Ac =Area of throat = π/4 x (.06 x.06)=.002826m2

H=Constant head =3 m

H s e p =2.5 m, Ha=10.3m

Q max= Ac x $\sqrt{2gH}$ =.002826 $\sqrt{2\ x\ 9.81\ x\ 3}$

= .002826 x $\sqrt{58.86}$

= .002826 x 7.672

= .02168 m3/s

Let Ao =area of outlet and Do=diameter of outlet

As per ratios of areas of outlet and throat by equation (7.17)

$$\frac{Ao}{Ac} = \sqrt{1+\frac{Ha-Hs}{H}}$$

$$(\frac{Do}{Dc})^2 = \sqrt{1+\frac{10.3-2.5}{3}} = \sqrt{3.6} = 1.897$$

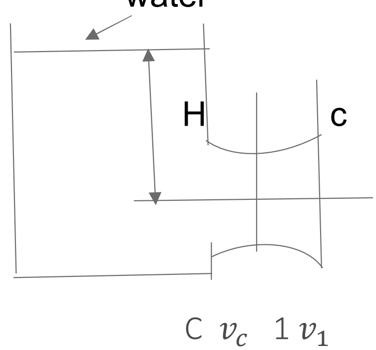
Do=$\sqrt{1.897}$ x D c= 1.377 x 6cm=8.262cm

= maximum diameter for maximum discharge.

Q 24

Throat and exit diameter of a convergent -divergent mouthpiece are 40 mm and 80 mm respectively. It is fitted to thr vertical side of a tank ,containing water. Find the maximum head of water of steady flow. The maximum vacuum pressure is 8m of water. Take atmospheric pressure head 10.3m of water. [Ans .533m]

Solution



c=throat 1 =outlet.

H=Constant head

Convergent-divergent mouth piece

Dia of throat =dc =4 cm

Dia of exit = de =8cm

Atmospheric pressure =Ha =10.3 m

Max, vacuum pressure at throat only .

Pressure head at throat =8m (vacuum)

Or, H c= Ha—8m

=10.3—8 =2.3m(abs.)

Max head of water over mouth piece = H m of water

Ratio of areas a_1 and a_c ,at outlet and throat of con-div mouth piece as given in equation (7.17)

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}}$$

$$\frac{\pi/4(d_{1})^2}{\pi/4{d_c}^2} = \sqrt{1 + \frac{10.3-2.3}{H}}$$

$$\left(\frac{d_1}{d_c}\right)^2 = \sqrt{1 + \frac{8}{H}}$$

16 =1 +8/H

H =8/15 = ,533 m =max head of water

over mouth piece.

END OF CHAPTER 7

Fluid Mechanics and Hydraulic Machines

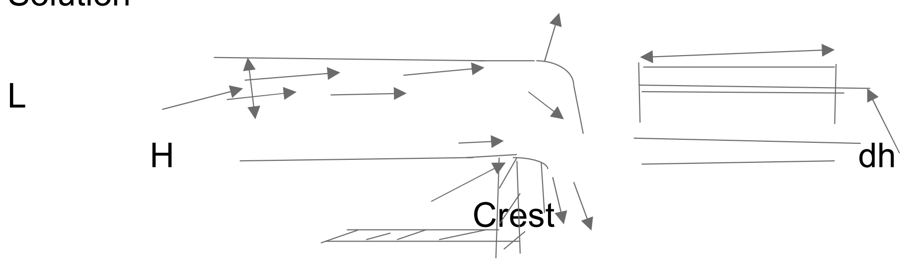
Notches and Weirs

Chapter 8

Q1

Find the discharge of water flowing over rectangular notch of 3m length when the constant head of water over the notch is 40cm. Take Cd=0.6 [Ans 1.344m3/s]

Solution



Rectangular notch or weir

Q =Discharge = ?

L=length of notch=3m

H= constant head of water =.4m

Cd=.6

Q =Discharge for whole notch (equation-8.1)

$= 2/3 \text{ Cd x L x } \sqrt{2g}\ \ (H)^{3/2}$

$=2/3 \text{ x } .6 \text{ x } 3 \text{ x } \sqrt{2x9.81} \text{ x } (.4)^{1.5}.$

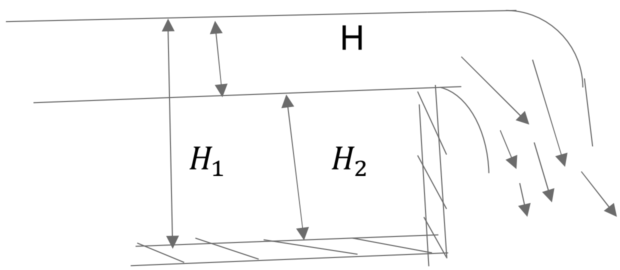
=1.2 x 4.429 x .2529

=1.344 m3/s

Q 2

Determine the height of a rectangular weir of length 5m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.5 m and discharge is 1.5m3/s. Take Cd =.6 and neglect contraction.[Ans 1.194m]

Solution



$H_1 = Max\ depth\ of\ water = 1.5m$,

$H_2 = height\ of\ recangular\ weir = ?$

H = Width of weir = ?

L= length of weir =5m

Cd =.6

Q = Discharge = 2/3 Cd x L x $\sqrt{(2g)}$ x $(H)^{1.5}$

1.5 = 2/3 x .6 x 5 x 4.429 x $(H)^{1.5}$

= 4 x 4.429 x $(H)^{1.5}$

$(H)^{1.5} = \frac{1.5}{8.8588} = .16932$

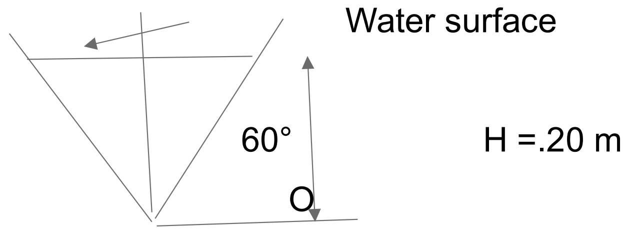
H= $\sqrt[1,5]{.16932}$ = .306m

H_2= H_1—H=1.5 --.306= 1.194 m

Q3

Type equation here.

Find the discharge over a triangular notch of angle 60°when the head over the triangular notch is 0.20 m, Take Cd=0.6 [Ans .0164 m3/s]



Triangular notch

As per equation no 8.2

Q =Discharge of triangular notch

$= \frac{8}{15} x Cd\ x tan\ \frac{\theta}{2}\ \sqrt{2g} x H^{5/2}$

$=8/15 \times .6 \times \tan 30° \times \sqrt{(2 \times 9.81)} \times (.2)^{2.5}$

=.5333 x .6 x .5773 x 4.429 x .01788

= .81777 x .017888

= .0146 m3

Q 4

A rectangular channel 1.5 m wide has a discharge of 200lit /s which is measured by a right angled V notch weir. Find the position of apex of notch from the bed of the channel. if maximum depth of water is not to exceed 1 m [Ans .549m]

Solution

Width of rectangular channel= 1.5m

Q =Discharge = 200lit/s=.2m3/s

H_1=Depth of water =1m

To find height of water over V notch i .e H

Type equation here.

Θ=90°(right angled notch)

By equation of (8.2)

$$Q = \frac{8}{15} Cd\ x\sqrt{2g}\ x\tan\frac{90}{2}\ xH^{5/2}$$

$$.2\ \ =.533\ x\ .6\ x\ \surd(2\ x9.81)\ \ x\ 1\ x\ H^{2.5}$$

$$H^{2.5} = \frac{.2\ X\ 15}{8\ X.62\ X4.429} = \frac{3}{21.967} = .13656$$

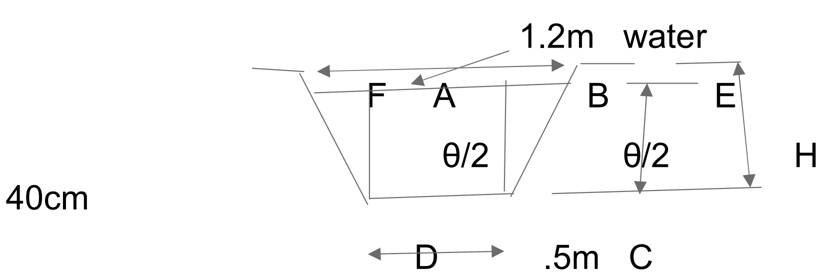
$$H = \sqrt[2.5]{.13656} = .450$$

So position of apex of notch from the bed of channel = depth of water of water in channel over V notch

= 1m -- .450m = .55m

Q 5

Find the discharge through a trapezoidal notch which is 1.2m wide at the top and 0.5m at the bottom and is 40 cm in height. The head of water on the notch is 30cm .Assume Cd for rectangular portion .62 and for triangular portion .6[Ans ..228m3/s



Solution

AB=DC=.5m

FA=BE=.70/2=.35 m

Height of notch=.4m

Head of water =.3m =H

Cd1=.62 for rectangle   Cd2=.6 (triangle)

Tan θ/2 = FA/AD=.35/.3 =1.166

We will find discharge of both triangles and rectangles by using the following

= Q for rectangle + Q for triangles

= 2/3 x Cd1 x L x √(2g) x $H^{3/2}$

+8/15 Cd2 x tan θ/2√(2g) x ($H^{5/2}$)

=.66 x .62 x.5 x $\sqrt{2x9.81}$ x(.3)$^{1.5}$+.533x.6x1.166x$\sqrt{2x9.81}$ x$(.3)^{2.5}$

= .66 x.62 x.5 x 4.429 x .1643+.533 x.6 x1.166 x4.429 x .0492

= 0.14888+.0812 =.230 m3/s

Q 6

A rectangular notch 50 cm long is used for measuring a discharge of 40 lit/sec. An error of 2mm was made in measuring the head over the notch . Calculate the % error in discharge . Take Cd=.6. [A 2.37%]

Solution

Q =40 Lit/s =40000cm3/s%

L=50 cm =.05m

Cd=.6

Error =d H=2mm=.2cm = .002m

Q =2/3 Cd x L x √(2g) x$H^{3/2}$

40000   =2/3 x .6 x 50 x √(2 x 981) x $H^{1.5}$

$$H^{1.5} = \frac{40000\ x3}{2x\ .6\ x50\ x\ 44.294} = \frac{120000}{2657.64}$$

= 45.15

H =$(45.15)^{2/3}$=12.36 cm

Using equation (8.6) ,% age error is as follows ;

$\frac{dQ}{Q} = \frac{3}{2}\ x\ \frac{dH}{H}$ =1.5 x $\frac{.2}{12.36}$ =0.0242 = 2.42 %

Q 7

A right angled V notch is used for measuring a discharge of 30 lit /s An error of 2mm was made in measuring the head over the notch. Calculate the %age error in discharge Cd=.62 [An;2.37%]

Angle of V notch =90°=θ

Discharge =30 lit/s=30000 cm3/s

Error in Head while measuring=.2 cm =Dh

Let the head over V notch=H.

Cd= ,62

Discharge = 8/15 Cd x tan(90/2) x √(2g))x $H^{5/2}$H

30000 = 8/15 x .62 x1 x 44.29 x $H^{\frac{5}{2}}$

$H^{5/2}$ =$\frac{30000\ x\ 15}{8\ x\ .62\ x\ 1\ x\ 44.29}$ =2048.44

H =$(2048.44)^{\frac{2}{5}}$=21.114 cm

Using equation 8.7

$\frac{dQ}{Q}$= 5Dh/2H= $\frac{5X\ .2}{2X\ 21.114}$ =.02368 =2.368 %

II To find limiting error/value of head.

$\frac{dQ}{Q}$=2.36%=5/2 x $\frac{dH}{H}$

∴d h= .236/2.5 x H=.0236/2.5 x 21.114=.1993

Limiting value of Head =21.114 ±.1993 =21.31 , 20.91

Q 8

Find the time required to lower the water level from 3m to 1.5m in a reservoir of dimension 70m x 70m by rectangular notch of length 2.0 m. Take Cd=.6 [Ans; 11m 1 s]

Initial height =3m =H_1

Final height =1.5m =H_2

Area of reservoir = 70 x70 =4900 m2

Length of notch =2m , Cd=.6

As per equation (8.8)

T =Time for lowering water level

$= \frac{3A}{CdxLx}\frac{}{\sqrt{2g}}[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}]$

$=\frac{3X4900}{.6X2X\sqrt{2x9.81}} [\frac{1}{\sqrt{1.5}} - \frac{1}{\sqrt{3}}]$

$= \frac{14700}{5.31} [\frac{1}{1.2247} - \frac{1}{1.732}]$

=2768.36 (.81499--.57736)

=2768.36x .23763

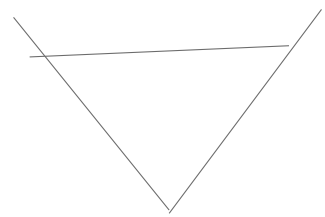
=10min 57.845 sec.

Q9

If in the problem 8, instead of rectangular notch a right angle V notch is used find the time required Take all other data same.

[Ans 13min 31s]

Solution



Θ=90°

Triangular notch.

T =time to lower the water level from 3m to 1.5m

$$= \frac{5A}{4Cd.tan90/2x\sqrt{2g}}x[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}]$$

$$=\frac{5X490}{4x\ .6x1x\sqrt{2x9.81}}[\frac{1}{1.5^{1.5}} - \frac{1}{3^{1.5}}]$$

=2304.88 [.5445--.1924]

= 2304.88 x .3521

= 811.54 seconds

= 13min 31 sec.

Q10

Water is flowing in a rectangular channel 1.2m wide and .8m deep . Find the discharge over a rectangular weir of crest length of 70 cm if the head of water over the crest of weir is 25 cm and water from the channel flows over the weir. Cd=.6, neglect end contraction but consider velocity of approach. [0.1557m3/s]

Solution

A = Area of channel =width x depth =1.2m x ,8m=.96m2

L =length of weir=70cm=.7m

H_1=Hea d of water =25cm= .25m

Cd=.6

Discharge over a rectangular weir without velocity of approach= 2/3 x Cd x L x √(2g) x $H_1^{3/2}$

=.66 X .6 X.7 X $.25^{1.5}$=.153464m3/S

Va=velocity of approach= Q/A=.15346/.96

=.1598 m/s

Additional head =ha = V_a^2/2g =$\frac{(.1598)^2}{2x\ 9.81}$ =.0013.

The discharge with velocity approach is by equation (8.10)

Q= 2/3 Cd x L x √(2g) [$(H_1 - ha)^{\frac{3}{2}} - h^{\frac{3}{2}}$]

=.66 x.6 x .7 x √(2 x 9.81)[(.25--.0013)$^{1.5}$—(.0013)$^{1.5}$]

=.2772 x 4.429[(.2487)$^{1.5} - (.0013)^{1.5}$]

= 1.22771 x .123979

=.15221m3/s

Q11 Find discharge 0ver a rectangular weir of length 80m.The head of water over the weir is 1.2m. The velocity of approach is given as 1.5m/s. Take Cd =3.6 [Ans 208.11m3/s]

Solution

L=80m. H_1=1.2m Va=velocity of approach =1.5m/s

Cd=3.6 ha =additional head =v_a^2/2g

$= \frac{1.5\ x1.5}{2\ x9.81}$ =.114679m

Q =Discharge considering velocity approach

=2/3 Cd x L x √(2g) [$(H_1 + ha)^{3/2} - (ha)^{3/2}$]

=.666 x.6 x 80 x 4.429 [(1.2 + ,114679)$^{1.5}$ – $(.114679)^{1.5}$]

=141.586 [(1.314679)$^{1.5}$--.038835]

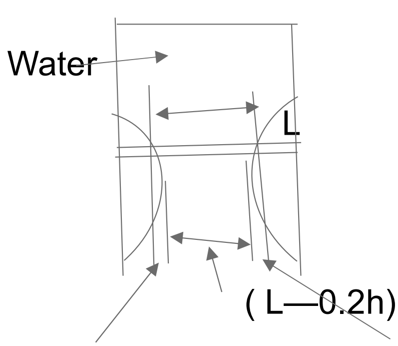
=141.586[1.5074 ---.038835]

=141.586 x 1.468565 = 207.928 m3/sec.

Q12

The head of water over the rectangular weir is 50 cm .The length of the crest of the weir with end contraction suppressed is 1.4m. Find the discharge using following formulae ;

1 Francis formulae 2. Bazin's formulae. [Ans 1.) .91m3/s 2.).901m3/s]



0.1H 0 .1 H

Due to contractions the effective length reduces to

= L –2 x 0.1H =(L --.2H)

H =Head =50 cm=. 5m

L =Length of weir =1.4m

1 Fransis formula for end contractions suppressed is given by (8.12)

$Q = 1.84\ L\ H^{3/2}$ $=1.84 \times 1.4 \times (.5)^{3/2}$

= 1.84 x 1.4 x .35355

= .91m3/s

2 Bazin's formulae

Q =m x L x √(2g) x $H^{3/2}$

Where m =0.405 +$\frac{.003}{H}$=0.405+.003/.5

=.405 +.006 =0.411

So, Q= 0.411 x 1.4 x 4.429 x $(.5)^{1.5}$

= 2.548 x 0.35355 =.9008 m3/s

Q 13

A discharge of 1500m3/s is to pass over a rectangular weir . The weir is divided into a number of openings each of span 7.5 m . If the velocity of approach is 3m/s, find the number openings needed in order the head of water over the crest not to exceed 1,8m. [Ans 37.5 say 3b]

Solution

Velocity of approach= 3m/s

Head of weir =1.8m

Let number openings =N

Head due to velocity of approach

= ha =$\frac{v}{2g}a^2$ = $\frac{3\ x3}{2\ x\ 9.81}$= .4587 m

For each opening contractions are =2.

Hence discharge for each opening is given by Francis 'formula

$$Q = 1.84\,[L - O.I \times 2(H+ ha)]\,[(H+ h)^{\frac{3}{2}} - (ha)^{\frac{3}{2}}]$$

$$= 1.84[7.5 - -.2(1.8+.4587)]\,[1.8+.4587)^{1.5} - (.4587)^{1.5}]$$

$$= 1.84[\,7.5 - -.45174]\,[3.394 - -\ .31]$$

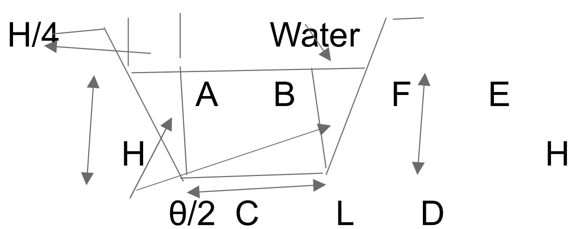
$$=1.84 \times 7.0482 \times 3.084=39.995437$$

$$\text{Number of opening} = \frac{total\ discharge}{discharge\ for\ one\ opening}$$

$$=\frac{1500}{39.9954337} = 37.5 \text{ say 38 nos.}$$

Q14

Find discharge over a Cipolletti weir of length 1.8m when the head over the weir is 1.2 m , take Cd =.62.



Cipolletti weir.

Length of weir =1.8m

Head of weir =1.2 m = H

C d =0.62

Using equation 8.16 for Cipolletti's weir as below;

$$= \ Q = 2/3 \times Cd \times L \times \sqrt{(2g)} \times H^{3/2}$$

$$= .66 \times .62 \times 1.8 \times \sqrt{(2 \times 9.81)}\ (1.2)^{1.5}$$

$$=3.2622 \times 1.314534$$

= 4.2879

=4.288m3/s

Q 15

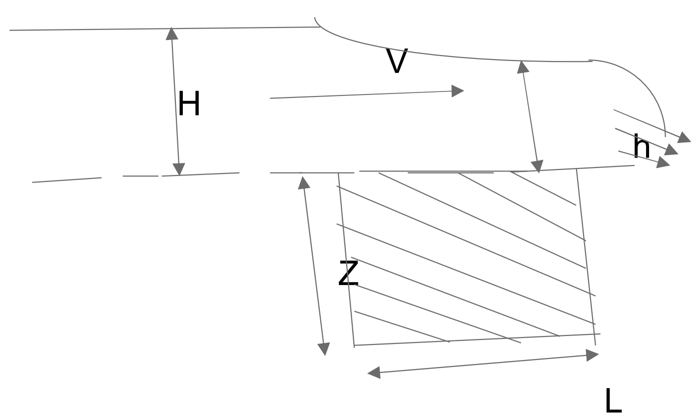
(a) A broad crested weir of length 40m has 400mm height of water about its crest Find the maximum discharge take Cd =.6. neglect velocity of approach[ans;10.312]

(b) If the velocity of approach is taken into account find the max discharge when the channel has a cross section area of 40m2 on the upstream side. [Ans; 10.475m3/s]

Solution

Given

Broad crested weir



A L=40m .H=400mm =.4 m Cd=.6

Equation (8.19)

Q max=1.705Cd L.$H^{3/2}$

$=1.705 \times .6 \times 40\ (.4)^{1.5}$

$= 1.705 \times .6 \times 40 \times .25298$

$=10.3519\text{m}$

B Velocity of approach when taken into consideration

Va= Q/A =10.3519/40= .2588m/s =.26m/s(say)

Head due to Va , ha=$\frac{.26^2}{2g}$ =$\frac{.0676}{2\ x 9.81}$ =.00344m

Q max =1.705 x Cd x L x $[(H+ ha)^{3/2} - (ha)^{3/2}]$

$=1.705 \times .6 \times 40 \times[(.4+.0035)^{1.5} - (.0035)^{1.5}]$

= 40.92(.25625---.0002)

=40.92 x.2560

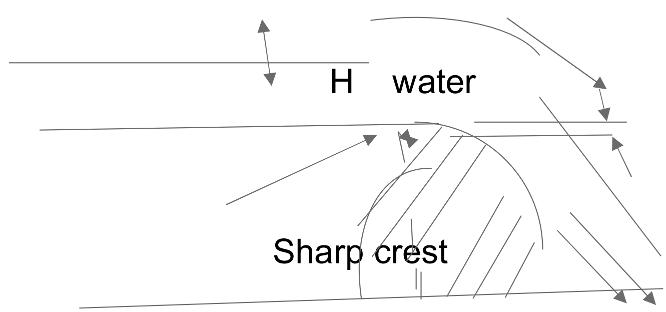
= 10.475 m3/s

Q 16

An Ogee Weir 4 m long has a head of 500 mm of water If Cd =0.6 find the discharge over the weir.
{Ans 2.505m3/s]

Solution

.115 H



.

Ogee Weir.

Length of weir =4m
Head of water =500mm=.5 m

Cd=.6

Discharge over Ogee Weir is given by equation (8.21)

Q =2/3 C d x L x √(2g) x $H^{3/2}$

=.666 x .6 x 4 x √(2 x9.81) x $(.5)^{1.5}$

= 1.59984 x 4.42946 x .35355

=2.5054 m3/sec

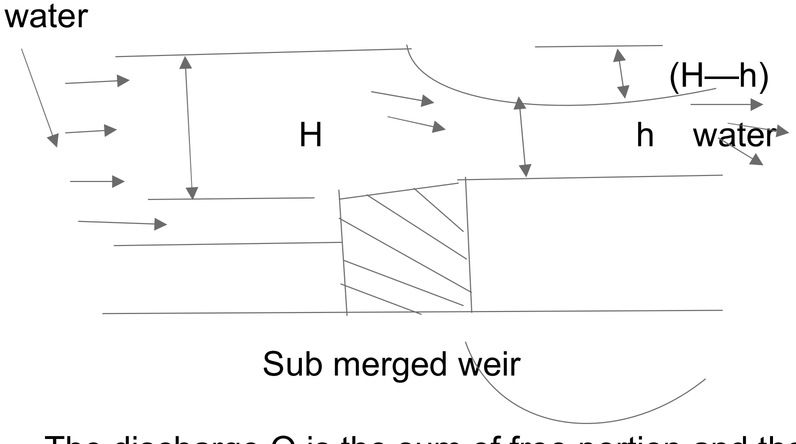
Q 17

The heights of water on the upstream and downstream side of submerged weir of length 3,5m,300mm and 150 mm respectively. If Cd for free and downstream portion are .6 and .8 respectively. Find the discharge over the weir [Ans 1.0807m3/s]

Solution Height of water on upstream side =H= .3m

Do- downstream side =h =.15m

L=Length of weir =3.5m Cd1=.6 for free, Cd2=.8 downstream



Sub merged weir

The discharge Q is the sum of free portion and that of downstream portion.

This is given by equation (8.22)

Q =2/3 Cd1 x L x √(2g)(H--h)$^{3/2}$ +Cd2 x (L x h)$\sqrt{2g(H-h)}$

=.666 x.6 x 3.5 √(2x9.81) (.3--.15)$^{1.5}$

+.8 x (3.5x.15)√$\{2x9.81(.3-.15)\}$

=.666 x.6 x3.5x 4.429x .0581+.8x3.5 x .15 x 1.6 997

= .35989 +.713874

=1.073874 m3/sec.

End of Chapter 8

Type equation here.

Fluid Mechanics and Hydraulic Machines

Viscous Flow

Chapter 9

Q1

A crude oil of viscosity .9 poise and s p gr. .8 is flowing through a horizontal circular pipe of diameter 80mm and of length 15 m. Calculate the difference of pressor at the two ends of the pipe if 50 kg of oil is collected in a tank in 15 seconds [Ans .559 N/cm2]

Solution

Given

,μ =.9 poise =.9/10=.0.09

S p gr=.8, ρ =density =.8 x 1000 =800

D= dia. Pipe =80mm =.08 m

L= length of pipe =15m.

To find difference of pressure = P1 –P2

$\bar{u}$ =average velocity =$\frac{Q}{AREA}$ = .004/ π/4 x $(.08)^2$

= $\frac{.004\ x4}{\pi x\ .0064}$ =.816 m/s

[Q =Mass of oil/s = $\frac{50}{15\ x800}$ =.004m3/s]

For Laminar flow or viscous flow Re number should be <2000

So Re=$\frac{\rho vD}{\mu}$ =$\frac{.8\ X1 \quad X.816\ X.08}{.09}$ = 580.26.<2000

∴ flow is laminar or viscous flow.

Pressure difference at 2 ends of circular pipe

= P1—P2 =$\frac{32\ x\mu\ xL\ x\ \bar{u}}{D^2}$

=$\frac{32\ x\ .09\ x\ 15\ x.816}{.08\ x.08}$

= .5508 N/cm2

Q 2

A viscous flow is taking place in a pipe of diameter 100mm. The maximum velocity is 2 m/s. Find the mean velocity and the radius at which this occurs Also calculate the velocity at 30 mm from the wall of the pipe [Ans 1m/s, rad =35.35, $\bar{u}$ =1.68 m]

Solution

Given

1 Diameter of pipe=R =100mm =.1m

U max=2m/s , $\bar{u}$ =U max/2 =2/2 =1m/s

Let r radius at which mean velocity occurs

velocity U $=-\frac{1}{4\mu}x\frac{dp}{dx}[R^2-r^2]$

$=-\frac{1}{4x}x\frac{dp}{dx}R^2[1-(\frac{r}{R})^2]$

From equation (9.4)

U max =--1/4γμ x d p/dx x R^2

So from above

U=U max [1—(r/R)^2

U=$\overline{U}$ =.75

$.75=1.5[1-(\frac{r}{D/2})^2]$

$=1.5[1-(r/.1/2)^2]$

$=1.5[1-\frac{r^2}{.0025}]$

$(\frac{r}{.05})^2=1-\frac{.75}{1.5}=.5$

r/.05 =√.5 =.7071

r =.7071 x .05 =.03535 m

=.03535 x1000 mm=35.35 mm

iii. to calculate velocity at 30mm from wall of the pipe that is (50mm—30mm)=20 mm from center of pipe

so velocity = U max[1—(r/R)^2]

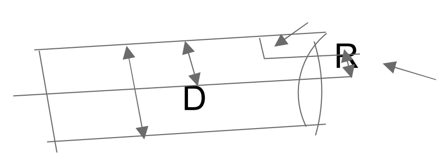
$=1.5[1-(20x1000/50x1000)^2]$

$=1.5(1-(.4)^2]=1.5x(1-.16)$

=1.5x .84

=1.26m/s

30mm



20mm=r

R=50mm

Pipe

Q 3

A fluid viscosity .5 Ns/m2 and specific gravity1.2 is flowing through a circular pipe of diameter 100mm. The max shear stress at the pipe wall is given as147.15N/m2, find a) the pressure gradient b) average velocity c) Reynold number of the flow.

[Ans a)—5886 N/m2 b) 3.678m/s c)882.72]

Solution

Given

(a)

Pressure gradient =dp/dx =P1 –P2

,τ=-- $\frac{dpxR}{dxX2}$

147.15 =– $\frac{dP}{dx}X\frac{.05}{2}$

$\frac{dp}{dx} = -\frac{147.15x2}{.05}$

= --5886 N/m2 =pressure gradient

(b) Average velocity $\overline{U}$ =1/2 x U max

=1/2 x [--$\frac{1}{4\mu}$ x$\frac{dp}{dx}R^2$

=1/8 x$.05^2 x\frac{5886}{.5}$

=3.678 m/s

(c) Reynold number = $\frac{\rho VD}{\mu}$ =$\frac{(1.2\ X1000)X3.678\ x\ .i}{.5}$

=$\frac{1200\ x3.678\ xc.1}{.5}$

=882.72

Q 4

Determine a)pressure gradient b)shear stress at the two horizontal parallel plates and c) the discharge /m width (ω)for the laminar flow of oil with maximum velocity of 1.5 m/s between two parallel fixed plates which are 80mm apart. Take viscosity of oil as 1.962 Ns/m2.[Ans a) 3678.7N/m2 b)147.15N/m2 c).08m3/s]

Solution

Given

To find

a) pressure gradient =d p/dx

b) shear stress at the wall = τ

C)discharge /m width =Q

U max=1.5 m/s, , μ =1.962 Ns/m2

t =distance between 2 plates=80mm=.08 m

a) U max =-- $\frac{1}{8\mu}\ x\frac{dp}{dx}\ xt^2$

1.5=---$\frac{1}{8\ x1.962}\ x\frac{dp}{dx}\ x(.08)^2$

$\frac{dp}{dx} = -\frac{1.5\ x8\ x1.962}{.08\ x.08}$

= --- 3678.75N/m2 per min

b) Shear stress τ = --1/2 x$\frac{dp}{dx}$ x t

= ---1/2 x {--3678.75) x.08

= 147.07N/m2

c) Discharge perm width for laminar flow

Q=Mean velocity x area

= 2/3 x U max x (t x width)

=2/3 x 1.5 x (.08 x 1m)

= 0.08 m3/s

Q 5

Water is flowing between two large parallel plate which are 2.00 mm apart .Determine a) maximum velocity b) pressure drop/unit length and c) shear stress at walls of the plates if the average velocity is .4m/s. Take viscosity of water at .oi poise.

[Ans a) .6m /s b) 1199.7N/M2 N/m2 c)1.199N/m2]

Solution

Given

, t = distance between two plates =2mm =$2\ x 10^{-3}\ m$

Average velocity =$\bar{u}$ =.4 m/s

,μ=.01poise =.01/10 =.001 Ns]m2

L= 1m

a) Maximum velocity = U max=3/2 x $\bar{U}$

=1.5 x .4 =0.6 m/s

b) Pressure drop/m =$(p_1 - p_2)$ = $-\frac{12\mu L\bar{U}}{t^2}$

=-- $\frac{12\ x .001\ xi\ x .4}{..002^2}$

=---1200N/m2

c) Shear stress at the wall

=τ =--1/2 x d p/dx x t

=(---1/2) x (--1200) x .002

= 1.2 N/m2

Q 6

There is a horizontal crack 50 mm wide and 3mm deep in a wall of thickness 150mm Water Leaks through the crack Find the rate of leakage of water through the crack if the Type equation here.difference of pressure

between the 2 cracks is 245.25N/m2.Take the viscosity of water as .01 poise.
[Ans 183.9cm3/s]
Solution

Given

. b =width of crack =50mm =.05m

. t =depth of crack =3mm=.003m

L= length of crack =150mm =.150m

Pressor drop = P1—P2 =245.25N/m2

.μ of water =.01 poise =.001 Ns/m2

To find Q of leakage ;----

$$p_1 - p_2 = \frac{12\mu L\bar{U}}{t^2}$$

$$\bar{u} = \frac{245.25\ x(.003)^2}{12\ x.001\ x\ .15}$$ = 22/18 =1.222 m/s

Q =Rate of leakage (discharge)

= $\bar{u}\ xArea\ of\ cross\ section\ of\ crack$

=1.222 x (b x t) =1.222 x (.05 x.003)

=.0001833 m3/s =.0001833 x 1000000cm3/s

=183.3 cm3/s

Q 7

A shaft having diameter 10 cm rotates centrally in a journal bearing having diameter 10.02 cm and length 20cm .The annular space between is filled with oil having viscosity 0.8 poise . Determine the power absorbed in the bearing when speed of rotation is 100 rpm . [Ans 343.6w]
Solution

Given

Dia of shaft =10 cm=..1m

Dia of bearing =10.02 cm =.1002 m

L =length of shaft =20cm = .2m

, μ of oil =.8poise =.8/10 =.08 Ns/m2

Bearing speed =N =500 rpm

To find power absorbed by bearing ?

Thickness of the filling = t = $\frac{D_1-D}{2}$ =(.1002--.1)/2 = .0001m

Tangential velocity of shaft V=$\frac{\pi DN}{60}$=$\frac{\pi\ x\ .1\ x\ 500}{60}$ =2.61799m/s

Shear stress =τ =μ dv/dy =μ v/t =.08 x2.61799/.0001

=2094.392 N/m2

Shear force F =τ x area =2094.392 x π D L

=2094 x π x .1 x .2

=131.59N

Resistance torque = T = F x D/2=131.59 4x .1/2

= 13.1594/2

=6.5797Nm

Power absorbed by the bearing = $\frac{2\pi NT}{60}$

= $\frac{2\ x\ \pi\ x500\ x\ 6.5797}{60}$

=$\frac{20670.7371}{60}$

=344.512 watt

Q 8

A shaft of 150mm diameter runs in a a bearing of length 300mm, with a radial clearance of 0.04mmat 40 rpm Find the viscosity of the oil if the power required to overcome the viscous resistance is 220.725 w. [6.32poise]

Solution

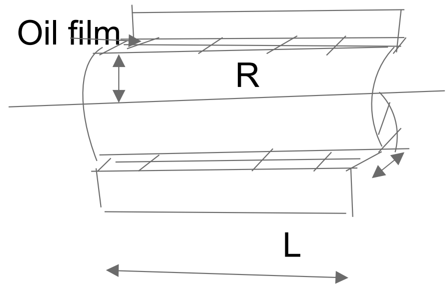
Given

D of shaft =150mm =.15m

L of shaft =300mm =.3m

, t= Radial clearance =.04mm=.04 x 10^{-3}m

Speed of shaft =40rp



t=film thickness

Power required to overcome viscous resistance

=220.725 w

, $\tau = \mu \frac{\pi DN}{60 \; xt}$

Shear force or viscous resistance

$= \tau$ x area of surface of shaft

$= \tau \, x \pi \, D \, x \, L$

$= \frac{\mu \pi^2 D^2 NL}{60 \, x \, t}$

Torque required to overcome the various resistance

T =Viscous resistance x D/2

$T = \frac{\mu \pi^2 D^3 N \quad L}{120t}$

P =power absorbed =220.725 $= \frac{2\pi NT}{60} = \frac{\mu \pi^3 D^3 N^2 L}{60 \, X 60 \;\; x \, t}$

$\mu \quad = \frac{220.725 \, x \, 60 \, x 60 x .04 \, x 1 \quad ^{-3}}{\pi^3 .15^3 40^2 x .3}$

$= \frac{31784.4 \, x 10^{-3}}{31 \, x .003375 \, x 1600 \, x ..3}$

$= \frac{31.79}{50.22}$

=.633 Ns/m2

=6.33 poise.

Q 9

Find the torque required to rotate a vertical shaft of diameter 8 cm at 800 rpm The lower end of the shaft rests in a foot step bearing The end of the shaft and surface of the bearing are both flat and re separated by oil film of thickness 0.075cm.The viscosity of oil is given as 1.2 poise.

Solution

Given

Shaft diameter D=8cm =.08m

R =.04m

, t=film thickness =.075cm=.00075m

, μ =1.2 poise =.12Ns/m2

N=800 rpm

T=Torque required to rotate the shaft

$=\frac{\mu}{60t}\pi^2 \, xN \; x \; R^4 Nm$

$=\frac{.12}{60\,x.00075}x\pi^2 x \; 800 \; x(.04)^2$=.0538 N

Q 10

A collar bearing having external and internal diameters 20cm and 10 cm is used to take the thrust of a shaft A n oil film of thickness .o3cm is maintained between the collar surface and bearing Find the power lost in overcoming the viscous resistance when the shaft rotation at 250rpm Take μ=0.9 poise [30.165w]

Solution

Given

$D_1 = 10CM = .1m$ D_2=20cm=.2m

R2=10cm =.1m R1= 5cm =.05m

,t=thickness of oil film = .03cm=.0003m

, μ=.9poise=.09Ns/m2 ,speed =N=250rpm

Total torque required to overcome the viscous resistant

$$T=\frac{\mu}{60t}\pi^2 N[R_2^4 - R_1^4]$$

$$=\frac{.09}{60\ \ .0003}x\pi^2\ x250[(.1)^4 - (.05)^4]$$

$$=\frac{.09\ x\pi\ x\pi\ x250\ x}{60\ x.0003}\text{x}(.0001\text{-- }.00000625)$$

$$=\frac{.09\ x\pi\ x\pi\ x2\ \ \ \ x.9375}{60\ x3}=\frac{208.1869}{180}=1.156$$

Nm

Power lost in overcoming the viscous resistance

$$P=\frac{2\pi NT}{60}=\frac{2\ x\ \pi\ x250\ x1.156}{60}=\frac{1815.840}{60}$$

=30.26 w

Q11

Water is flowing through a 150mm diameter pipe with a coefficient of friction f=.05. The shear stress at a point 40mm from the pipe wall is 0.01962 N/cm2.Calculate the shear stress at the pipe wall [Ans 0.04198 N/cm2]

Solutions

Given

Pipe diameter =150 mm=.15 m

Coefficient friction =.05

Shear stress at r =75 –40=35mm

, τ =0.01962 N/cm2

The liquid is viscous or not ?

For that friction f =$\frac{16}{Re}$ so Re= 16/f=16/.05

= 320 <2000

So liquid is viscous.

So $\tau = \frac{dp}{dx}\frac{r}{2}$ as there is no variation of pressure or x ,therefore $\tau \propto r$.

,r =75—40 =35 mm from center

At the pipe wall R=75mm

Shear stress at pipe wall =τ_0

On proportional basis

$\frac{\tau}{3.5} = \frac{\tau_0}{7.5}$ So $\tau_0 = \frac{\tau\, x 7.5}{3.5} = \frac{0.01962\, x 7.5}{3.5}$

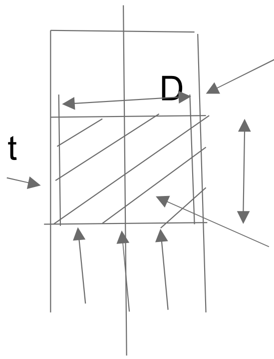
τ_0 = 0.0420 N/m2

Q 12

An oil dash pot consists of piston moving in a cylinder having oil. The piston falls with uniform speed covers 4.5cm in 80 seconds If an additional weight of 1.5N is placed on the top of the piston ,it falls through 4.5cm in 70 seconds with uniform speed The diameter of piston is 10cm and its length is 15cm The clearance between piston and the cylinder is 0.15cm which is uniform through Find the viscosity of oil .[0.177 poise]

Solution

Given



Dash pot

L L= length of piston

D=Dia . of piston

Piston , t=gap between piston and dash Pot.

Distance covered by piston due to self weight =4.5cm

Time taken = 80 sec

Additional weight placed =1.5 N

Time taken to cover 4.5cm =70 sec.

Dia of piston =D =10cm = .1m

Length of piston = L = 15cm =.15m

, t =clearance between piston and cylinder =.15cm=.0015m

Let the viscosity =μ

Weight of piston =w

V=Velocity of piston without additional weight

V_1 =velocity of piston with additional weight.

Using equation (9.24)

$$\mu = \frac{4wt^3}{3\pi D^3 L\, xV} = \frac{4(W+1.5)t^3}{3\pi D^3 L\, V_1}$$

(Cancelling $\frac{4t^3}{3\pi D^3 L}$ from both sides)

$$\frac{w}{v} = \frac{w+1.5}{v_1}$$

$$\frac{V}{V_1} = \frac{W}{W+1.5} \quad \text{---------- I}$$

But V= Distance /time =4.5/80 cm/s

V_1=4.5/70 cm/s

V/V_1 = 70/80 =.875 ----------- II

Equating I and II

$$\frac{W}{W+1.5} = .875$$

W =.875(W +1.5)

W =$\frac{.875X\ 1.5}{.125}$ =10.5 N

Also , V =$\frac{4.5}{80}$ x 1/100=.0005625 m/s

Using equation (9.24)

, μ =$\frac{4Wt^3}{3\pi D^3 L.V}$ =$\frac{4\ x\ 10.5\ x(.0015)^3}{3\ \ x\ \pi\ x(.1)^3\ x\ .15x\ .0005625}$

= $\frac{.00000014175}{.000000795}$ =.178Ns /m2

=1.78 poise. =viscosity of oil.

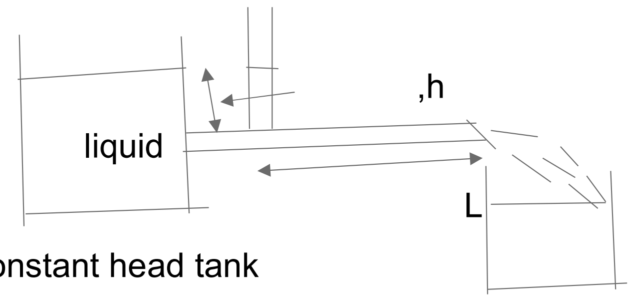
Q13

The viscosity of oil s p gr 0.8 measured by capillary tube of diameter of 40mm . The difference of pressure head between two points 1.5m apart is 0.3m of water. The mass of oil collected in a measuring tank is 40 kg in 120seconds. Find viscosity of oil.

[Ans 2.36 poise]

Solution

Given (capillary tube viscometer)



constant head tank

measuring tank

s p gr of oil =.8

L= length of tube =1.5 m

,h =difference of pr. head = 0.3 m of water.

M=mass of oil =40kg

. t =collection time for oil =120 s

D= dia. Of tube =40mm=.04 m

Density of oil =.8 x1000= 800kg/m2

Mass of oil/s = 40/120= 1/3 kg/s

Discharge Q= Mass of oil per second/density

$$= \frac{1/3}{800} = .0004125 \text{ m3/s}$$

Using equation (9.25)

$$, \quad \mu = \frac{\pi\rho g h D^4}{128QL} = \frac{\pi\, x800x\; 9.81\; x0.3\; x(.04)^4}{128\; x.0004125\;\; x\; 1.5}$$

$$= \frac{.018935}{.0792}$$

=.2390 Ns/m2

= 2.39 poise.

Q 14

A capillary tube od diameter 4mm and length 150mmis used for measuring viscosity of a liquid . The difference of pressure between two ends of a tube is .7848 N/cm2 and viscosity of this liquid is 0.2 poise . Find the rate of flow of liquid through this tube'. [Ans 16,43m3/s]

Solution

Given

Dia. Of capillary tube =D =4mm =$4\ x10^{-3}$

L = Length of tube =.15m

Viscosity =μ =.2 poise= .02N/m2

Difference of pressure = h = .7848 N/cm2

=.7848 x 10^{4}N/m2= .7848 x $\frac{10^{4}}{\rho g}$

N/m2

Let rate of flow of liquid = Q

Using equation (9.25)

, μ $=\frac{\pi\ x \rho\ x g\ x\ h\ D^{4}}{128QL}$

.02

$==\frac{\pi\ x\ \rho\ x g\ \ x.7848x\ 10^{4}x\ (4\ x10^{-3})^{4}}{128\ x.Q\ \ x.15X\ \rho\ x\ g}$

Q $=\frac{\pi\ X.7848\ X10^{4}\ x(4\ x1\ \ ^{-3})^{4}}{128\ x\ .15\ x.02}$

$=\frac{631.17}{.384}\ \ x10^{-8}$m3/s

= 1643.67x 10^{-8}m3/s

=1643 x$10^{-8}x10^{6}$cm3/s

=16.43 cm3/s rate of flow of liquid.

Q15

A sphere of 3mm diameter falls 100mm in 15 seconds in a viscous liquid . The density of the sphere is 7000 kg/m3 and that of liquid is 800kg/m3. Find the coefficient of viscosity of the liquid. [Ans 45.61 poise]

[Note: falling time taken 15 sec instead of 1.5 sec to match the answer given in the text.]

Solution

Given

Dia of sphere =3mm=.003m

L =Distance travelled by sphere = 100mm/1000 = .1m

Time of fall =15 s

U= Velocity of fall of sphere =.1/15=.0066 m/s

Density of sphere =ρ_s =7000kg/m3

Density of fluid =ρ_f =800kg/m3

Using equation (9.26)

, μ=$\frac{gD^2}{18U}[\rho_S - \rho_f]$

= $\frac{9.81(10^{-3})^2}{18\,x.0066}$[7000---800]

=$\frac{9.81\,x9\,x10^{-6}}{18\,x\,.0066}$ x 6200

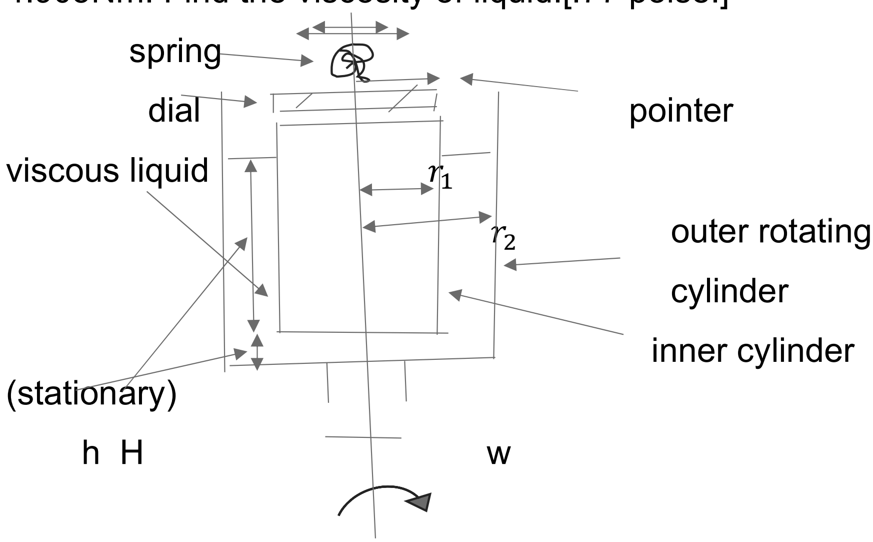
=$\frac{547398\,x10^{-6}}{.1188}$ =$\frac{.547398}{.1188}$ =4.60Ns/m2

=4.6 x 10 poise

=46 poise.

Q 16

The viscosity is determined by Rotating cylinder method in which case the inner cylinder of diameter 25 cm is stationary. The outer cylinder of diameter 25.5cm contains the liquid up to height of 40 cm. The clearance at the bottom of 2 cylinder is .6cm.The outer cylinder is rotated at 300rpm.The torque registered on torsion meter attached to the inner guide is 4.905Nm. Find the viscosity of liquid.[.77 poise.]



,r_1 =radius of stationary inner cylinder=25/2cm=.125m

r_2= radius of rotating outer cylinder.=25.5/2cm=.1275m

Liquid height outer side   from bottom,=40cm=.4m

h =clearance at the bottom of 2 cylinder= .6cm=.006m

H= Height of inner cylinder immersed in liquid=40 --,6=39.4cm

=.394m

N=Speed of rotating cylinder =300rpm

, ω $=\frac{2\pi N}{60}=\frac{2\pi\ X300}{60}$ =31.41Radian

T= Torque measured = 4.905 Nm

Using the equation(9.27)

,μ $=\frac{2(R_2-R_1)xh\ xT}{\pi R_1^2\ \omega[4HhR_2+R_1^2\ X(}\ \frac{1}{R_2-R_1)}$

$=\frac{2(.1275-.125)X.006\ x4.905}{\pi\ x(.125)^2\ x\ 31.41\ x[4\ x.394\ x.006\ x.1275+(.125)^2(.1275-.125)]}$

$=\frac{.000147}{1.54[.0012\quad .000039]}$

$=\frac{.000147}{0.0019}$

=.077Ns/m2

=.77poise.

Q17

Calculate a)pressure gradient along the flow b)average velocity, c) discharge for oil of viscosity 0.02Ns/m2 flowing between two parallel plate 1m wide maintained 10mm apart The velocity midway between plates is 2.5m/s. [a)4000N/m2,b)1.667m/s c).01667m3/s]

Solution

Given

, μ =.02 Ns/m2

Width =b =1m

, t =distance between plates =1omm=.01m

Velocity in the midway =u_{max} =2.5 m/s

a)

pressure gradient (d p/dx)

equation (9.10)

$$U_{max} = -\frac{1}{8\mu} \times \frac{dp}{dx} t^2$$

$$2.5 = \frac{-1}{8x.02} \frac{dp}{dx} x.01^2$$

$$\frac{dp}{dx} = \frac{-2.5\ x8\ x.02}{.01^2}$$ =-- 4000N/m2

b)

average velocity

$\frac{U_{max}}{\bar{u}} = \frac{3}{2}$ so $\bar{u} = \frac{2\ xu_{max}}{3} = \frac{2\ x2.5}{3}$

= $1.\dot{6}$ m/s

c)

discharge Q

Q=area of flow x average velocity

=(b x t) x $\bar{u}$

= (1 x .01) x 1.67

=.0167 m3/s

Q18

Calculate

I .The pressure gradient along the flow

2 the average velocity

3 discharge for oil of viscosity .03Ns/m2 flowing between the stationary plates which are parallel and at 10 mm apart. Width of the late is 2m.The velocity midway between the plates is 2.0m/s. [no answer is provided in this sum in the original book.]

Solution

Given

, μ = .03 Ns/m2

.b =width =2m

t =distance between plates =10mm =.01m

U_{max}=velocity in midway =2m/s.

I pressure gradient (d p/d x)

U max=--1/8μ (d p/d x) x t x t

2=1/8x.03 (d p/d x) x (.01 x.01)

,d p/dx = --2 x8 x.03 /.01 x.01

=-- (16 x.03)/.0001

=-- 4800 N/m2

I I

Average velocity

U max/$\overline{U}$ =3/2

2/$\overline{U}$ =3/2

$\overline{U}$ =2 X2/3 =1 .33 m/s

I I I

Discharge Q =Area of flow x $\bar{u}$

= b x t x $\bar{u}$

= 2 x .01 x 1.33

=.0266m3/s

$**$ students may note that sufficient hints are given in the last 4 questions 19,20,21,22, in the original book , hence no solutions have been provided .

End of Ninth Chapter

Fluid Mechanics and Hydraulic Machines

Turbulent Flow

Chapter 10

Q 1

A pipe line carrying water has average height of irregularities projecting from the surface from the boundary of the pipe as 0.20mm . What type of boundary it is ? The shear stress development is 7.848 N/m2. Take value of kinematic viscosity for water as 0.01 stokes [boundary is in transition]

Solution

Given

Average height of irregularity =k =0.20 mm

=0.20 x 10^{-3}m

Shear stress developed τ_0 =7.848 N/m2

Kinematic velocity =v = 0.01 stokes = .01cm2/s

V =.01 x 10^{-4}m2/s

Density of water =1000kg/m3

Shear velocity $=u_*=\sqrt{\tau_0/\rho}$ $=\sqrt{\frac{7.848}{1000}}$

$\sqrt{.007848}$ =.088m/s

So Roughness Reynold number$=\frac{U_* xk}{V}=\frac{.088\ X.20\ X10^{-3}}{.01\ X10^{-4}}$

$=\frac{.088\ X0.20\ X\ 10}{.01}$ =17.6

Since Roughness Reynold number is 17.6 which is between 4 and 100 hence the boundary is in transition

Q 2

Determine the average height of roughness for a rough pipe of diameter 10cmwhen the velocity at a point 4 cm from the wall is 40% more than the velocity at a point 1 cm from pipe wall.

[Ans .94 cm]

Solution

Given

Dia ,of rough pipe =10cm =.1m

Let velocity of flow at 1 cm from wall =U

Velocity of flow at 4cm from wall = 1.4 U

Velocity distribution of rough pipe by equation (10.21)

U/$U_\circ$ =5.75 Log10 y/k +8.5

K is height of roughness

$u_\circ$ =shear velocity.

For a point y=1cm from pipe wall

$\frac{u}{u_\circ}$ =5.75log10(1/k)+8.5 --------- i

For point y=4cm

$\frac{1.4U}{U_\circ}$ =5.75 log10(4/k)+8.5 --- ii

Dividing (ii/i) we get

$$1.4 = \frac{5.75log10\left(\frac{4}{k}\right)+8.5}{5.75log10\left(\frac{1}{k}\right)+8.5}$$

By solving the above we get as follows

8.05log10(1/k) --- 5.75log 10(4/k)=8.5 –11.9=-- 3.4

8.05[$log_{10}1 - log_{10}^{k}$]---5.75[$log_{10}^{4} - log_{10}^{k}$] =-- 3.4

8.05[0—log k]—5.75[.602059 –log k]=-- 3.4

--- 2.3 log k=--3.4 +3.4618 = .0618

log_{10}^{k}=$\frac{.0618}{-2.3}$ =--.02687 =$log_{10}^{.94}$

So, k-=.94 cm=height of roughness.

Q3

A smooth pipe of diameter 10 cm and 1000m long carries water at the rate of 0.70m3/min. Calculate 1. The loss of head 2wall shearing stress 3. Center line velocity

and shear stress at 3cm from pipe wall Also calculate thickness of laminar sublayer.

Take kinematic viscosity of water 0.015 stokes and value of coefficient of friction f as f=$\frac{.0791}{R_e^{1/4}}$ where Re=Reynold number

[20.05m,4.9N/m2,1.774m/s,19.62N/m2, 0.248mm]

Solution

Given

Dia. Of smooth pipe d= 10cm =.1m

Length of pipe =1000m

Kinematic viscosity =.015stokes

=.015 x 10^{-4}

Q=Discharge =.70 m3/min

=.011m3/s

, ρ =1000kg/m3, V =$\frac{Q}{A}$ =$\frac{.011}{\frac{\pi}{4}X(.1)^2}$=.044/.0314 =1.40127m/s

Reynold number=$\frac{V\,Xd}{v}$

=$\frac{1.4\,x.1\,x10^4}{.015}$ =93300

Since Reynold number is more than 4000 so the flow will be turbulent in nature.

Coefficient of friction f=$\frac{.0791}{R_e^{1/4}}$ = $\frac{.0791}{9.33\,x10^4)^{\frac{1}{4}}}$

= $\frac{.0791}{17.47}$ ='0045277

I loss of head (eq, 10.2) h_f =$\frac{4\,f\,L\,V^2}{d\,x2g}$

= =

$= \frac{4\,x.0045277x100 \quad x(1.40127)^2}{.1\,x2\,x9.81}$

=$\frac{4\,x4.5277\,x1.96355}{1.96}$

=35.56/1.96

. = =18.14 m

II

Wall shear stress (10.5 equation)

$$=\frac{f\, x \rho\, x v^2}{2} = \frac{.0045277\, x 1000\, x (1.40127)^2}{2}$$

=8.89/2 =4.445 N/m2

III Central line velocity u_{max} (equation 10.20)

$$\frac{u}{u_\circ} = 5.75 \log 10 \ \frac{u_\circ}{V} y \ +5.55\text{-------}1$$

$U_\circ$ = Shear stress velocity= $\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{4.43}{1000}}$ =0.066

Velocity will be maximum when y = d/2=.1/2 =.05 m

And, U=U_{max}

Substituting the value in 1 we get

$$\frac{U_{max}}{U_\circ} = 5.75\ log_{10}\ x \frac{.066\ x .05}{.015\ x 1\ \ ^{-4}} + 5.55$$

$$=5.75\ log_{10} \frac{\prime 0033}{.0000015} + 5.55$$

=5,75$log_{10}2200$+5.55

=5.75 x 3.34 +5.55

=19.205+5.55

=24.755

$$u_{max} = u_\circ\ x\ 24.755 = .066558\ x 24.755$$

=1.647 m/s

IV a) Shear stress at 3 cm from pipe wall =τ

$$\frac{\tau}{\tau_0} = \frac{r}{R}$$ r= 5—3 =2 cm=.02 m

T $=\frac{.02}{.05} X\tau_0$ = 2/5 X 4.43 =8.86/5=1.772 N/m2

b.) velocity at 3cm from wall U

y=3cm, =.03 m

$\frac{U}{U^\circ}$ =5.75 $log_{10}\frac{u_\circ}{v}\ xy + 5.55$

= 5.75 $log_{10}\frac{.0665x..03}{.015\ x1\ \ ^{-4}}$. + 5.55

5.55 + 5.75 log_{10}1330

= 5.55 +5.75 x3.123=5.55+17.95

=23.5

U=23.5 X $U_\circ$ = 23.5 X.066

=1.56 m/s

V. Thickness of laminar sublayer given by

$,\delta = \frac{11.6\ x\ V}{U_\circ} = \frac{11.6\ X.015\ X1\ \ ^{-4}}{.066}$

$= \frac{.174}{660}$

$= 0.263\text{mm}$

------- -------- --------

Q4

The velocity of water through a pipe diameter 10cm are 4 m/s and 3.5m/s at the center of the pipe and 2 cm from the pipe center respectively. Determine the wall shearing stress in pipe for turbulent flow [15.66 kgf/m2]

Solution

Given

Dia of pipe =10cm =.1m

R of pipe = .05m

Velocity at center =4 m/s =u_{max}

Velocity at .02 m away from center = 3.5 m/s = u

Let wall shearing stress = τ_0

As the flow is turbulent velocity distribution in terms of u_{max} ,(central line velocity) as per equation (10.18)

$$\frac{u_{max}-u}{U_\circ}=5.75\ log_{10}\frac{R}{Y}$$

[U=3.5m/s. y = R—r= .05m--.02m=.03m]

Or, $\frac{4m-3.5}{U_\circ}$ $=5.75\ log_{10}\frac{.05}{.03}=5.75\ log_{10}1.666$

Or, $\frac{.5}{U_\circ}$= 5.75 X.2216=1.2742

$\therefore U_\circ=\frac{.5}{1.2742}=.3924$ m/s

Wall shear stress $=U_\circ=\sqrt{\frac{\tau_0}{\rho}}$

Squaring both sides and adjusting we get

$(.3924)^2 X\rho=\tau_0$

$\tau_0=$ .3924 x .3924 x1000

=153.977 N/m2

=153.977/9.8 kgf/m2

=15.7 kgf/m2= wall shea stress.

Q5

For turbulent flow in a pipe of diameter 200mm find the discharge when the center line velocity is 3.0 m/s and velocity at a point 80 mm from the center as measured by a pilot tube is 2m/s.

[64.9 lit/s]

Solution

Given

D =Pipe d I a, = 200mm= .2 m

R = .1m

Center line velocity = U_{max}=3m/s

Velocity at(r =80mm=.08m) distance=2m/s

Y=R—r=.1--.08 =.02m

Velocity in comparison to U max Eq (10.18)

$$\frac{U_{max}-U}{U*}=5.75\ log_{10}^{R/Y}$$

Or, $\frac{3-2}{U\circ}$ =5.75 $log_{10}^{.1/.02}$=5.75 log 5

=5.75 x 0.69897

=3.893

$u_* = \frac{1}{3.893}$ =.25687 m/s ---- (-I)

Using equation (10.26)which gives relation between velocity at any point and average velocity ,we have

$$\frac{U-\bar{U}}{U*}=5.75\ log_{10}\frac{y}{R}\ \ +3.75$$

At y=R Velocity u becomes U max

$$\frac{Umax-\bar{u}}{U*}=5.75 log_{10}\frac{R}{R}+ 3.75$$

-= 5,75 log 1 +3.75= 0 +3.75

From above after putting values of U max U∗ we get

$\bar{u}$ =3,0—3.75 x .25687 =2.03m/s

In above situation the discharge is arrived as follow

= Q =area of pipe x average velocity

= π/4 x $(.2)^2$ x 2.03

=0.2551/4

=.0637m3/s

$= .06377 \text{ x } 1000$ lit/s

$=63.77$ lit/ s

Q 6

For problem 5 above ,find the coefficient of friction and average height of roughness projections [.029, 25.2mm]

Solutions

Given

$R=.1m, \quad U*=.25687 \quad \overline{U = 2.03}$

1 For coefficient of friction $= U* = \overline{\overline{U}}\sqrt{f/2}$

$.25687 = 2.03\sqrt{f/2}$

$\frac{f}{2} = (\frac{.25687}{2.03})^2 = .0160$

, f =.0160x2=.032 =coefficient of friction.

2 height of roughness projection K (equation (10.36).

$\frac{1}{\sqrt{4f}} = 2log_{10}\frac{R}{K} + 1.74$

$1/\sqrt{(4 \; X.032)} = 2 \; log_{10}\frac{.1m}{k} + 1.74$

1/.3577 --1.74 =2log10 .1/k

2log10 .1/k =2.7956---1.74 =1.0556

Log .1/k =.5278

.1/k = 3.371

So $k=\frac{.1}{3.371} =$ =..0296 x1000 =29.6 mm

Q 7

Water is flowing through a rough pipe of diameter40 cm and length 3000 m at the rate of .4m3/s. Find power required to maintain this flow, Take the average height of roughness at K is .3mm. [278..5KN]

Solution

Given

K=0.3 d=40m=.4m, R= .2m. L=3000m

Q =.4m3/s

To find $\bar{u}$ =V ? h_f =?

$h_{f=}$ $\frac{4.f.LV^2}{d\ x2g}$ power required

$$P = \frac{wx\ \ f}{1000}\text{kw}$$

To find for above to calculate

$$\frac{1}{\sqrt{4f}} = 2log_{10}\frac{R}{K} + 1.74$$

$$\frac{1}{\sqrt{4F}} = 2log_{10}\frac{.2}{.3} + 1.74 = 2\ \log\ \frac{}{10}\frac{2\ x100}{3} + 1.74$$

=2log 666.6 +1.74

=2 x 2.8238+1.74

=5.6476+1.74 =7.3876

$$4\text{f} = \left(\frac{1}{7.3876}\right)^2 = (.1353)^2 = .0183$$

∴ f =..004575=frequency

Average velocity $\overline{U}$ =Q/area $= \frac{.4}{\pi/4d^2} = \frac{.4}{\pi/4x,4^2}$ =3.18m/s

Type equation here.

$$h_f = \frac{4.x.004575\ x3000\ x(3.18)^2}{.4\ x2\ x9.81} = 71.12\text{m} = \text{head loss.}$$

So power required to maintain the flow

$$= \frac{w\ xh_f}{1000} = \frac{\rho\ XG\ X\ Q\ Xh_f}{1000}$$

$$= \frac{1000\ x9.81\ x\ .4\ x71.12}{1000}$$

$$= 279.07 \text{ kw}$$

Q8

A smooth pipe of diameter 300mm and length 600m carries water at a rate of 0.04 m2/s. Determine the head loss due to friction wall shear stress, central line velocity and thickness of lamination sublayer. Take the kinematic viscosity of water 0.018stokes. [0.588m ,0.72N/cm2,0.665m/s,0.779mm]

Solution

Given

D =Dia, =300mm=.3m

R =.3/2 =.15m, L=600 m, Q=.04m3/s

$$\text{V}=\overline{U} = \frac{Q}{A} = \frac{.04}{\frac{\pi}{4} X(.3)^2} = .5659\text{m/s}$$

Kinematic viscosity =v =.018stokes=.018cm2/s=.018 x$10^{-4} m2/s$

$$\text{Reynold number} = \frac{V.D}{v} = \frac{.5659\ x.3}{..018x10^{-4}} = 9431.6$$

Flow is turbulent.

Now to find out the friction

$$,\quad \text{f} = \frac{\prime 0791}{R_e^{1/4}} = \frac{.0791}{(9.43x1\quad ^4)^{1/4}} = \frac{.0791}{(9.43)^{\frac{1}{4}}\ x1} = \frac{.0791}{1.75\ x10}$$

$=.00452$

1

Head lost due to friction $=h_f = \frac{4fLV^2}{D\ X2g} = \frac{4\ x\ .00452\ x\ 600\ \ x(.5659)^2}{.3\ x9.81}$

$= \frac{3.47}{5.886} \quad = .592$ m

2 wall shear stress $=\tau_0 = \frac{f\ x\ \rho\ xV^2}{2} = \frac{.00452\ X1000\ X.5659\ X.5659}{3}$

$= .7237$ N/m2

3

Central line velocity $= \frac{Umax}{U*} = 5.75 log_{10}^{\frac{U*.R}{V}} + 5.55$

[$U* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{.72}{1000}} = .0268$ m/s]

$=5.75\ log_{10} \frac{.0268\ x.15}{.018\ x10^{-4}} + 5.55$

$= 5.75\ log_{10}\ \frac{40.2}{.018} + 5.55$

$=5.75\ log_{10} 2233 + 5.55$

$= 5.75\ x3.349 + 5.55$

$= 19.256 + 5.55$

$=24.80$

U max $=U*\ X24.80 = .0268\ \ X\ 24.80$

$= .66464$m/s

Thickness of laminar sublayer $=\delta = \frac{11.6\ xv}{U*} = \frac{11.6\ X.018\ X10^{-4}}{.0268}$

$= \frac{11.6\ X.018}{268}$

$=.779$ mm

Type equation here.

Q9

A rough pipe of diameter 300mm and length 800m carries water at the rate of .4 m3/s. The wall of roughness is 0.015mmDetermine the coefficient friction, wall shear stress ,central velocity and velocity at a distance of 100mm from pipe wall.

[f=.00263 To=42.08N/cm2Umax= 6.457m/s U=6.249M/s]

Solution

Given

D=rough pipe dia. =300mm=.3m

R=radius of above=.15m

L=length of above=1000m

Q =discharge=.4m3/s

K =wall roughness =0.015 mm

To find 1 f ,2 To,

3 U max & 4.U at 100mm from wall.

1 coefficient of friction f (Eq 10.36)

$1/\sqrt{(4f)}=2\ log_{10}\ R/K+1.74$

$= 2log_{10.15/.015x10^{-3}} + 1.74$

$=2\ log_{10}10000+1.74$

$=2\ x4 +1.74 =9.74$

$\sqrt{(4f)} =1/9.74=0.10266$

„ f =(0.10266 x 0.10266)/4

=.01053/4

=.00263

2) wall shear stress

$$To = \frac{f \rho V^2}{2}$$

[V=Q/π/(4X dx d) = .4/(π/4 x .3x.3) =5.659m/s]

$$So\ To = \frac{.00263\ x1000\ x5.659\ x5.659}{2}$$

=2.63 x5.659 x 5.659 /2

=42.11 N/m2

3) center line velocity U max.

U max /U∗ =5.75 $log_{10}\frac{R}{K}$ +8.5

=5.75 log_{10} .15/.015 x10^{-3}+8.5

[U∗=√(To/ρ) =√(42.11/1000) =√.04211=0.025m/s]

=5.75log_{10} (150/.015) +8.5

=5.75 log 10000 +8.5

=5.75x 4 +8.5 =31.5

U max=31.5 xU∗=31.5 X 0.205

=6.4575 m/s

(4)

Let Velocity at point 100mm from pipe wall=U m/s

U/U∗ =5.75 $log_{10}\frac{Y}{K}$+8.5

=5.75 $log_{10}\frac{.1}{.015\ x10^{-3}}$+8.5

=5.75 log_{10} 100/.015 +8.5

=5.75 log_{10} 6666.6+8.5

=5.75 x 3.824+8.5

=22.08+8.5

=30.58

U=U∗ X30.58=.205 X30.58

=6.269 m/s

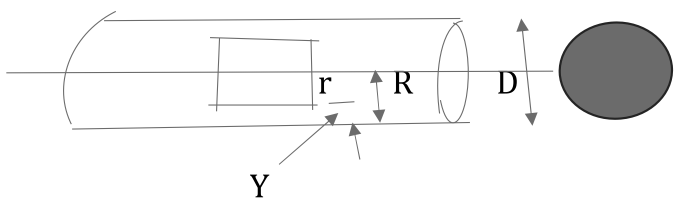
Q 10

Determine the distance from the center of pipe at which the local velocity is equal to the average velocity for turbulent of pipe.

$\overline{[\,Ans\ \ 0.7772R]}$

Solution

Given



, r =R—Y (relation from above fig.)

We have find r distance where local velocity equal to average velocity .

U= Local velocity $\overline{U}$ =Average velocity.

We use the following relation (10.25)

$\frac{\overline{U}\ -U}{U*}$ =5.75 $log_{10}\frac{Y}{R}$ +3.75 [as local velocity = average velocity

$$0 = 5.75\ log_{10}\frac{y}{R} +3.75$$

$$5.75log_{10}(\frac{Y}{R}) =\text{--}3.75$$

$$log_{10}y/R =\text{---}\frac{3.75}{5.75} =0..6521$$

$$\frac{Y}{R} = .22279$$

$$Y = R \times .22279$$

Now $r = R — Y = R --- Rx\ .22279$

$$= R(1 -- .22279)$$

$= .7772\ R =$ distance from central line of pipe.

END OF CHAPTER TEN

Fluid Mechanics and Hydraulic Machines

Flow through Pipes

Chapter 11

Q 1

Find the head loss due to friction in a pipe of diameter 250mm and length 60m through which water is flowing at a velocity of 3.0m/s using 1. Darcy's formula & 2.Chezy's formulae for which C =55 take v=kinematic viscosity=.01stokes.

[1.1.182 m 2.2.856m]

Type equation here.

Solution

Given

Dia. Of pipe =d =250mm=.25m

Length =L=60m

Velocity of flow =3m/s

Crazy's constant C= 55

Kinematics viscosity

v=.01stokes=.01cm2/s=.01x10^{-4}m2/s

Head loss =h_f=$\frac{4\ f\ L\ V^2}{dx2g}$

We have to find f f0r that we to find Reynold's number Re

Re=(V x d)/v =(3 x ..25)/.01x10^{-4}=75 x 10^5

, f =$.079/Re^{1/4}$ =$\frac{.079}{(75x1\ \ 5)^{1/4}}$ =$\frac{.079}{75^{1/4}\ x10}$ =.002687

So h_f = $\frac{4\ \ x\ .002687\ x60\ x\ 3x3}{.25\ x\ 2\ x9.81}$ =$\frac{5.788}{4.9}$

=1.1812 m head loss due to friction.

11 Crazy formula – V =C$\sqrt{mi}$;

m=d/4=.25/4=.0625

i =$\frac{h_f}{L}$ =$\frac{h_f}{60}$

So, $3=55\sqrt{.0625\frac{h_f}{60}}$

$\sqrt{.0625\frac{h_f}{60}}$ =3/55

Type equation here.

$$.0625 \text{ x}\frac{h_f}{60} \quad =(.054)^2=.002916$$

$$h_{f=} \ \frac{.002916\ x6}{.0625} = 2.8 \text{ m} = \text{Head loss.}$$

Q2

Find the diameter of a pipe of length2500 m when the rate of flow of water through the pipe is 0.25 m3/s and head loss due to friction is 5m . Take C=50 in Chezys formulae.

[Ans 605mm.]

Solution

Given

Length of pipe =2500m , Q =.25m3/s.

h_f=5m = Head loss due to friction.

Crazy's constant=50, dia. Of pipe =d= ?

Velocity of flow V=$\frac{Q}{A}$ =$\frac{.25}{\frac{\pi}{4}Xd^2}$ = $\frac{4\ x.25}{\pi\ xd^2}$ ---(-A)

By Crazy's formula=V=C $\sqrt{m\ i}$

.[m =d/4 , i =$\frac{h_f}{L}$ =$\frac{5}{2500}$ =.002]

$$\left(\ \frac{4\ X.25}{\pi\ Xd^2}\right)^2 = 50\left(\sqrt{\frac{d}{4}\ x.002}\ \right)^2$$

Or $[\frac{2\ x4\ x.25}{\pi\ x50\ x}]^{2=}$ $[d^2\sqrt{d\ x.002}\ ]^2$

$(\ 2/157)^2 = d^5\ x.002$

d^5 =(.000161/.002)

.d =$(.0805)^{1/5}$=.604 m

Diameter of pipe = 604 mm

Q3

An oil of kinematic viscosity .5stokes is flowing through a pipe diameter 300mm at the rate of 320 lit /s. Find head lost due to friction for a 60 m pipe [Ans 5.14 m]

Solution

Given

Kinematic viscosity=v=.5stokes=.5cm2/s= .5x 10^{-4} m2/s

, d =dia. of pipe=300mm=.3m,

Q=320 lit/s=320/1000 =.32m3/s

V=Velocity of flow =Q/A=.32/π/4 x$(.3)^2$=4.529m/s

Reynold number=Re=$\frac{V\,Xd}{v}=\frac{4.529\,x.3}{.5\,x10^{-4}}$=27162

Flow is turbulent.

Coefficient of friction=$\frac{.079}{(Re)^{1/4}}=\frac{.079}{(27162)^{1/4}}=\frac{.079}{12.83}$

=.00615

So Head loss due to friction=$\frac{4\,xf\,xL\,x\,V^2}{d\,x2g}$

$=\frac{4\,x.00615\,x60\,x\,(4.529)^2}{.3\,x9.81}$

=29.889/5.886

=5.077m

Q4

Calculate the rate of flow through a pipe of diameter 300mm,

When the difference of pressure head between the two ends of the pipe 400m apart, is 5m of water. Take the value of friction f =.009 in the formula [Ans.101me/s]

$$h_f = \frac{4\ f\ L\ V^2}{d\ x\ 2g}. \quad \text{--------------}1$$

Solution

Given

,d=300mm=.3m

L=400m

Difference of pressure head=5m of water

, f =.009, V to find out '

putting the values in 1 above we get

$$\frac{4xf\ x\ L\ XV^2}{d\ x2g} = h_{f=}\frac{4\ x\ .009\ x400\ x\ V^2}{..3\ X\ 2\ X9.81} \qquad = 5$$

$$\frac{14.2}{5.866}V^2 = 5$$

V^2 $=\frac{5\ X5.886}{14.2}$ =29.43/14.2 =2.0425

V=$\sqrt{2.0425}$ =1.429m/s

Q= Discharge= V X A

=1.429 x (π/4) x d^2

=1.429 x (π/4)x $(.3)^2$

=.10101 m3/s rate of flow of water

Type equation here.

Q 5

The discharge through pipe is 200lit/s. Find the loss of head when pipe is suddenly enlarged from 150mm to 300mm diameter.

[Ans 3.672m]

Solution

Given

Diameter of small pipe=d1=150mm=.15m

A1=area of small pipe =π/4 x (.15 x.15)

= .01767m2

Diameter of large pipe = d2=300mm=.3 m

A2 =area of large pipe =π/4 x (.3m x.3m)

=.0706m2

Q =Discharge =200lit/s=200/1000 m3/s=.2m3/s

Velocity V1 through smaller pipe=Q/A1

=.2/.01767=11.3 m/s

Velocity V2 through larger pipe=Q/A2

=.2/.0706 =2.83 m/s

Loss of head due enlargement of diameters(eq.11.5)

$$h_c = \frac{(V1-V2)^2}{2g} = \frac{(11.3-2.83)^2}{2\ x9.81} = \frac{(8.47)2}{19.62} = 3.656\text{m}$$

Q6

The rate of flow through a horizontal pipe is .3m3/s. The diameter of pipe is suddenly enlarged from 250mm to 500mm .The pressure intensity in the smaller pipe 13.734N/cm2.

Determine 1 loss of head due to enlargement 2. Presser intensity due to large pipe 3. Power lost due to enlargement.

[Ans 1) 1.07m 2)14.43N/cm2 3) 3.15 kw]

Solution

Given

Q =.3m3/s ,d1=250=.25m. A1 =π/4 x ,25 x.25 =.04908m2

Large pipe d2=500mm=.5m A2=π/4 x .5 x.5 =.1963 m2

Pressure in small pipe=P1=13.734 N/cm2=13.734 x 10^4N/m2

V1=Q/A1=.3/.04908 =6.11m/s

V2=Q/A2 =.3/.1963 =1.53 m/s

1 loss of head $=h_c=\frac{(V1-V2)^2}{2g}=\frac{(6.11-1{,}53)^2}{2.9.81}=\frac{4.58^2}{19.62}=1.07\text{m}$

2 pressure in large pipe=P2

Applying Bernoulli's equation

Z1=Z2

$$\frac{P1}{\rho g}+\frac{V_1^2}{2g}=\frac{P2}{\rho g}+\frac{V_2^2}{2g}+h_C$$

$$\frac{P2}{\rho g}=\frac{13.734\ x1\quad 4}{1000\ x\ 9.81}+\frac{6.11^2}{19.62}-\frac{1.53^2}{19.62}-1.07$$

= 14+1.90--.1192—1.07

=14.71

P_2=14.71 x 1000 x9.81x 10^{-4}

=14.43 N/cm2

3. power lost due to sudden enlargement

$$\text{P}=\frac{\rho gQ\ x\ h_C}{1000}=\frac{1000\ \ x\ 9.81\ x.3\ x\ 1.07}{1000}$$

=9.81 x.3 x1.07

= 3.148 kw

Q 7

A horizontal pipe of diameter 400mm is suddenly contracted to diameter 200mm. The pressure intensity in large and small pipe is given as 14.715 N/cm2 and 12.753 N/cm2 resp.

If Cc=.62, find the loss of head due to contraction. Also determine the rate of flow of water. [i..571m ii. 171.7 lit/s]

Solution

Given

Diameter of large pipe D1=400mm =.4m

A1 =Area of large pipe =π/4 .4 x.4 =.1256m2

Diameter of small pipe =D2=200mm=.2m

A2 =area of smaller pipe= π/4 x .2 x.2 =..0314m2

P1=P r in large pipe =14.715 x 10^4N/m2

P2 =P r in small pipe =12.753x 10^4N/cm2

Cc=.62

1. head loss due to contraction=$h_c = \frac{V_2^2}{2g}$ [1/Cc ---1 $]^2$

$=[1/.62 -1]^2 \frac{V_2^2}{2G}$

h_c=.375 V_2^2/2g ------------ a)

From the continuity equation A1V1=A2 V2

V1=A2.V2/A1=(D2/D1$)^2$ $x\ V2$

=(.2/.4$)^2$X V2=V2/4 ---- b)

Applying Bernoulli's formula

Z1=Z2

$$\frac{P1}{\rho g}+\frac{V_1^2}{2g}=\frac{P2}{\rho g}+\frac{V_2^2}{2g}+h_c \text{ ----------- c)}$$

Putting equations a)and b) from above we get

$$\frac{14.715\ x10^4}{9.81\ x10}+(\frac{V_2}{4})^2/2g=\frac{12.753\ x10^4}{9.81}+\frac{V_2^2}{2g}+\frac{V_2^2}{2g}\ x.375$$

Computing above we get

$$15+\frac{V_2^2}{16\ X2g}=13+\frac{V_2^2}{2g}(1.375)$$

$$\frac{V_2}{2g}(1.375\text{--}.0625)=15—13=2$$

$$V_2=\sqrt{\frac{2\ x2g}{1.3125}}=\sqrt{\frac{2\ X2\ x9.81}{1.3125}}\quad =5.467m/s$$

$$h_c=.375\frac{V_2^2}{2g}=.375\ x\frac{5.467\ x5.467}{2x9.81}$$

= ,571= head loss due to contraction

Q=Rate of flow=A2 X V2=.03141 X5 .467

=.17171 m3/s

=.17171 x 1000 lit/s

=171.71 lit/s

Q 8

Water is flowing through a horizontal pipe of diameter 300mm at a velocity of 4m/s. A circular solid plate of diameter 200mm is placed in the pipe to obstruct the flow. If Cc=,62 find the loss of head due to the obstruction in the p ipe [2.953m]

Solution

Given

D=diameter of pipe =300mm=.3m

A=Area of pipe =π/4 x (.3 x .3)

=.070685m2

Velocity of water =4m/s

Diameter of obstruction =200mm=.2m

Area of obstruction =π/4 x (.2 x.2)

=.0314m2 m2

Cc=0.62

Head loss due to obstruction by equ(11.10)

$$=\frac{V^2}{2g}\left[\frac{A}{Cc\ .(A-a)}-1.0\right]^2$$

$$=\frac{4^2}{19.62}\left[\frac{.0706}{.62(.0706-,0392)}-1\right]^2$$

=2.942 m

Q 9

Determine rate of flow of water through a pipe of diameter 10 cm and length60mwhen one end of the pipe is connected to a tank and other end of the pipe is open to atmosphere. The height of water in the tank from the center of pipe is 5 cm. The pipe is given as horizontal and value of f=.01 Consider minor losses.

[Ans 15.4 lit/s

Solution

Given

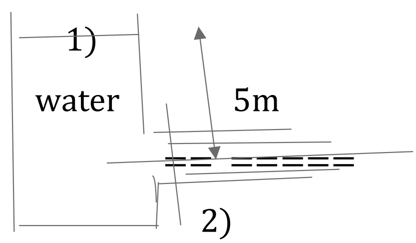
Dia . of pipe =d=10cm=.1m

Length

Height of water =5m

Coefficient of friction=.01

Let the velocity of water in pipe =V m/s



Bernoulli's equation considered at section as the top of water and section 2 at outlet of pipe . for sec 1, V1=0 ,and at sec 2,V2

$$\frac{P1}{\rho g}+\frac{V_1^2}{2g}+Z1=\frac{P2}{\rho g}+\frac{V_2^2}{2g}+Z2$$

Or, $0+0+5=0+V_2^2/2g+0+(h_i+h_f)$

$$5=\frac{V_2^2}{2g}+(h_1+h_f) \text{ ----- } \quad i)$$

From e q u (11.8) $,h_i = .5\frac{V^2}{2G}$

And from e q u (11.1) , $h_f = \frac{4\, f\, L\, V^2}{d\, x2g}$

Computing above 2 equations with (i)

$$\frac{V^2}{2g}+,5\frac{V^2}{2g}+\frac{4.xf\ xL\ V_2^2}{d\ x2g}=5$$

$$\frac{V_2^2}{2g}\ [1 + .5 + \frac{4\ x..01\ x60}{.1}\ ]=5$$

$$[1+.5+2.4\}\ \frac{V_2^2}{2g} = 5$$

$$V_2^2 = \frac{5\ X2\ 9.81}{25.5} = 98.1/25.5 = 3.847$$

$$V_2 = 1.961 m/s = V$$

Q =Discharge = A.V=π /4 x (.1 x .1) x 1.961

=..015399 m/s=.015399 x 1000 lit/s

= 15.399 lit/sec.=rate of flow through pipe

Q10

A horizontal pipe line 50m long is connected to a water tank at one end and discharging freely in the atmosphere at the other end. For the first 30 m of its length from the tank the pipe is 200mm diameter and its diameter suddenly enlarged to 400mm.The height of water level in the tank is 10m above center of the pipe. Considering all minor loss, determine the rate of flow. Take f=.01for both of pipe. [Ans 164.13lit/s]

Solution

Given

L=Pipe length=50 m

L1 =1ST Part pipe =30m

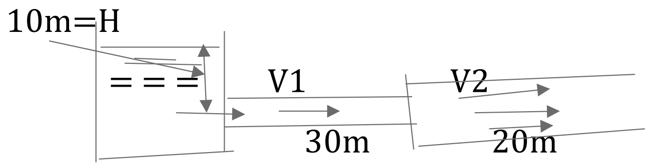
L2=2ND par=20 m

, d1=d I a of 1st=200mm=.2m,

, d2=d I a of 2nd part=400mm=.4m

Height of water =10m=H

Coeff. Friction =.01 =f, V1Type equation here.



Water tank

Bernoulli's equation application;--

$\frac{V_1^2}{2g}+\frac{p_1}{\rho g}+\ \ z1=\frac{V_2^2}{2g}+\frac{P2}{\rho g}+z2+all\ losses$ as below(i)

$0+0+10=\frac{V^2}{2g}+h_i+h_{f1}+h_e+h_{f2}$ ------- (i)

From continuity equation

A1V1=A2V2, $\mathrm{V1}=\frac{A2,V2}{A1}=\frac{(\ d2/d1)^2}{}\mathrm{V2}=(\frac{.4}{.2})^2V2=4V_2$

Losses $h_{i=}$inlet loss$=.5\frac{V_1^2}{2g}.=.5\frac{16\ \ _2^2}{2g}=\frac{8V_2^2}{2g}$

h_{f1}=loss due to friction$=\frac{4\ f\ L\ 16\ \ _2^2}{(d1)2g}=\frac{4.x.01x30x16V_2^2}{.2\ X2g}=96\frac{V_2^2}{2g}$

h_e=loss due to sudden enlargement$=\frac{(4v2-v2)^2}{2g}=\frac{9V_2^2}{2g}$

h_{f2} =due to friction $=\frac{4\ f\ L.V_2^2}{d2x\ 2g}=\frac{4x.01\ x20xV_2^2}{.4x2g}=2V_2^2/2\mathrm{g}$

Substituting theses values I equation (i) we get

$$10=\frac{V_2^2}{2g}+8\frac{V_2^2}{2g}+96\frac{V_2^2}{2g}+9\frac{V_2^2}{2g}+2\frac{V_2^2}{2g}$$

$$10=\frac{V_2^2}{2g}\{1+8+96+9+2\}$$

$$=\frac{V_2^2}{2g}\mathrm{X}\ 116\quad,\ V_2^2=\frac{196.2}{116}$$

$$V_2 = \sqrt{\frac{196.2}{116}}$$

$$V_2 = 1.3 \text{m/s}$$

Rate of discharge =(A2)X(V2) =π/4 X$(.4)^2$X 1.3

=.1634 m3/s

=.1634x1000 lit/s

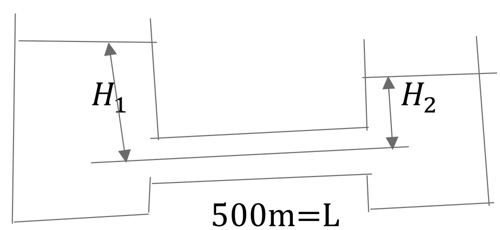
=163.4 lit/sec

Q11

Determine the difference in elevation between the water surfaces in two tanks which are connected by a horizontal pipe of diameter 400 mm and length 500m .The rate of flow of water through the pipe is 200lit/s . Consider all losses and take the value of f =.009. [Ans 11.79 m]

Solution

Given



Water tanks

Dia of pipe = .4m

Q=Discharge rate =200lit/sπ.2m3/s

Friction f = .009

Velocity =Q/A =.2/π/4 x,$(.4)^2$=1.6m/s

Difference of elevation=$H_1 - H_2$

Applying Bernoulli's equation

$$H_1 = H_2 + h_i + h_f + h_0$$

Losses hi=loss of head at entrance=$.5\frac{V^2}{2g}$=.5 x $\frac{1.6^2}{2\,x9.81}$=.0652m

h_{f1} =loss for friction=$\frac{4\,f\,L\,V^2}{dx2g}=\frac{4.x.009x500x(1.6)^2}{.4\;x2\;x9.81}$=5.89m

h_0=loss of head due to outlet =$\frac{V^2}{2g}=\frac{1.6^2}{2X\;9.81}$=.130m

H_1--H_2=$h_1+h_f+h_0$

=.0652+5.89+.13

=6.08 m

Note Students may please verify the process and calculation as the above answer is not matching From my side it is alright.

Q 13

A syphon of diameter 150mm connects two reservoir having a difference in elevation of 15 m. The length of the system is 400 m and summit is 4 m above the water level in the upper reservoir. The length of the pipe from upper reservoir to the summit is 80m.

Determine the discharge through the syphon and also pressure at summit Neglect minor losses. Coefficient of friction is .005.

[41.52lit/s,--7.281m]

Solution

Given

Diameter of system =d=150mm=.15m

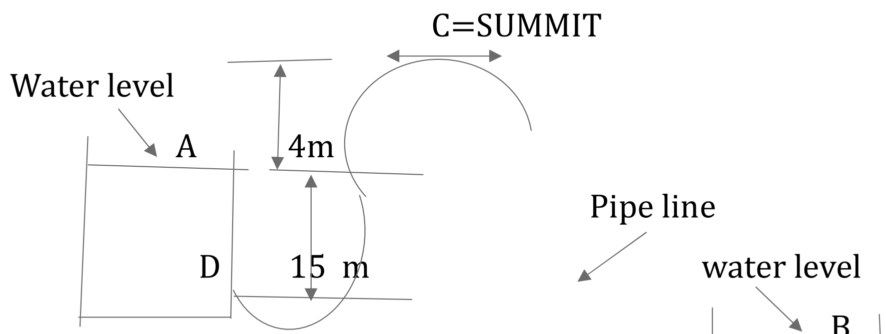
H=difference of level of reservoirs=15 m=Za—Zb

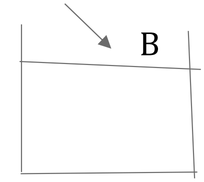
Length of syphon =L=400m

Height of summit =h=4m for upper reservoir

Length of syphon = L1=80 m

Coefficient of friction=f .005.





Vertical distance A to C is 4m

A to d=15m

Pipe line actual length from D to C is 80m.

A and B are tanks.

Considering loss of due to friction only we pros=ceed as follows ;----

Bernoulli's equation

$\frac{Pa}{\rho g}+\frac{Va^2}{2g}+Za=\frac{Pb}{\rho g}+\frac{Vb^2}{2g}+Zb+h_f$ [Va=Vb=0 &Pa=Pb=atmospheric pr]

Considering above

$0+0+\ Za=0+0+Z\,b+h_f$

$Za—Z\,b=h_f=\frac{4xfxL\quad ^2}{dx2g}$

$15 \quad =\frac{4\ x.005\ x400\ xV^2}{.15\ x2\ x9.81}$

$2.718\ V^2=\ 15$

$V=\sqrt{\frac{15}{2.718}}=2.35$

So Q =Discharge = velocity x area

$=V\ x\ \pi/4\ x\ (.15^2)$

$=2.35 \times \pi/4 \times .0225$

$=.0415\ m3/s$

$=.0415 \times 1000\ lit/s$

$=41.5\ lit/sec.$

Q 14

A Syphon of diameter 200mm connects two reservoir having a difference of elevation 20m. The total length is 800 m and the summit is 5 m above the water level in the upper reservoir. If the separation takes place 2.8mof water absolute ,

Find the maximum length of sphere from upper reservoir to the summit.

[. Take f=.004.and atm pr.10.3mof water.] [Ans 87.52m]

Solution Given

Dia of syphon=d =200mm=.2m

Difference of level of reservoir =20m

Total length of pipe =800m

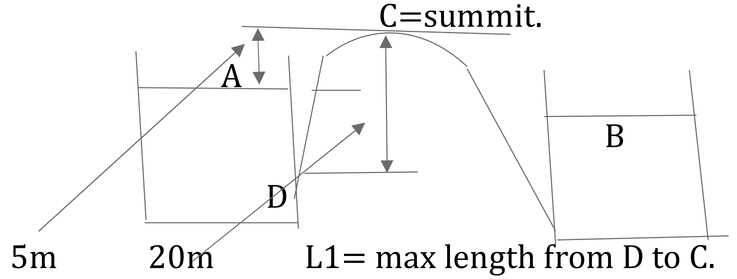
Height of summit = 5m above water level.

Pressure head at summit $=\frac{p_C}{\rho g}$ =2.8 m of water absolute.

Atmospheric pressure head=10.3 m of water absolute.

Friction coefficient = f =.004 , A=Upper &B lower reservoir,

C=summit.



L1= max length from D to C.

Bernoulli's equation to points A &C taking the datum line passing through A

$$\frac{Pa}{\rho g}+\frac{Va^2}{2g}+Za=\frac{Pc}{\rho g}+\frac{Vc^2}{\rho g}+Zc+loss\ due\ to\ friction.$$

$$10.3+0+0=2.8+\frac{V^2}{2\,g}+5+h_f$$

$$h_f=2.5---\ \frac{V^2}{2g}\quad -------\quad \text{I.}$$

For part A & B on similar equation

Pa/ρ g =Pb/ρ g, Va=Vb=0 ,Za=20,Zb=0

Putting the values we get

$$0+0+20=0+0+0+h_f$$

$$h_f=20\text{ m}=\frac{4.f.L.V^2}{dx2g}=\frac{4.x.004\ x800xV^2}{.2X2\ X9.81}$$

$$V=\sqrt{\frac{20\ x.2x2\ x9.81}{4\ x.004\ x800}}\ =2.475\text{m/s}$$

Substituting the value of V in equation i)

$$h_{f1}=(2.5---\frac{V^2}{2g})=(2.5---\frac{6.13}{19.62})\ =2.1876\quad -------\quad \text{ii)}$$

But $h_{f1}=\frac{4\ xf\ xL_1\,xV^2}{d\ x2g}$ L1=Length of syphon from upper reservoir to the summit

$$=\frac{4\ x.004\ x\ L1\ x\ 6.13}{.2\ x2\ x9.81}=.02499\text{L1}\ ---------\ \ \text{III)}$$

Equating ii and iii

.02499L1=2.1876

$$\text{L1}=\frac{2.1876}{.02499}=87.53\text{m}$$

Q 15

Three pipes of lengths 800m,600m and 300m and of diameter 400mm,300mm and 200mmrespectively are connected in series. The ends of compound pipes are connected to two tanks

Whose water surface levels are maintained at a difference of 15m. Determine the rate of flow of water the pipes if f=.005. what will be diameter of single pipe of length 1700m of f=.005 which replaces the three pipes, [.0848m3/s, 266.5mm]

Solution

Given

Neglecting minor losses except friction f.=.005

For all three pipes f=f1=f2 =f3=.005 (eq ,11.13)

$$H=\frac{4\,x.f\,xL1xV\ ^2.}{d1\,x\,2g}+\frac{4\,x\,f\,x\,L2\,X\,V2^2}{d2\,x2g}+\frac{4\,X\,f\,XL3XV3^2}{d3\,x2g}$$

$$15\ =\frac{4\,x.005\,x800\,xV_1^2}{.4\,x2g}+\frac{4\,x.005\,x600\,xV_2^2}{.3\,X2g}+\frac{4V_3^2\,x.005\,x300\,x}{.2\,X2g}\quad(1)$$

As per continuity equation

A1 X V1=A2 XV2 =A3 XV3

Putting values respective areas we get

$$V2=\frac{A1\,VV}{A2}\ = 1.77V1$$

$$V3=\frac{A1V1}{A3}\ =4V1$$

In equation replacing V2 & V3 with above equivalent values of V1 in (eq.1) we get finally

$$15=\frac{V_1^2}{2g}(40+125.2+480)=\frac{V_1^2}{2g}(645.2)$$

$$V_1^2=\frac{294.3}{645.2}=.4561$$

$$V_1=.6753,$$

Q= Discharge =A1 X V1=.6753 X π/4 X .4 X.4

= .0848 m3/sec.

ii) to find diameter of single pipe when the length is 1700m

= 800m+600m+300m

„ d1=.4m, d2=.3m , d3=.2m

As per equation (11.17)

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{1700}{d^5} = \frac{800}{.4^5} + \frac{600}{..3^5} + \frac{300}{.2^5}$$

$$\frac{1700}{d^5} = \frac{800}{.01024} + \frac{600}{.00243} + \frac{300}{.00032}$$

$$\frac{1700}{d^5} = 78125+249913+937500$$

$$= 1265538$$

$$„\ d^5 = \frac{1700}{1265538} = .001343$$

[d=$(.001343)^{1/5}$=.26644 m

266.4mm= diameter of single pipe

Q16

Two pipes of lengths 2500m each and diameter 80 cm and 60cm respectively are connected in parallel. The coefficient of friction for each pipe is .006 The total flow is equal to 250 lit/s. Find the rate of flow in each pipe. [0.1683m3/s & .0817]

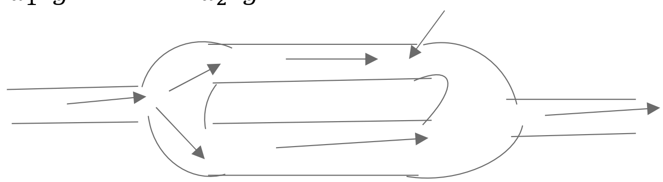
Solution

Given

Q=Q1+Q2

Loss of head of each pipe is same

So $\frac{4\,x f_1 x L_1 x V_1^2}{d_1 2g} = \frac{4\,x f_2 x L_2 x V_2^2}{d_2 2g}$ $\quad L_2 d_2 V_2$



$L_1 d_1 V_1$

$L_1 = L_2$ =2500 m

, f is same for both.=.006

Since f is same for both, =.006

$\frac{L_1 V_1^2}{d_1 2g} = \frac{L_2 V_2^2}{d_2\ \ 2g}$ -------- 1)

$L_1 = L_2 = 2500m$

d_1 =80 cm=.8m, d_2=60cm =.6m

From 1) Equation since L is same

$\frac{V_1^2}{.8} = \frac{V_2^2}{.6}$ SO V_1^2=4/3 V_2^2 SO V_1=1.153 V_2

Q=Q1 +Q2

.25= Q1 +Q2 ----- 2)

Now we know

Q1=A1 V1=π/4 (.8 x.8) x V1=π/4(.8X.8) x1.153 V2

Q2=A2 V2=π/4 (.6 x .6) x V2

Putting above in equation 2)

$.25= V_2[\pi/4(.8 X.8)X\ 1.153 +\pi/4\ X\ (.6 X.6)]$

$=V_2 \frac{\pi}{4}[.73792+.36]$

$V_2=\frac{.25}{.8623}=.2899m/s \quad ,V_1 = 1.153V_2=.3342m/s$

$Q_2=A_2V_2=(\pi/4\ x\ .36)x\ .2899=.0819m3/s$

$Q_1 =.25---.0819= .1679\ m3/s$

Q 17

A pipe of diameter 300mm and length1000 m connects two reservoir, having difference of water level 15m.Determine the discharge through the pipe . If an additional pipe of diameter 300 mm and length 600m is attached to the last 600m length of the existing pipe find the increase in the discharge, f=.o2,neglect main loss.[.0742m3/s ,.0258m3/s]

Solution

Given

1. Q1=Q2 +Q3 , d1=.3m, d2=.3m d=.3 m

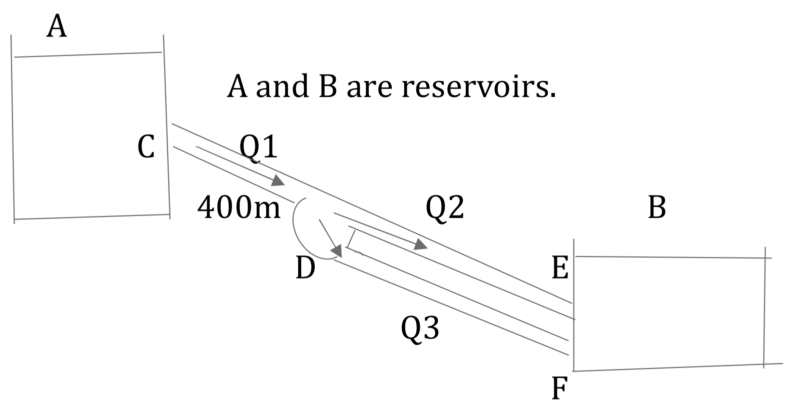
L1=400m, L2 =600 L3=600M

1^{ST} case;----

Discharge through the pipe to two reservoirs by a single pipe

$$H=Head=\frac{4\ f\ LV^2}{d\ x\ 2g}=\frac{4\ f\ L}{dx2g}(\frac{Q}{\pi/4Xd^2})^2=\frac{32xf \quad 2}{\pi^{2.}\ Xgxd^5}$$

$$Q=\sqrt{\frac{Hg.\pi^2 \cdot d^5}{32.f.L}}=\sqrt{\frac{15x9.81x.3^5\pi^2}{32x.02x1000}}=\sqrt{\frac{3.529}{640}}=\sqrt{.0055}=.07416m3/sec$$



CD=400m, DF=DE=600m, CD+DE=1000m

Stage 2

Q1=Discharge CD L1=400m

Q2=Discharge DE, L2=600 m

Q3=discharge DF , L3=600m

Diameter of Q2 and Q3 are equal so Q2=Q3.

Q1=2Q2 ,Q2=Q1/2

Bernoulli's applied to point A & B and taking the flow through CDE we have

$$H=15=\frac{4fL_1V_1^2}{d_1 2g}+\frac{4fL_2V_2^2}{d_2 2g} \text{ ---------------------- } (1)$$

Where $V_1=\frac{Q1}{\pi/4(.3)^2}=\frac{4Q1}{\pi X.09}$ $V_2,,\frac{Q2}{\pi/4X.3^2}=\frac{4XQ1/2}{\pi X.09}=\frac{2Q1}{\pi x.09}$

Replacing value of V_1 $andV_2$ in terms of Q1 we get from

$$15=\frac{4.x.02x400x(\frac{4Q1}{\pi X.09})^2}{.3x2x9.81}+\frac{4x.02x600}{.3x2x9.81}(\frac{2Q1}{\pi x.09})^2$$

Computing above we get ultimately;

$15 = Q_1^2(1088.0+408.25)$

$Q_1 = \sqrt{\frac{15}{1496.25}} = \sqrt{.0100} = .1 m3/s$

So increase in discharge $= Q_1 - Q$

=.1 --- .0741

=.0259 m3/s

Q18

Two sharp ended pipes of diameter 60mm and100mm respectively each of length 150m are connected in parallel between two reservoirs which have different level of 15m.If coefficient of friction for each pipe is .08, calculate the rate of flow of each pipe and also diameter of single pipe 150m long which would give the same discharge if it were substituted for original two pipes . [Ans 0.0017,.00615,110mm]

Solution

Given

Diameter of 1st pipe=d1=60mm=.06m

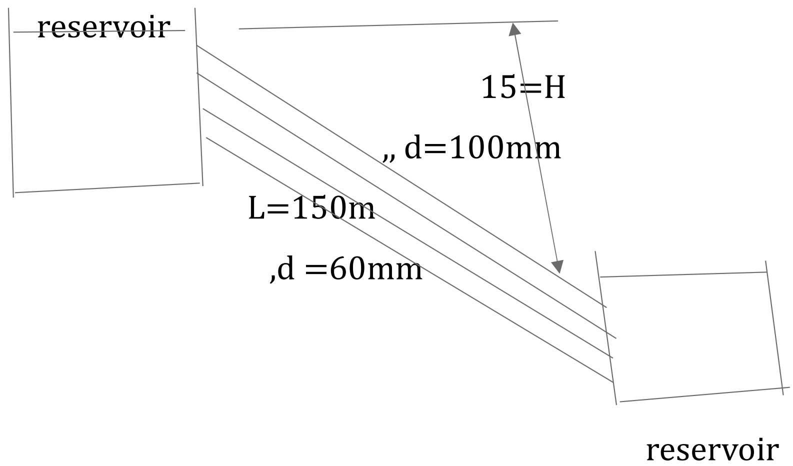
Length L1=150m

2nd pipe d2=100mm=.1m

Length L2=150m

Head =15m, friction =.08

V1=Velocity of pipe 1 , V2= Velocity of pipe 2



When pipes run parallel the loss of head is same for both pipes.

For 1st pipe

$$H=\frac{4fL1\ xV_1^2}{d1x2g}=\frac{4x.08x150xV_1^2}{.06X2X\ \ .81}$$

$$15=\frac{48\ \ _1^2}{1.177}\ .\quad V_1^2=.3678\quad V_1=.666\text{m/s}$$

Rate of flow=Q1=V1 A1= .606 x $\pi/4x.06^2$=.0017m3/s

For 2nd pipe

$$H=\frac{4fL_2V_2^2}{d_2 2g}=\frac{4\ x\ .08\ x150xV_2^2}{.1\ X2\ X9.81}=\frac{48\ \ _2^2}{1.962}=15$$

$$V_2^2=\frac{15\ X1.962}{48}=.613$$

V2=.7829 m/s

Rate of flow Q2=V_2A_2=.7829 x π/4 x (.1 x .1)

Type equation here.

$= .7829 \times .7854 \times .01$

$= .00615 m3/sec$

2nd stage; -----

Replacing by a single pipe 150m length but we have to find diameter so that total discharge remain the same.

Let d i a =D m H=15 m L=150m V= Velocity of flow

Q = Total discharge = Q1+Q2=(.0017+.00615)=.00785m3/s

$$V=\frac{Q}{Area}=\frac{.00785}{\frac{\pi}{4}x\ D^2}=\frac{.00785}{.7854D^2}=\frac{.00999}{D^2} \quad ------ \quad I)$$

Now

$$H=\frac{4fLV^2}{D\ X\ 2g}=\frac{4\ x\ .08\ x150\ xV^2}{D\ X2g}=\frac{4\ x.08\ x150}{DX2g}(\frac{'00999}{D^2})^2=15$$

$$D^5=\frac{4\ x.08\ x150x\ (.00999)^2}{15x2x9.81}=\frac{.00479}{294.3}$$

$$D^5=.00001628$$

$$D=(.00001628)^{\frac{1}{5}}=.11m=.11\ x1000\ mm$$

= 110mm =single pipe diameter.

Q19

Three reservoirs ABC are connected by a pipe system having lengths 700m,1200m & 500m and respective diameters are 400mm 300mm 200mm . The water level in A & B from a datum line 50m and 40 m .The level of water in reservoir C is below the level of B. Find the discharge into or from B&C if the rate of flow from reservoir A 150 lit/s. Find the height water level in reservoir C. Take f=.005 for all pipes.[A; .005m3/s, .095 m3/s,24.16m]

L1=AD=700m , Dia =d1=.4m, Q1=150 ,l/s, Za=50m

L2=DB=1200m , Dia=d2=.3m , Q2=? Z b=45m

L3=DC=500m , di a=d3=.2m , Q3=? Z c= ?

Coefficient of friction = f =.005 for all pipes.

Bernaulli's equation for points A & D

$$Za=Zd+\frac{P_D}{\rho g}+h_f \text{ ------------- } 1.$$

$$\text{Also } h_f=\frac{4fL1\ X\ V_1^2}{d1x2g} \text{ ---------------2.}$$

$$V_1=\frac{Q1}{\pi/4_{d_1^2}}=\frac{.15}{\pi/4x(.4)^2}=1.193\text{m/s}$$

$$V1^2=1.4232\text{m2}$$

Equation 2 above

$$h_{f1}=\frac{4\ x.005\ x7\quad x1.4232}{.4\ x2\ x9.81}=2.5385\text{ m---------- }2(\text{i})$$

Putting values in equation 1.

$(Z_D+\frac{P_D}{P_S}) =(50--- 2.5385) =47.46$--------------2(ii)

So Piezometric head at D=47.46m

Z_B=45m so water flows from D to B. ***

Therefore Q1=Q2 +Q3 ---------------3

Again applying Bernoulli's equation

$$Z_D + \frac{P_D}{\rho g} = Z_B + h_{f_2} \quad \text{------------------ 4}$$

$$47.46 = 45 + h_{f_2}$$

$$h_{f_2} = 2.46 \text{ m}$$

$$\text{But } h_{f_2} = \frac{4f.xL_2.xV_2^2}{d_2x2g} = \frac{4\,x.005\,x1200\,xV_2^2}{.3\,X2\,X\,9.81}$$

$$2.46 = \frac{24V_2^2}{5.886}$$

$$V_2 = \sqrt{.6029} = .7764$$

$$\therefore\ Q2 = V_2 \text{ x } \pi/4(d_2^2) \quad = .7764 \text{ x}.7854 \text{ x } (.3 \text{ x}.3)$$

$$= .054 \text{ m3/s}$$

Bernoulli's equation

$$Z_D + \frac{P_D}{\rho g} = \text{Z c} + h_{f_3}$$

$$47.46 = \text{Zc} + h_{f_3} \text{------------- } 5$$

$$V_3 = \frac{Q3}{\frac{\pi}{4}Xd_3^2} = \frac{Q_1 - Q_2}{.7853\,X(.2)^2} = \frac{.15-.054}{.7853X.04} = \frac{.096}{.0314} = 3.057\text{m/s}$$

$$h_{f_3} = \frac{4f\,xL_3\,xV_3^2}{d_3\,x2g} = \frac{4\,x.005x500\ \ (3.057)^2}{.2\,x2\,x9.81} = 23.815 \text{ m}$$

So $\text{Z c} + h_{f_3} = 47.46$

1 Z c=47.46—23.815=23.645 m (answer is basically matching)

2 Q3=Q1—Q2=.15--.054=.096 m3/s

3 Q2=.054 m3/s (this answer is correct)

Q2O

A pipe of diameter 300mm and length 3000m is used for the transmission of power by water. The total head at the inlet of the pipe is 400m. Find the maximum power available at the outlet of the pipe. Take f=.005. [Ans 667.07kw]

Solution

Given

Diameter of pipe=300mm=.3m

Length of pipe=3000m

Total head at inlet =400m

. f =.005

At maximum transmission of power (11.23)

$$, h_f=\frac{H}{3}=400/3=133.3\text{m} \quad \text{-----------} \quad 1$$

$$h_f = \frac{4fLV^2}{dx2g} = \frac{4\ x.005\ x3000\ xV^2}{d\ x2g} = 10.91V^2 \text{------- } 2$$

Type equation here.

From 1 and 2

$$V=\sqrt{133.3/10.19}=3.616\ m/s$$

Q = Discharge =V x area=3.616 x $\pi/4$ x $(.3)^2$

=0.2554m3/s

Head available at the end of the pipe=H--h_f

=H –H/3

=2/3 H=2/3 x 400

=266.7m

Maximum power available at the outlet

$$=\frac{\rho g\ xQ\ XHead\ at\ end\ of\ pipe}{1000}$$

$$=\frac{1000\ x\ 9.81\ x.2554\ x266.7}{1000}$$

=9.81 x.2554 x266.7 kw

=668.20 kw

Q 21

A Pipe line of length 2100m is used for transmitting 103 kw.

The pressure at the inlet of the pipe 392.4 N/cm2.If the efficiency of transmission is 80 % find the diameter of the pipe.

Take f=.005 [Ans 136mm]

Solution

Given

1. pressure head at inlet $= H = \frac{P}{\rho g} = \frac{392.4\ X 10^4}{1000\ X\ 9.81} = 400m$

2. pressure drop =

Let diameter of the pipe = d having efficiency μ 80%=.08

„μ = given by e q u (11.22) = $\frac{H-h_f}{H}$ = .8

$H - h_f = .8\ H = .8 \times 400 = 320$

$h_f = 400 — 320 = 80$ m ---- (i)

3 now P=103 KW given.

L=2100m

, f =.005

Using equation for power transmission

$$P = \frac{\rho g\ xQ\ x(H-h_f)}{1000}$$

$$= \frac{1000\ x9.81\ xQ\ X(400- \quad)}{1000}$$

$$Q = \frac{P}{9.81\ x\ 320} = \frac{103}{9.81\ x320} = .0328 \text{ m3/s}$$

4 $Q = \pi/4 \text{ x } d^2V = .0328$

$$V = \frac{.0417}{d^2}$$

5. head loss due to friction $h_f = \frac{4fLV^2}{d\ x2g}$

$$\text{Putting value, } 80 = \frac{4\ x.005\ x2100\ x.0417}{2\ x\ 9.81\ xd\ x(d^2)^2}$$

$$80 = \frac{.0730}{19.62 \quad 5}$$

$$\therefore \quad d^5 = \frac{.0730}{1569.6} = .0000465$$

„ d= $(.0000465)^{1/5}$=.13598 m

= 135.98mm=diameter of pipe.

Q22

A nozzle is fitted at the end of the pipe of length 400m and of diameter 150mm for maximum transmission of power through nozzle. Find the diameter of nozzle. Take f=.008 [Ans 41.5mm]

Solution

Given

L= Pipe length=400m

Diameter=D =150mm=.15m

, f =.008

Let diameter of nozzle at max. efficiency μ_{max}=d

For max transmission of power the diameter of nozzle is given by equation relation (11.29)

$$, d=\left(\frac{D^5}{8fL}\right)^{\frac{1}{4}} = \left\{\frac{.15^5}{8\,x\,.008\,x400}\right\}^{\frac{1}{4}}$$

$$= \left(\frac{.000076}{25.6}\right)^{\frac{1}{4}}$$

$$= (.000002968)^{.25}$$

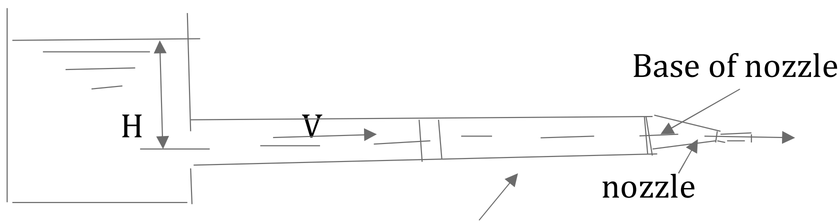
=.04149 m

= 41.49 mm = diameter of nozzle.

Q23

The head of water at the inlet of a pipe of length 1500m and of diameter 400mm is 50m. A nozzle of diameter 80 mm at the outlet is fitted to the pipe.. Find the velocity of water at the outlet,

If f=.01 for the pipe. [28.12 m/s]



H=Total head at inlet of pipe=50m

V=Velocity of flow in pipe

D=diameter of pipe =400m =.4m

, d=diameter of nozzle=80mm =.08m

„ v =velocity of flow at outlet of nozzle

A=area of pipe

L=Length of pipe=1500m

„ f =.01 for pipe.

Velocity of water at outlet is given by equation(11.25)

$$„\ v=\sqrt{\frac{2gH}{(!+\frac{UfL}{D}(\frac{a}{A})^2}} = \sqrt{\frac{2x9.81x5}{1+\frac{4x.01x1500}{.4}(\frac{\pi/4d^2}{\pi/4D^2})^2}}$$

$$=\sqrt{\frac{981}{1+150\{(\frac{.08}{.4})^2\}^2}} = \sqrt{\frac{981}{1+150(\frac{.0064}{16})^2}}$$

$$=\sqrt{\frac{981}{1+.24}} = \sqrt{791.129}$$

=28.12 m/s

Q24

The rate of flow of water in a pipe of length 1500m and diameter 800mm is 2m3/s . At the end of pipe a nozzle of outside diameter 200mm is fitted Find the power transmitted through nozzle if the head of water at the inlet of pipe is 180m and f =.01 for pipe [2344.7kw]

Solution

Given

Length of pipe =1500m

Dia of pipe=800mm=.8m

Discharge of pipe =2m3/s=a. v

Dia of nozzle =d=200mm=.2 m

Head of pipe =H=180 m

„f =.01

Area of pipe =π/4 x DXD=π/4 .8 x.8=.5026m2

Velocity of water through pipe =V=Q/A

=2/.5025

=3.98 m/s

Power transmitted through the nozzle(11.27)

$$P=\frac{\rho g(a.v)}{1000}[H--\frac{4fLV^2}{DX\,2g}]$$

$$=\frac{1000\,x9.81\,x2}{1000}[180-\frac{4\,x.01\,x1500\,x(3.98)^2}{.8\,x2\,x9.81}]$$

$$=19.62\ [180---\frac{950.4}{15.696}]$$

=19.62 x119.45

=2343.609 kw

Q25

The water is flowing with a velocity of 2m/s in a pipe of length 2000m and of diameter 600mm. At the end of the pipe valve is provided. Find the rise in pressure if the valve is closed in 20second. Take the value of C=1420m/s [20 N/cm2]

Solution velocity of water =V=2m/s ,, length = 2000m

Diameter of pipe =D=600mm=.6m, time to close valve=20sec.

Value of C=1420

Let the rise in pressure =P

The ratio -=$\frac{2L}{C}$ =$\frac{2\ X2000}{1460}$ =2.74

From equation (11.33) we have

T> 2L/C

20> 2.74 Hence valve is closed gradually.

For gradually closed valve the rise in pressure is guided by equation (11.31)

P=ρ v L/T=$\frac{1000\ X2\ X2000}{20}$ = 200000 N/m2

=$\frac{200000}{10^4}$ N/cm2

=20 N/cm2

Q26

If the valve in prob 25 is closed in 1.5 secs find the rise in pressure. Take bulk modulus of water 19.62 x 10^4 and consider pipe is rigid one. [Ans 186.75N/cm2]

Solution

Given

Velocity =2m/s

Valve closing time=T=1.5 sec.

D=600mm=.6m

K= bulk modulus of water=19.62 x10^4N/cm2

= 19.62 x10^8N/m2

Velocity of pressure wave C=$\sqrt{\frac{K}{\rho}} = \sqrt{\frac{19.62\ X10^8}{1000}}$

$=\sqrt{19.62\ x\ 10^5}$

$=\sqrt{1962000}$

=1400.71

Ratio =2L/C= 2 x2000/1400.7

=2 .85 ---------- 1

2.85 > 1.5 (T) given

From Eq=(11.34) if T< $\frac{2L}{C}$, Valve closes suddenly, here 1 satisfies the same and also pipe is rigid as given in question, hence

Rise in pressure will follow equation 11.35 or 11.36 –

So P=V$\sqrt{K\rho}$ =2$\sqrt{19.62\ x10^8\ x1000}$

=2 x$10^4\sqrt{19.62\ x10^3}$

=2 x140,07 x 10^4 N/m2

=280 -14 N/cm2

Type equation here.

Note ; I have followed the process and checked thoroughly even result has not matched. Students may recheck .

Q27

If in problem 25 thickness of the pipe is 10mm and valve closed suddenly find the rise in pressure if the pipe considered to be elastic Take value of E=19.62 X 10^6N/cm2 for pipe material k=19.62 x 10^4N/cm2. Calculate circumferential stress and longitudinal stress in pipe wall. [p=221.47 fc=6644.1 fl=3322N/cm2]

V=velocity of water=2m/s

L= length of pipe =2000m

D= 600m=.6m

, t =thickness of pipe =10mm=.01m

E=19.62 X10^6N/cm2=19.62 x $10^6 x 10^4$N/m2

K=19.62 x$10^4 10^4 N/cm2$ =19.62 x 10^8 N/m2

a) For closer of valve for an elastic pipe the rise in pressure is given by equation (11.37)

$$P= V x \sqrt{\frac{\rho}{(\frac{1}{k}+\frac{D}{E.t})}} =2\sqrt{\frac{1000}{\frac{1}{19.62\ x10^8}+\frac{.6}{19.62\ x10^{10}\ x.01})}}$$

$$=2\sqrt{\frac{1000}{5.09\ x10^{-1}\ +3.05\ x1\ ^{-10}}} =2\sqrt{\frac{1000}{10^{-10}(5.09+3.05)}}$$

$$=2\sqrt{\frac{1000\ x10^{10}}{8.14}} =2\ x10^6 x\sqrt{1.228}$$

$$=2216000N/m2$$

Type equation here.

$=221.6$ N/cm2

b) circumferential stress

$$=\mathrm{fc}=\frac{P\ xD}{2t}\ =\frac{221.6\ x.6}{2\ x.01}$$

$$=\frac{132.96}{.02}=6648\ \mathrm{N/m2}$$

Longitudinal stress

$$=f_l=\frac{P\ xD}{4t}=\frac{221.6\ x.6}{4x.01}$$

$$=\frac{132.96}{.04}$$

$$=3324\mathrm{N/m2}$$

END OF CHAPTER 11

Fluid Mechanics and Hydraulic Machines

Boundary Layer Flow

Chapter 13

Q1

Find the ratio of displacement thickness to momentum thickness and momentum thickness to energy thickness for the velocity distribution in the boundary layer given by

$\frac{u}{U}$ =2(Y/δ)—(Y/δ)^2 U=Free stream velocity.

,u=velocity in boundary layer.

,δ =boundary layer thickness.

b) find the displacement thickness, momentum thickness and energy thickness for the velocity distribution in the boundary layer given by

u/U=2 (Y/δ)—$(Y/\delta)^2$

solution

1.displace thickness is given by equation (13.2)

$$\delta * = \int_0^\delta (1 - \frac{u}{U}) dy$$

$$= \int_0^\delta (1 - \{2(\frac{y}{\delta} — (\frac{y}{\delta})^2\} dy$$

$$= \int_0^\delta dy - 2\int_0^\delta \frac{y}{\delta} dy + \int_0^\delta (\frac{y}{d})^2 dy$$

$$= \delta — 2/\delta \text{ x} \frac{\delta^2}{2} + \frac{1}{d^2}\ x \frac{d^3}{3} + c$$

$$= \delta — \delta + \delta/3 + c$$

$$= \frac{\delta}{3} + c$$

Momentum thickness $\theta = \int_0^\delta \frac{u}{U}(1 - \frac{u}{U}) dy$

Putting the given value of u/U we get

$$=\int_0^{\delta}\{2(\frac{y}{\delta})-(\frac{y}{\delta})^2\}[1-\left\{2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2\right\}\Big]\,dy$$

$$=\int_0^{\delta} 2(\frac{y}{\delta})-\left(\frac{y}{\delta}\right)^2\Big\}-[4(\frac{y}{\delta})^2+\{(\frac{y}{\delta})^2\}^2-2.2\left(\frac{y}{\delta}\right)x(\frac{y}{\delta})^2]\mathrm{dy}$$

$$=\int_0^{\delta}[2\,\frac{y}{\delta}-5(\frac{y}{\delta})^2+4(\frac{y}{\delta})^3-\left(\frac{y}{\delta}\right)^4\Big]\,dy$$

$$=\ 2/\delta[y^2/2]_0^{\delta}{-}{-}{-}\frac{5}{\delta^2}\frac{y^3}{3}]_0^{\delta}+\frac{4}{\delta^3}\,[\frac{y^4}{4}]_0^{\delta}\ {-}{-}{-}1/\delta^4[\frac{y^5}{5}]_0^{\delta}$$

=δ –5/3 δ+4δ/4 –δ/5

$$=\frac{120\delta--112}{60}\ =(8/60\,)\ \ \mathrm{x}\ \ \delta\ \ =\frac{2}{15}\,\delta$$

Hence the Ratio of $\frac{displacement\ \ thickness}{momentum\ thickness}=\frac{\frac{\delta}{3}}{\frac{2\delta}{15}}$

= δ/3 x 15/2δ

=15/6

=2.5

Note; Energy thickness is left for students to try out.

Q2

For the velocity profile in laminar boundary layer gives us as

u/U=3/2(Y/δ)---1/2(y/δ $)^2$ find

1, the thickness of boundary layer and shear stress 1.8m from leading edge of a plate. The plate is 2.5 m long and 1.5m wide and placed in water which is moving with velocity of 15cm/s.. Find the drag on one side of the plate if the viscosity of water .01 poise

[1.6cm,.0014N/m2, .0889N]

Solution

Given

L=2.5m=long plate

. b=1,5m=width of plate

U=15 cm/s=.15m=velocity of plate in water

, μ =.01 poise =.001Ns/m2

X=1'8m from leading edge.

Re= Reynold number at the end of plate

$$Re= \frac{\rho UL}{\mu} = \frac{1000\ X .15 X\ \ .5}{.001} = 375000$$

1.Euation (13.17)

, δ =5.48 x/√Re =5.48 x 1.8/√(375000) =.0160m

=1.6cm =thickness of boundary layer.

11.

Velocity distribution

u/U =[3/2(Y/δ) ---1/2(Y/δ)^2]

Equation13.10

$$,\frac{\tau}{\rho U^2} = \frac{d}{dX}\left[\int_0^{\delta} \frac{u}{U}(1 - -\frac{u}{U})dy\right]$$

Putting values of u/U on above equation we get

$$\frac{T_0}{\rho U^2} = \frac{d}{dx}\int_0^{\delta}\ \left[\left\{\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right\}\right]\left[1 - \left\{\frac{3}{2\left(\frac{y}{\delta}\right)} - 1/2\left(\frac{y}{\delta}\right)^2\right\}\right]$$

Continuing the calculation further we get

$$=\frac{\delta}{\delta x}[\int_0^{\delta}\frac{3y}{2\delta}\text{---}11/4\frac{y^2}{\delta^2}+3/2\frac{y^3}{\delta^3}\text{---}\frac{y4}{\delta^4}]_0^{\delta}$$

After completing the integration we get intermediate result of our calculation

$$=\frac{\delta}{\delta x}[3/4 \text{ x } \delta \text{---}11/12 \text{ x } \delta + 3/8 \text{ x } \delta \text{---} 1/20 \text{ x } \delta]$$

$$=\frac{\delta}{\delta x}[\frac{90-110+45-6}{120}]\delta$$

$$=\frac{\delta}{\delta x}(\frac{19}{120})\delta = \frac{19}{120}\frac{\delta}{\delta x}(\delta)$$

$$T_0 = \frac{19}{120}\rho U^2 \frac{\delta}{\delta X}(\delta) \qquad (13.15)$$

As per Newton's law of viscosity

$$T_{0=} = \mu\,(\frac{du}{dy}) \quad y=0 \quad \text{-------} \quad ii$$

As given in question

„u=U[3/2(Y/δ) –1/2 (y/δ)^2]   (Y=0)

by Differentiation above we get

$$\frac{du}{dy} = U[\frac{3}{2\delta} - \frac{2Y}{2\delta^2}] \qquad Y=0\delta$$

$$=3U/2\delta$$

$$T_0 \quad = \mu\frac{3U}{2\delta} = \frac{3\mu\, U}{2\delta} \quad \text{-------} \quad iii$$

Equating 13.15 and iii above

$$\frac{19}{120}\rho\, U^2\frac{\delta}{\delta X}(\delta) = 3Mu/2\delta$$

$$\int\frac{\delta}{\delta x}(\delta) = \int\frac{\delta}{\delta x}\frac{3\mu}{2\delta}x\frac{120}{19\rho U^2} = \int\frac{\delta}{\delta x}\frac{180\mu}{19\rho U} \qquad iv$$

On integrating we get

$$\frac{\delta^2}{2} = \frac{180}{19} \text{x} \frac{\mu X}{\rho U}$$

$$;;\delta = \sqrt{\frac{180X2}{18} \cdot \frac{\mu X}{\rho U}} = \sqrt{10\ x2\ \frac{\mu x}{\rho U}} = \sqrt{20} \quad \sqrt{\frac{\mu X^2}{\rho U_X}}$$

$$= \sqrt{20}\ \text{X}\ \frac{1}{\sqrt{R_{Ex}}} = \frac{4.472}{\sqrt{R_{Ex}}}$$

Now taking the equation iii above as follows;

$$T_0 = \text{Shear stress} = \frac{3\mu U}{2\delta} \quad \text{---} \quad \text{iii}$$

$$= \frac{3\ x.001x.15}{2\ x.0161} = .01397 \text{N/m2}$$

III

$$\text{Drag force } F_D = .73\ \text{b}\ .\mu\ \text{U} \sqrt{\frac{\rho UL}{\mu}} \quad \text{(13.18) equation)}$$

$$= .73\ \text{x}1.5\ \text{x}\ .001\ \text{x}.15 \sqrt{\frac{1000\ x.15\ x2.5}{.001}}$$

$$= 0.73\ \text{x}1.5\ \text{x}.001\ \text{x}612.37\ \text{x}.15$$

$$= .670\ \text{x}.15 = .1005\text{N}$$

Q3

Air is flowing over a smooth plate with a velocityof8m/s. The of plate is1.5m long and width 1 m. If the laminar boundary exits up to a value of Reu =5 x10^5, find find the maximum distance x from leading edge up to which laminar layer exists. Find also ii)maximum thickness δ of laminar body boundary layer if the velocity profile is given by

$u/U = (Y/\delta)$ ---$(Y/\delta)^2$ [Ans -9375m,7.26mm]

Take v=kinematic viscosity for air=0.15stoke =.15x $10^{-4} m2/s$

μ/ρ=v =kinematic viscosity of air.

Solution

Given

Velocity of air =U=8m/s

Length of plate =L=1.5m

Width of plate =b=1m

Re =Reynold number =5 x 10^5

1 To find x =maximum distance from leading edge up to which laminar boundary exists ;--

$$Re=\frac{\rho UX}{\mu} = \frac{UX}{\frac{\mu}{\rho}} = \frac{UX}{v} = 5 \text{ x}10^5$$

$$X=\frac{5x10^5 x.15x1^{-4}}{8} = \frac{7.5}{8} = .9375\text{m}$$

11

To find δ =thickness of laminar boundary;

Equation (13.17) states

$$,\delta = 5.48 \frac{x}{\sqrt{Re}} \qquad [Re=\frac{\rho UX}{\mu} [$$

Since Re is a number hence is a constant and hence thickness is proportional to distance upon square root of Re.

$$, \quad \delta = \frac{5.48\ x.9375}{\sqrt{5\ x10^5}} = .007265\text{m}$$

$$=7.26\text{mm}$$

Q4

In problem 3 the velocity profile is given by $\frac{u}{U} = Sin(\frac{\pi}{2} x \frac{y}{\delta})$ and density of air 1.24kg/m2 find

i)maximum thickness of boundary layer ii)Shear stress at 20cm (x)from leading edge iii) drag force on one side of the plate assuming laminar boundary layer over the entire length of the plate. [Ans 0,635 cm ii)0.099 N/m3 iii).0871N]

Solution

Given

i) value of δ ,thickness of boundary layer at distance x = .9375m in Q3

So δ (eq ; 13.32)

$$= \frac{4.795x}{\sqrt{Rex}} = \frac{4.79 \quad ..9375}{\sqrt{5x10^5}}$$

=.635 cm

ii) velocity profile

=u/U=sin (π/2 x y/δ)

Shear stress =To=$0.327\frac{\mu U}{X}\sqrt{Re}$x

$$Rex = \frac{\rho Ux}{\mu} = \frac{UX}{\frac{\mu}{\rho}} = \frac{Ux}{v}$$

$$= \frac{8\ x\ .\ 2}{.15\ x10^{-4}} = 1.066 \text{ x } 10^5$$

$$To = .327\frac{.186\ x10^{-4} x8\ x\sqrt{1.066\ x10^5}}{.2}$$

=.079455 N/m2

iii) drag force on one side of the plate(basis velocity profile)

$$=0.655\mu Ub\sqrt{\frac{\rho UL}{\mu}} = .655\ X.186x\ 10^{-4}\ x8\ x1\sqrt{\frac{UL}{v}}$$

$$-=0.655x\ .186x10^{-4}x8x1\sqrt{\frac{8\ x1.5}{.15\ x10^{-4}}}$$

$$=.97464\ x10^{-4}\ x\sqrt{800000}$$

$$=\frac{.97464x894.427}{10^4}$$

$$=\frac{871.744}{10^4}$$

$$=.0871N$$

Q5

A thin plate is moving in still atmospheric air at a velocity of 4m/s. The length and width of the plate are .5 and .4m .

Calculate i. Thickness of boundary layer at the end of the plate .

II .Drag force on one side of plate. Density of air =1.25kg/m3.kinematic of viscosity .15stokes=.15x 10^{-4} m2/s

[Ans i..672cm ii 00728]

Solution

Given

U=4m/s   L=.5m   b=.4m

, ρ =1.25kg/m3, v=k viscosity=.15 $x10^{--4}$

$$Re = \frac{UL}{v} = \frac{4\ x.5}{.15\ x\ 10^{-4}} = 133333.33 < 5\ x10^5$$

Hence boundary layer is laminar over entire length.

i) as per Blasius solution

boundary layer for entire length δ

$$=\frac{4.91}{\sqrt{Rex}} = \frac{4.91\ xL}{\sqrt{133333.33}}$$

$$=\frac{2.455}{365.15} = .00672\text{m}$$

$$= .672\text{cm}$$

Ii) Drag force on one side (13.14)

C_D=Co efficient of drag=$\frac{F_D}{\frac{1}{2}x\rho AU^2}$

$$F_D = C_D\ x\frac{1}{2}\ x\rho AU^2$$

C_D= as pr Blasius solution=$\frac{1.328}{\sqrt{133333.33}}$ =.003636

So $F_D = C_D \frac{x1}{2} x\ \rho AU^2$

=.003636 X ½ x 1.25 x(.5 x.4) x 4^2

=.003636 x.5 x1.25 x.2 x16

= .00727 N

←∗Students to note ; I f no velocity profile is given and boundary layer is found laminar then only Blasius solution should be applied.

Q6

Find i) frictional drag on one side of the plate 200mm wide and 500mm long placed longitudinally in a steam of crude oil(sp .925,v=k .vis= .9stokes =.9 x 10^{-4}m2/s) flowing with undisturbed velocity of 5m/s .Also find ii) thickness of boundary layer iii) shear stress at the trailing edge of the plate [I 9.34 N,14.75 mm]

Solution

Given

L=500mm=.5m

. b=200mm=.2m

Crude oil s p g r=.925=s. ρ=density =.9x1000

, v =k.v= .9stokes =.9x 10^{-4}m

,U =.5m/s

Re $=\frac{UXL}{v}=\frac{5x.5}{.9x10^{-4}}$ =27777.77<5x10^5

So boundary layer is laminar.

i) drag force =1/2 ρ A$U^2 x C_D$

=1/2x900x(.5x.2)x$(5)^2$xC_D

[$C_D=\frac{1.328}{\sqrt{ReL}}=\frac{1.328}{\sqrt{27777.7}}$ =.007968]

F_D =1/2 x 900x (.5 x.2)x $(5)^2 x.007968$

=8.964 N

ii) thickness of the boundary at the end of plate from Blasius

solution. „$\delta=\frac{4.91}{\sqrt{ReL}}$ X=L=.5

$=\frac{4.91X.5}{\sqrt{27777.7}}=\frac{4.91\quad .5}{166.66}$ =.01473m

=14.73mm

iii) Shear stress =0.332 $\frac{\rho U^2}{\sqrt{ReL}}=\frac{.332X25X90}{166.66}$

=44.83 N/m2

Q7

A smooth flat plate of length 5 m and width 2m is moving with a velocity of 4m/s in stationary air of density 1.25 kg/m3 and kinematic viscosity 1.5 x 10^{-5}m2/s.

Determine i) thickness of the boundary layer at the trailing edge of the smooth plate. Find also ii) total drag on one side of the plate assuming boundary layer turbulent from the very beginning.

[Ans I 110mm ii 0.43 N

Solution

Given

L=5m, b=2m, U=4m/s,ρ=1.25kg/m3
,v=1.5x10^{-5} ,v=μ/ρ ,μ= v x ρ =1.5 x 10^{-5}x1.25

1. to find thickness of turbulent boundary layer on flat plate

At trailing edge x=L (equation13.40)

,, $\delta = 0.37(\frac{\mu}{\rho U})^{1/5} x X^{4/5}$ (X=L, μ/ρ=v)

$= 0.37(\frac{1.5\,x1\quad^{-5}}{4})^{\frac{1}{5}} x 5^{\frac{4}{5}}$

$= 0.37\ \text{x}(.00000375)^{\frac{1}{5}} x(5)^{\frac{4}{5}}$

=0.11018m

=110.1mm

ii)$Re_L = \frac{\rho UL}{\mu} = \frac{UL}{\frac{\mu}{\rho}} = \frac{UL}{v} = \frac{4x5}{1.5x10^{-5}} = \frac{20}{1.5\,x1\quad^{-5}} = \frac{2}{1.5}\ x10^6 = 1.33x10^6$

$since\ ReL\ above\ > 5\ x10^5 but$
$< 10^7\ so\ equation\ 13.43\ will\ be\ applicable\ as\ Von\ Karma\ integral\ equati$
$-$

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2} = \frac{.072}{(ReL)^{\frac{1}{5}}} = \frac{.072}{1.33\ x10^6)^{\frac{1}{5}}} = .0043$$

$$F_D = C_D \frac{1}{2} \rho A U^2 \ = .0043 \text{ x.5 x1.25x10 x 16}$$

$$= .43 \text{ N}$$

Q 8

Water is flowing over a smooth thin plate of length 4.5m and width 2.5m at a velocity of .9m/s. If the boundary layer flow changes from laminar to turbulent at R no5 x 10^5find i) distance from leading edge up to which boundary layer is laminar ii) thickness of boundary layer at the transition point and iii) drag force on one side of the plate. Dynamic viscosity of water as .01 poise =.01 x10^{-1} Ns/m2

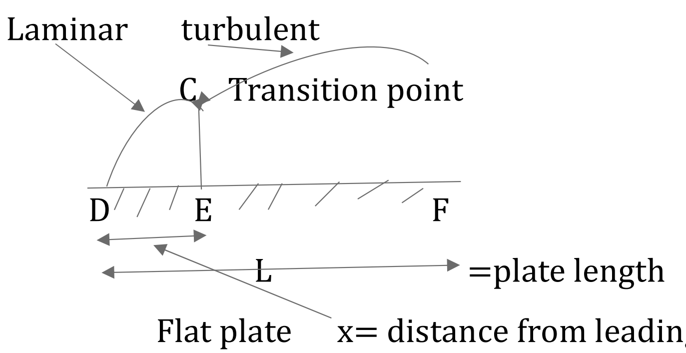
Solution

Given

L=4.5m b=2.5m U= .9m/s ,Re=5 x10^5

X=distance from leading edge up to which laminar layer exists

, μ =.01 poise=.001 Ns/m2



x= distance from leading edge up to which boundary layer laminar

i) x= ?

$\mathrm{Re} = \frac{UX}{v}$ so $x = \frac{Rev}{U} = \frac{Re\mu}{U\rho} = \frac{5\,x10^5\,x.001}{.9\,x100} = \frac{5}{.9}x\frac{1}{10}$

x =.555m=555mm

ii) δ =?Type equation here.

We will follow Blasius solution to find out layer thickness at transition point C above;

, $\delta = \frac{4.91}{\sqrt{Rex}} = \frac{4.91x.555}{\sqrt{5\,x10^5}} = \frac{2.72505}{707.1}$ =.00385m=3.85mm.

iii) Drag force on one side of the plate

a) F_D due to laminar B ,L + b) F_D due to Turbulent B.L

a) $F_D = \frac{1}{2}\rho A\, U^2\, xC_D$

[Blasius solution $C_D = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{5x10^5}}$ =.001878

A=up to transition point (.555) x 2.5=1.3875m2]

=1/2 x1000 x1.3875 x(.9x .9)x .001878 =1.0553N

b) F_{DF} (Due to turbulent boundary layer (D to F) ---F_{DE} drag force due to turbulent flow from D to E

F_{EF} =1/2 X ρ .A, $U^2 C_D$

[$C_D = \frac{.072}{ReL)^{\frac{1}{5}}}$, $\mathrm{Rel} = \frac{\rho UL}{\mu} = \frac{1000X.9X4.5}{.001}$ =4.05X 10^6

$= \frac{.072}{(4.05\,X10^6)^{\frac{1}{5}}} = \frac{.072}{20.96}$ =.003435]

$F_{DF} = \frac{1}{2}X\rho AU^2\; XC_D = \frac{1}{2}$ X 1000 X 4.5 X2.5 .9X.9X.003435

=15.650 N

$F_{DE} = \frac{1}{2}\; X\; \rho A\; U^2 C_D$

[A=.555 X2.5=1.3875m2]

$$[\ C_D = \frac{.072}{R_{DE}^{\frac{1}{5}}} = \frac{.072}{(5X10^5)^{\frac{1}{5}}} = \frac{.072}{13.979} = .00522]$$

$$= \frac{1}{2} x1000x1.3875\ x(.9x.9)x.00522$$

$$= 2.933\text{N}$$

Therefore drag force due to turbulent boundary layer from EF

$$= F_{DF} - F_{DE} = 15.65 --- 2.933 = 12.717\text{N}$$

Drag force on the plate on one side

= drag force due to laminar boundary layer from D up to E +drag force due to turbulent boundary layer from E to F area.

=1.0553N+12.717N=13.76N

Q9

For the velocity profile given below state whether the boundary layer has separated or on the verge of separation or will remain attached with surface [i remain attached ii has separated iii. Verge of separation

Velocity profile i) $\frac{u}{U} = 2(\frac{y}{d}) - (\frac{y}{d})^2$

ii) $\frac{u}{U} = -2\left(\frac{y}{d}\right) + \frac{1}{2}(\frac{y}{d})^3$

iii) $\frac{u}{U} = \frac{3}{2}(\frac{Y}{d})^2 + (\frac{Y}{d})^3$

i) $\frac{u}{U} = 2(\frac{y}{d}) - (\frac{y}{d})^2$ OR, u= 2U(y/d)--- U(Y/d)^2

du/d y =2(U/d) – 2U(y/d) x 1/d

At y=0 $\frac{du}{dy} = \frac{2U}{d}$ --- 0= $\frac{2U}{\delta}$ -------- 1.

As 1 above is positive hence flow will not separate or flow will remain attached.

ii) $u = -2U(\frac{Y}{d}) + \frac{U}{2}(\frac{y}{d})^3$

$$\frac{du}{dy} = -2U\left(\frac{1}{d}\right) + \frac{U}{2}x3(\frac{y}{d})^2(\frac{1}{d})$$

At y=0 =--2U($\frac{1}{d}$)+U/2 X 0 X 1/d

=--- $\frac{2U}{\delta}$ which is negative. So flow has separated.

iii) „ u =$\frac{3u}{2}(\frac{y}{d})^2 + \frac{U}{2}(\frac{Y}{d})^3$

$$(\frac{du}{dy}) = \frac{3U}{2}X2\left(\frac{Y}{d}\right)x\frac{1}{\delta} + \frac{U}{2}x3(\frac{Y}{d})^2x\frac{1}{\delta}$$

At y=0 , $(\frac{du}{dy})_{y=0}$=3U($\frac{0}{\delta}$)$x\frac{1}{\delta} + \frac{3U}{2}(\frac{0}{d})^2$x 1/δ

= o + o

=0

Since $(\frac{du}{dy})_{y=0} = 0$, the flow is on the verge of separation.

Q10

Oil with free stream velocity of 1.5m/s flow over a thin plate1.4m wide and 2.2 m long.

Calculate i) boundary layer thickness ii) shear stress at the trailing end point and iii)determine total surface resistance of the plate. Take s p g r=.80 and kinematic viscosity 0.1 stoke-=10^{-5}

[Ans 1.88cm ii 1.04N/cm2 iii 12.8N]

Solution

Given

U=1.5m/s, b=1.4m. L=2.2 m A=L x b =1.4 x2.2 =3.08

S p=.8 ρ=1000 x. .8=800kg/m3 , v=.1stoke=10^{-5} m2/s

Reynold no=$Re_L = \frac{UL}{v} = \frac{1.5\ x2.2}{10^5} = 3.3 \text{ x } 10^5$ -------- i)

Since Re_L in i) above $< 5 \text{ x}10^5$

Boundary layer is laminar.

Hence Blasius solution will be applicable

i) B. Layer δ=thickness=$\frac{4.91}{\sqrt{Re_L}} = \frac{4.91X2.2}{\sqrt{3.3X10^5}} = \frac{4.91\ X2.2}{574.456} = .0188\text{m}$

$= 1.88\text{cm}$

ii) Shear stress at the trailing end (end of the plate) (L=2.2m)

$=.'332\frac{\rho U^2}{\sqrt{Re_L}} = \frac{.332X1.5^2X(1000\ \ .8)}{\sqrt{3.3x10^5}} = \frac{597.6}{574.456} = 1.04 \text{ N/m2}$

iii) total surface resistance of the plate =2xF_D [F_D=Drag force]

$=2 \text{ x}(\frac{1}{2}\ \rho\ A\ U^2 x C_D) = (\rho \text{ A } U^2 C_D)$

[Where $C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{3.5\ X10^5}} = .0023$]

=800 x(1.4 x2.2) x (1.5x1.5) x .0023

=12.7512 N

Q 11

For velocity profile u/U =3/2(y/d) ---1/2(y/d)^2

Calculate the coefficient of drag (C_D)in terms of Reynold's no.

b) A smooth plate of 0.3m width and 1.0m length moves 4.0m/s velocity in still atmospheric air of density 1.20 kg/m3 and kinematic viscosity1.49x 10^{-5} ii} calculate the drag force on this plate. [.00716 N]

Solution b)

Given

Plate width=b=.3m, L=1m.   U=4.0 m in still atmosphere.

Air density=1.49 x$10^{-5} m3/s$, k. viscosity=1.20kg/m3

The drag force on the plate = $F_D = 1/2\ \rho\ AU^2 C_D$

$$R_{e_L} = \frac{UL}{v} = \frac{4x1}{1.49x10^{-5}} = 2\text{-}68 \times 10^5$$

$$C_D = \frac{1.328}{\sqrt{R_{e_L}}} = \frac{1.328}{\sqrt{2.68\ X10^5}} = \frac{1.328}{517.687} = .002565$$

F_D =Drag force =$\frac{1}{2}$ ρ A $U^2 C_D$

= ½ x 1.20 x (1 x .3) x(4 x4) x .002565

=.00738 N

Q 12

Find the displacement thickness momentum thickness and energy thickness for the velocity distribution in the boundary layer given by

$\frac{u}{U} = 2(\frac{Y}{\delta}) - (\frac{Y}{\delta})^2$ where δ is boundary layer thickness,

[Ans δ/3, 2δ/15 , 22/105 δ .]

Solution

In Q 1 of this chapter solution for displacement thickness and momentum thickness have been given and hence the same is not repeated here

However the calculation of Energy thickness is done as follows;-

Energy thickness δ∗∗ (equation 13.6)

$$=\int_0^\delta \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy \quad \text{(Putting value of u/U)}$$

$$=\int_0^\delta 2(\frac{Y}{\delta})---(\frac{Y}{\delta})^2[1--\left(\frac{u}{U}\right)^2\right] dy$$

$$=\int_0^\delta [\left(\frac{2y}{\delta}\right) - (\frac{y^2}{\delta^2})][1 - \{\frac{2y}{\delta} - \frac{y^2}{\delta^2}\}^2]\mathrm{dy}$$

$$=\int_0^\delta [\frac{2y}{\delta} - (\frac{y^2}{\delta^2}) \quad][1 - \{\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^{4}-} - \frac{4y^3}{\delta^3}\}]\mathrm{dy}$$

$$=\int_0^\delta [\frac{2y}{\delta} - \frac{8y^3}{\delta^3} - \frac{2y^5}{\delta^5} + \frac{8y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5}] \mathrm{dy}$$

$$=\int_0^\delta \frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{8y^3}{\delta^3} + \frac{12y^4}{\delta^4} - \frac{6y^5}{\delta^5} + \frac{y^6}{\delta^6}]dy$$

$$=[\frac{2y^2}{2\delta} - \frac{y^3}{3\delta^2} - \frac{8y^4}{4\delta^3} + \frac{12y^5}{5\delta^4} - \frac{6y^6}{6y^5} + \frac{y^7}{7\delta^6}]_0^\delta$$

Putting the value of y in terms of δ we get

$$=\frac{\delta^2}{\delta} - \frac{\delta^3}{3\delta^2} - \frac{2\delta^4}{\delta^3} + \frac{12\delta^5}{5\delta^4} - \frac{\delta^6}{\delta^5} + \frac{\delta^7}{7\delta^6}$$

=(δ---δ/3--2δ+12δ/5 –δ + δ/7)

$$=\frac{-210\delta - 35\delta + 252\delta + 1}{105}$$

Type equation here.

$$=\frac{22\delta}{105}$$

END OF CHAPTER 13

Type equation here.

Fluid Mechanics and Hydraulic Machines

Forces on Submerged Bodies

Chapter 14

Q1

A flat plate 2m x2m moves at 40km/hour in stationary air density 1.25 kg/m3. If the coefficient drag and lift are 0.2 and 0.8 respectively find i) the lift force ii) drag force iii) the resultant force iv) the power required to keep the plate in motion.

[Ans 246.86 ii 61.715 iii 254.4 N iv 0.684 kw]

Solution

Type equation here.

Given

A=area of plate=2mx2m=4m2

Velocity of plate=40km/hr=400/36=11.11m/s

Density of air =1.25kg/m3,Cd=co eff. Of drag=.2

Cl= co eff, lift=.8

i) lift force $=F_L= C_L A\rho\frac{U^2}{2}$ $=\frac{.8\ X4X1.25X(11.11)^2}{2}$=246.88 N

ii) drag force$=F_D= C_D Ax\rho\frac{U^2}{2}=\frac{.2\ x4\ x1.2511.11^2}{2}= 61.716$ N

iii) resultant force $=F_R=\sqrt{{F_L}^2+{F_D}^2}$ $=\sqrt{246.86^2+61.716^2}$

$=\sqrt{64748.71}$ =254.45 N

iv) power required to keep the forces in motion

$=\frac{F_D\ xU}{1000}$ $=\frac{61.71\ X11.11}{1000}=$ .685kW

Q2

Find dragging force difference on a flat plate of size 1.5 x1.5 when plate is moving at a speed of 5m/s normal to its plate first in water and second in air of density 1.24kg/m3. Coefficient of drag as 1.10 [Ans 30899 N]

Solution

Given

A =area =1.5m x 1.5m =2.25m2

U =velocity of plate = 5m/s

C_D=1.10 ρ of air=1.24 kg/m3, ρ of water =1000

1. F_D=Drag force $=C_D\ xAx\rho \frac{V^2}{2}$ plate moving in water.

=1.10 x2.25 x1000 x5x5/2

=30937.5N

2 F_D =Drag force when plate is moving in air

$=C_{D\ A\ X\frac{\rho}{2}}U^2$ =1.10 x 2.25 x1.24 x5^2

=38.3625N

So difference in dragging force

=30937.5----38.3625

=30899.14 N

Q 3

A truck having a projected area of 12 m2 travelling at 60km/hour has a total resistance of 2349N . Of this 25% due to rolling friction and 15% surface friction .The rest is due to form drag . Calculate the coefficient of form drag if the density of air 1.25kg/m3 [.847]

Solution

Given

A =area of truck=12m2

Speed(velocity) of truck =U=60km/hr=$\frac{60x1000}{60}$=16.66m/s

Air density=1.25 kg/m3, total resistance =F_T =$\frac{2943}{}$

Rolling friction (resistance 25%) = F_R = $\frac{2943}{4}$=735.7N

Surface Friction (Resistance 15%)=F_S = $\frac{15}{100}$ X 2943

= 441.45N

Form Drag =drag force (F_D) − F_R − F_S

=(2943—735.7---441.45) N

=1765.85N

Now using equation (14.3) of chapter 14

$$F_D = C_D\, xA\, x\, \frac{\rho U^2}{2}$$

$$C_D = \frac{1765.85\ X2}{12\ X1.25\ \ (16.66)^2} = \frac{3531.7}{4163.334} = .848$$

Q4

A circular disc 4m in diameter is held normal to30m/s wind of density 1.25 kg/m3. If the coefficient of drag of disc =1.1, what force required to hold the disc at rest. [Ans 7775.4N]

Solution…..

Given

Diameter of disc=4m

Area =$\frac{\pi}{4}4^2$ =4π=12.5663m2

Velocity of wind =U=30m/s

, ρ =density of air =1.25kg/m3

Coefficient of drag=C_D=1.1

The force required to hold the disc at rest =drag exerted by wind on the disc.

$$F_D = C_D A x \rho \frac{v^2}{2} = 1.1 \text{ x } 12.5663 \text{x} 1.25 \text{x } 30^2 \frac{1}{2}$$

$$=7775.39 \text{ N}$$

Q 5

Find the diameter of a parachute with which a man of mass of 80 kg descend to the ground from an aero plane against the resistance of air with velocity of 25m/s. Take C_D=0.5density of air=1.25kg/m3 [2.26m]

Solution

Given

Weight of man =80 kg=80 x9.81 N

U=25m/s, coefficient of drag =.5

Density of air =ρ =1.25 kg/m3

Let D =diameter of parachute A=π/4 X D^2

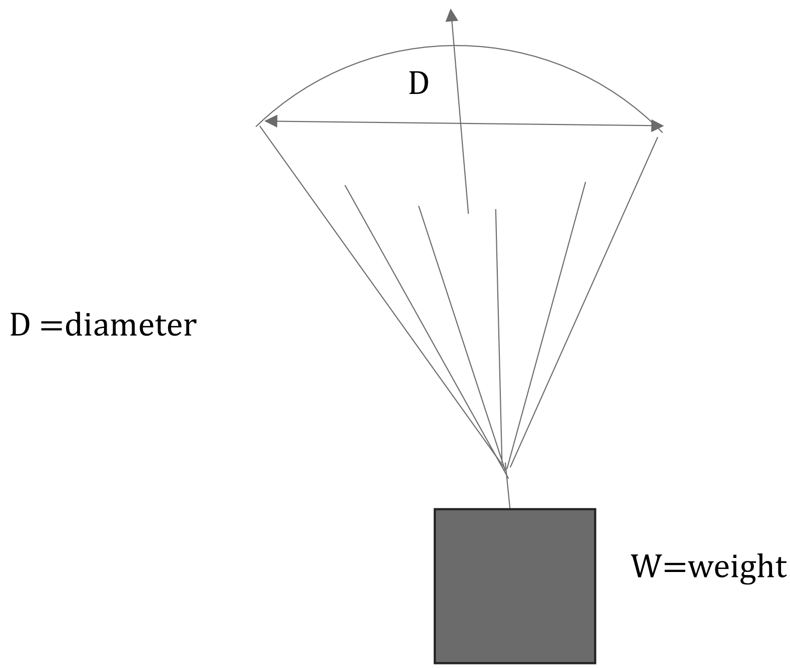
$$F_D = C_D A \frac{\rho}{2} U^2$$

$$80 \text{ x } 9.81 = .5\text{x } \frac{1.25}{2} \text{x} \frac{\pi}{4} D^2 x 25^2$$

$$D^2 = \frac{80\ X 9.81\ X 4\ X 2}{.5\ X \pi\ X 1.25\ X 25\ X 25} = \frac{6278.4}{1227.18}$$

$$=5.116$$

D =2.26m =diameter of parachute



Parachute

Q6

A man descend to the ground with the help of a parachute from an s=aero plane against the resistance of air with a uniform velocity of 10m/s. The parachute is hemispherical in shape and having a diameter of 5m. Find weight of man if $C_{D=.5}$ and density of air =1.25kg/m3 [613.52 N]

Solution

Given

Coefficient of drag=.5 ,dia. Of parachute=5m

Weight of man =W ? ρ =1.25 kg/m2 , U=10m/s , A=π/4 X (5X5)

$$F_D = Wx9.81 = C_D A\ x\ \frac{\downarrow \rho U^2}{2}$$

$$W = \frac{.5X\pi 5^2 X1.25X\ 10^2}{4X2\ X9.81} = 613.59N/9.81 = 62{,}5\ kg.$$

Q7

A kite of size 60cmx60cm weighing 2.943 N assumes an angle 10° to the horizontal. The string attached to the kite makes an angle of 45°t0 the horizontal. If the pull on the string is 29.43 N when the wind flowing at a speed of 40km/hr. Find the corresponding coefficient of drag and lift. Density of air is given as 1.25kg/m3 [Cd=.7489 Cl =.8548]

Solution

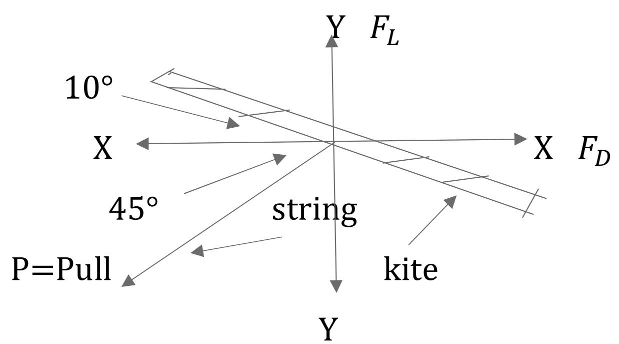
Given

Area of kite= 60 x60 cm2=.36m2

Weight of kite=2.943N ,angle made by kite with horizontal =10°

Angle made by string=45°, P=pull on the string =29.43N

Speed of wind =U=40 km /hr=11.11m/s ,ρ =1.25kg/m3



XX =Horizontal. w

Component of pull in XX direction=p cos 45°=F_D

=29.43 X .707=20.80N

F_L= Lift force in YY direction= psin45 °+weight of kite

=(29.43 x .707 +2.943) N=23.793N

We have to find C_D & C_L

1. $C_D = \frac{F_D X2}{AX\rho XU^2} = \frac{20.8X2}{.36\ X1.25\ X11.11}$ =.749

2. $C_L = \frac{F_L X2}{AX\rho XU^2} = \frac{23.793X2}{.36\ X1.25X1\ \ .11^2}$ = .856

Q8

Air is flowing over a cylinder of diameter 100mm and infinite length with a velocity of 150mm/s Find the total drag shear drag and pressure drag on 1 m length of cylinder if the total drag coefficient =1.5m and shear drag coefficient =0.25. Density of air =1.25 kg/m3. [Ans .00211N .000351N,.001756 N]

Solution

Given

Dia. Of cylinder =100mm=.1m

Length of cylinder to consider=L=1 m

Projected area =L XD=1X.1=.1m2

Velocity of air =U=150mm/s=.15m/s

Density of air =1.25 kg/m3 Total drag coefficient $=C_{D_T}$=1.5, Shear drag Coefficient=C_{D_S}=.25

I) Total drag $F_{D_T} = C_{D_T}$ x A x ρ $\frac{U^2}{2} = \frac{1.5\ X.1\ X1.25\ X\ (.15)^2}{2}$=.00211N

2) Shear drag $=F_{D_S} = C_{D_S} A \cdot \rho \frac{U^2}{2} = \frac{.25X.1X1.25 \quad (.15)^2}{2} = .0003515N$

3) Pressure drag = total drag ---Shear drag

=.00211N ---.0003515 N

=.00176 N

Q9

A body of length 2.5m has a projected area 1.8m2 normal to the direction of its motion. The body is moving through water with a velocity such that Reynold no =6x 10^6 and drag coefficient =.5. Find drag on the body . Take viscosity of water =.01 poise.

[2592N]

Solution

Given , length of body=L=2.5m ,ρ of water=1000

Projected area =A=1.8m2, μ=viscosity of water=.o1 poise =.001 Ns/m2 , coefficient of drag force=.5,Re=6X10^6

We will find velocity in water

$$Re=\frac{\rho UL}{\mu}$$

$$6x10^6=\frac{1000xUx2.5}{.001}$$

$$U=\frac{6X10^{10}X.001}{1000X2.5}$$

$$=\frac{6}{2.5}= 2.4 \text{ m/s}$$

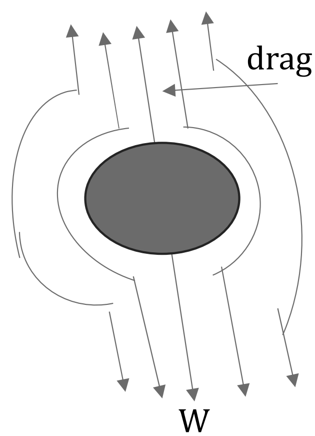
$$F_{D=C_D X\frac{A\rho U^2}{2}}$$

$$= \frac{.5\ x1.8\ x1000x(2.4)^2}{2}$$

=2592N=Drag on the body.

Q10

Calculate the weight of a ball of diameter 50 mm which is 10m/s. The density of air =1.25kg/m3 and kinematic viscosity =1.5 stokes. [Ans .0613 N]



diameter of ball =.05m

W=Weight of ball

Velocity of air =U=10m/s

Density of air=1.25kg/m3=ρ

„ v= k. viscosity=1.5 stokes=1.5 x 10^{-4}

Re no. $=\frac{U\,XD}{v}$ $=\frac{10\,x.05}{1.5x10^{-4}}$ =,3333 x10^{4}

=3333

The value of Re is between 1000 to 10^{5}

C_D=.5

When ball is supported in vertical air stream the weight of the ball =drag force shown in above sketch,

$F_D = C_D \; x A x U^2 \rho/2$

$= .5 \text{ x } (\pi/4 \text{ x}.0025) \text{ x}10^2 \text{x } 1.25/2$

$= .06135 \text{ N}$

Weight of ball= .06135 N.

Q11

A metallic sphere of s p gr 8.0 falls in an oil of density 800kg /m3. The diameter of sphere is 10mm and it attains a terminal velocity of 50mm/s. Find the viscosity of oil in in poise. [78.48 poise]

Solution

Given ; s p g r of metallic spere =8

Density of metallic sphere =s p g r x ρ of water

ρ_s = 8 x 1000=8000kg/m3

Density of oil =ρ =800kg/m3 , dia. Of sphere=D=1Omm=.01m

Terminal velocity =U=50mm/s=.05m/s.

Let viscosity of oil=μ

Weight of sphere =W=ρ_sx g x volume of sphere

$= 8000 \text{ x}9.81 \text{ x } \pi/6 \text{ x } D^3$

$= 8000 \text{ X } 9.81 \text{ X}\pi/6 \text{ x } (.01)^3$

=.04109 N

Buoyant force on sphere= F_B=density of oil x g x volume of sphere =800 x9.81 x π/6 $(.oi)^3$ =.004108 N

Drag force (equ.14.8) $F_{D=}$=3πμ D U= 3πμ.01 X .05

=.00471μ

Using e q u.(14.11)

$$W = F_D + F_B$$

.04109N =.00471μ +.004108N

$$„\mu = \frac{.036982}{.00471} = 7.85 \text{ Ns/m2}$$

= 7.85 x10 =78 .50 poise=viscosity of oil.

Expression given by e q u.(14.11) is valid only when Re is less than point .2. Hence it is necessary to calculate Re for the flow.

$$Re = \frac{\rho UD}{\mu} = \frac{800\ X.05\ X.01}{7.85} = .0509$$

$\therefore$ Re < .2

So the expression F_D=3πμ D U is valid.

Q12

A metallic ball of diameter 5mm drops in a fluid of s p g r 0.8 and viscosity 30 poise The specific gravity of the metallic ball is 9.0. Find i) drag force exerted by fluid on metallic ball ii) The pressure drag and skill(stream)friction drag iii) terminal velocity of ball in fluid[i).005264N ii).001754 N .003527 N iii) 3,7cm/s]

Solution

Dia of metallic ball= D=5mm=.005m

Specific gravity of fluid =.8

Density of fluid =ρ x .8 =1000 x.8=800kg/m3

Viscosity of fluid =30 poise =30/10 Ns/m2=3Ns/m2

S p g r of metallic ball=9.0 , density of metallic ball=1000x 9

=9000kg/m3

Weight of metallic ball=W=density x g x volume

=9000 x9.81 x π/6 x (.005)^2

=$\frac{.03467}{6}$ = ,005778N

Buoyancy force =F_B =Density of fluid x g x volume of ball

=800 x9.81 x π/6 x(.005)^3

=$\frac{.003082}{6}$ =.00051365N

When the metallic ball reaches the terminal velocity equation (14.11) is applicable

W=$F_D + F_B$

i) F_D= W-- F_B= .005778N--.00051365N

=.0052644N

F_D =3μπDU ------------ 1

iii) terminal velocity =U=$\frac{F_D}{3\pi\ X3\ X.005}$ =$\frac{'0052644}{.14137}$ =.0372=

3.72 cm/s

Now to check the Re number;

Re=$\frac{\rho UD}{\mu}$ =$\frac{800X.0372X.005}{3}$ =.0496 <.2

So Re <.2 so considering equation 14.8 is valid.

ii) a) pressure drag =1/3 F_D = 1/3 X.005264

= .001754N

b)skin friction drag=2/3 x .005264 =.003509N

Q13

A cylinder rotates at 200rpm with its axis perpendicular in an air stream which is having uniform velocity of 20m/s The cylinder is 2m in diameter and 8m long Assuming ideal fluid theory find i)lift force ii) position of stagnation points (iii circulation Take density of air 1.25kg/m3

[i)26309 N ii) θ=211.56 and 328.44°iii)131.57]

Solution

Given

Velocity of air U=20m/s, speed of rotation=N=200rpm

Dia. Of cylinder =D=2m. length of cylinder=L=8m

Density of air =ρ =1.25kg/m3

Tangential velocity of cylinder $U_\theta = \frac{\pi DN}{60} = \frac{\pi X\ .0X20}{60}$=20.94 m/s

i)circulation = t

$$U_\theta = \frac{t}{2\pi} \quad \text{(equation 14.14)}$$

„ t =$U_\theta\ x2\pi R$ =20.94 x2 x 2/2 x π=131.56 m2/s

ii) lift force F_L (14.16) =ρ x U x L x t (circulation)

=1.25 x 20 x8 x131.56

=26312N

iii) position of stagnation (14.19)

$$\sin\theta = \frac{t}{4\pi UR} = \frac{131.56}{4\pi x\ 20\ x2/2} = \frac{131.56}{251.327} = .52346$$

=sin (180+31.56) or sin (360—31.56)

θ =211.56° or 328.44° stagnation positions at these angles.

Q14

For problem 13,find the speed of rotation of cylinder which will give only give single stagnation point [381.97 rpm]

Solution

Given

For single point stagnation the applicable equation (14.20);

Circulation (t)=4πUR

=4π x 20 x D/2=4π x 20 x2/2

=251.327 m2/s

The speed of rotation corresponding to circulation (14.14)

$$U_{\theta} = \frac{t}{2\pi} = \frac{251.327}{2\ X\ \pi} = 40$$

But $U_{\theta} = \frac{\pi DN}{60}$, N $= \frac{U_{\theta} X 60}{\pi D} = \frac{40 X 60}{\pi X}$ =381.98 rpm

= speed of rotation and will give single stagnation point.

Q 15

The air having a velocity of 30m/s is flowing over a cylinder 1.4m diameter and length 10m, when the axis of the cylinder is perpendicular to the air stream. The cylinder is rotated about its axis and a total lift of 58860 N is produced. Find the speed of rotation and location of stagnation points. The density of air is 1.25 kg/m [Ans speed=486.87rpm, θ=216.5°,323.5°]

Solution

Given

$$Velocity\ of\ air = U = 30 m/s$$

$$Dia. 0f\ cylinder = D = 1.4m\ = 2r$$

$$Length\ of\ cylinder = L\ = 10m$$

Total lift =58860N , Density of air =ρ =1.25kg/m3

From equation (14.16) we have

F_L=58860 N=ρ L U t=1.25 x10 x30 t

. t =$\frac{58860}{1.25\ x10\ x3}$= 156.96 m2/s=circulation.

i) let the speed of rotation U_θ (corresponding to circulation t)

=$\frac{t}{\pi 2r}$ =$\frac{156.96}{\pi\ x1.4}$ =35.69 m/s.

Now U_θ=$\frac{\pi DN}{60}$ So N=$\frac{U_\theta X60}{\pi D}$ = $\frac{35.69X60}{4.398}$ = 486.9rpm

(U_θ =Tangential velocity =m/s)

ii) position of stagnation points (equation 14.19)

sin θ=$\frac{t}{4\pi UR}$=$\frac{156.96}{4\pi X30\ X1.4/2}$=.5948 =Sin (180+36.5)=sin 216.5

and =sin (360 --36.5)= sin 323.5

,,θ =216.5° & 323.5° = position of stagnation points.

Q16

A jet plane which weighs 19620N has a wing area of 25m2. It is flying at a speed of 200km /h r . When engine develop 588.6 kw 70% of this power is used to overcome the drag resistance of the wing Calculate the co-efficient of lift and drag for the wing , Take density of air 1.25 kg /m3 [Ans Coeff of lift and drag=.407,.114]

Solution

Given

Weight of plane =W=19620 N, wing area =A=25m2

Velocity of plane =U=$\frac{200x1000}{60\ x6}$ =55.55 m/s

Power develop by engine =588.6 kw

Power required to overcome the drag resistance =70%of 588.6kw

= 412.02 kw

„ ρ=density of air=1.25 kg/m3

Weight of plane =W=LIFT FOFRC=$\frac{C_{LX1.25X25}\ \ (55.55)^2}{2}$=19620 N

I) $C_L = \frac{19620}{48215.66}$=.4069 =coefficient of lift.

II) F_D =Drag force

Power required to overcome the resistance = $\frac{F_D U}{1000}$

$$412.02 \qquad = \frac{F_D X55.55}{1000}$$

$$F_D \quad = \quad \frac{412.02\ X1000}{55.55} = 7417.10\ \text{N}$$

$$= C_D AX \frac{\rho U^2}{2}$$

$$C_D = \frac{7417.10X2}{25X1.25\ \ (55.55)^2} \ = .153 = \text{Coefficient of drag}$$

Q17

Experiments are conducted in a wind tunnel with a wind speed of 50km/h r on a flat plate of size 2m long and 1m wide. The density of air is 1.15kg/m3. The plate is kept in such an angle that coefficient of lift and drag are 0.75 and .15 respectively.

Determine i) lift force ii) drag force iii) resultant force iv) its direction v) power exerted by the air stream on the plate.

[I, 166.4N II. 33.28 III.169.64N IV 78.7° V .461KW]

Solution

Given

Area of plate =2mx1m =2 m2

Velocity of air =50 km/h , density of air =1.15kg/m3

$= \frac{50x1000}{60x60}$ =13.89m/s

Value of C_D=0.15 C_L=0.75

i) lift force =$F_L = C_L\ xA\ x\ \rho\ x\ \frac{U^2}{2}$

=.75 x 2 x1.15 x$(13.89)^2$=166.404N

ii) drag force =$C_D\ xA\ x\rho x \frac{U^2}{2}$=.15 x2 x1.15 x $\frac{13.89^2}{2}$ =33.28N

iii) resultant force =$\sqrt{F_D{}^2 + F_L{}^2}$ =$\sqrt{33.28^2 + 166.404^2}$

=$\sqrt{28787.9089}$ =169.67

iv) direction of resultant =tan ϑ=$\frac{F_L}{F_D}$= $\frac{166.38}{33.257}$ =5

, θ =$tan^{-1}5$ =78.69°

v) power exerted by air on the plate ;

power= force in the direction of motion x velocity

$=F_D XU$ =33.28 x 13.89 =462.26 watt

=462.26/1000 kw

=.462 kW

END OF CHAPTER 14

Fluid Mechanics and Hydraulic Machines

Compressible Flow

Chapter 15

Q1

A gas is flowing through a horizontal pip of cross sectional area of 30cm2.At a point this is 30N/cm2 (gauge) and temp 20°c

At another section area of cross section is 15cm2 and pressure is 25 N/cm2 gauge. If mass rate of flow of gas is .15kg/s find the velocities of gas at these 2 sections, assuming an isothermal change Take R=287J/kg K and atmospheric pressure 10N/cm2

[Ans V1=10.71m/s V2=24.5m/s]

Solution

Given

„ t 1=20°c ,T1=20 +273=293°K

Area =39cm2 =30x10^{-4} m2

Cross section area =30cm2

π/4 x D^2 =30

D= 6.18cm=.0618m

Pressure =P1=30 +10=40 absolute=40x10^4

A2=.0015m2 ,P2=25 +10=35 (abs)=35x10^4(abs)

R=287J/kg K

Mass rate of flow of gas .15kg/s

Now $\frac{P_1}{\rho_1}$ =R T_1=287 x293=84091

From equation (15.2)

$$\rho_1 = \frac{P_1}{RT_1} = \frac{40X10^4}{28} = 4\ .757\ \text{kg/m3}$$

Mass rate of flow =,15kg=$\rho_1 A_1 V_1$=4.757x 30 x$10^{-4} V_1$

$$V_1 = \frac{0.15X10^4}{4.757X} = 10.51\ \text{m/s}\rho$$

For isothermal process temperature is constant and hence temperature at section 2—2 . is T_2=293°K

Using equation (15.2)

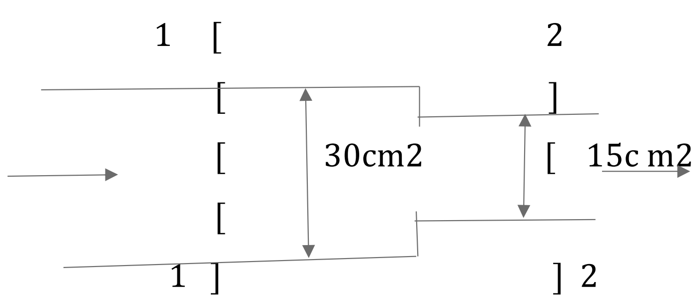
We get $\frac{p_2}{\rho_2}$=RT_2=287 X293

$$\rho_2 = \frac{P_2}{287X293} = \frac{35X10^4}{287X293} = 4.16\ \text{kg/m3}$$

Mass rate of flow =$\rho_2 A_2 V^2{}_2$

$$V_2 = \frac{.15}{A_2\rho_2} = \frac{.15}{4.16\ \ .0015}$$

$$= 24.04\text{m/s}$$



Q2

A gas with velocity of 350m/s is flowing through horizontal pipe at a section where pressure is 8N/cm2 (abs) and temp. at 30° c . The pipe changes in diameter and at this section pressure is 12 N/cm2 (abs). Find the velocity of the gas at this section if the flow of the gas adiabatic Take R =287J/k g Kand K =1.4

[Ans 218.63m/s]

Solution

Given

Section 1	section 2
V_1 =350m/s	V_2= Velocity sec 2
p_1= 8N /cm2 (abs)	P_2=12x10^4N/m2
t_1 =30°c T=303°K	R=287J/k g K

K=1.4 Adiabatic flow

Applying Bernoulli's equation for sec 1 & 2(15.13 equation)

$$\left(\frac{k}{k-1}\right)\frac{p_1}{\rho_{1g}}+\frac{V_1^2}{2g}=\left(\frac{K}{K-1}\right)\frac{P_2}{\rho_{2g}}+\frac{V_2^2}{2g} \qquad [\ Z_1 = Z_2\ ]$$

$$\left(\frac{K}{K-1}\right)\left[\frac{P_1}{\rho_1}-\frac{P_2}{\rho_2}\right]=\left[\frac{V_2^2}{2}-\frac{V_1^2}{2}\right]$$

Type equation here.

$$\left(\frac{1.4}{1.4-1}\right)\frac{P_1}{\rho_1}\left(1-\frac{P_2}{\rho_2}X\frac{\rho_1}{P_1}\right)=\frac{V_2^2}{2}-\frac{V_1^2}{2} \text{ --------} \qquad \text{I}$$

Using for adiabatic flow equation (15.4) we get

$$\frac{P_1}{\rho_1^K}=\frac{P_2}{\rho_2^K} \text{ OR } \frac{P_1}{P_2}=\left(\frac{\rho_1}{\rho_2}\right)^K \text{ OR } \frac{\rho_1}{\rho_2}=\left(\frac{P_1}{P_2}\right)^{\frac{1}{K}} \text{ --------} \qquad 2$$

Substituting the value of equation 2 in 1 above we get

$$\frac{1.4}{.4}x\frac{P_1}{\rho_1}\left[1-\frac{P_2}{P_1}\left(\frac{P_1}{P_2}\right)^{\frac{1}{K}}\right]=\frac{V_2^2}{2}-\frac{V_1^2}{2}$$

$$\frac{V_2^2}{2}-\frac{V_1^2}{2} \quad = \quad 3.5\frac{P_1}{\rho_1}\left[1\text{---}\left(\frac{P_2}{P_1}\right)X\left(\frac{P_2}{P_1}\right)^{\frac{-1}{K}}\right.$$

$$=3.5\frac{P_1}{\rho_1}\left[1\text{---}\left(\frac{P_2}{P_1}\right)^{\overline{1-\frac{1}{K}}}\right]$$

$$=3.5\frac{P_1}{\rho_1}\left[1\text{—}\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right] \text{ ---------} \qquad 3$$

Putting the value in 3 we get

$$=3.5\,\frac{p_1}{\rho_1}\left\{1-\frac{12\,x10^4}{8x10^4}\right)^{\frac{2}{7}}\right]$$

$$=3.5\frac{p_1}{\rho_1}\left[1-1.5^{.2857\}}\right]$$

$$= \quad (\text{-- } 3.5\text{x}.1228\,)\,\frac{p_1}{\rho_1}$$

$$\frac{V_2^2}{2}-\frac{350^2}{2} \quad =\text{---}.4298\frac{p_1}{\rho_1}$$

$$\frac{V_2^2}{2}=61250\text{ ---}.4298\left(\frac{P_1}{\rho_1}\right) \quad \text{-------} \quad 4$$

Now as per equation (15.2)of state i. e relation between pressure, temperature and specific volume of a gas;

$$\frac{P_1}{\rho_1}=\text{R}T_1=287\text{ x}303=86961 \text{ ----------} \qquad 5$$

Putting above value of 5 in 4 we get

$$V_2^2 = 2(61250 ---.4298 \text{ X } 86961)$$

$$= 47750$$

$V_2 = \sqrt{47750}$ =218.517 m/s =velocity at sec 2

Q3 Find the speed of sound wave in air at sea level where pressure and temperature are 9.81N/cm2(abs) and 20°C respectively. Take R=287J/kg K . [343.11m/s

Solution

Given

P= 9.81N/cm2,=9.81x 10^4N/m2

. t =20°c T=20 +273=293°k , R=287JkgK

K=1.4 C=Speed of sound.

Since nothing has been said about the process to be adopted from our side we have considered adiabatic process.

C = Speed of sound $= \sqrt{KRT} = \sqrt{1.4x287x}293$

$= \sqrt{117727.4}$

=343.11 m/s

Q4

Calculate the Mack number at a point on a jet propelled aircraft which is flying at 900km/h r at sea level where air temperature is 15° C , Take k=1.4 R=287 J/kg K

Solution

Given

,t=15° C T= 15 +273=288° (abs),k=1.4

R=287 °J/k g K Mack no =?

V=velocity of aircraft =900km/h r= $\frac{900x1000}{60x6}$

$= 250\text{m/s}.$

C=velocity of sound =$\sqrt{KRT}$

$=\sqrt{1.4x287\ x288}$

$=\sqrt{11578.4}$=340.174m/s

Mack no=$\frac{V}{C}$ =$\frac{250}{340.174}$ =.735 (Mack no.)

Q5

An aero plane is flying at a height of 20 km where tem$p\ is\ -20°\ C$

$The\ speed\ of\ plane\ is\ corresponding\ to\ Mack\ no1.8. Assuming\ k = 1.4\ \ and\ R = 287J/kg\ K. Find\ the\ speed\ of\ plane.$ [Ans 1982.63$m/h\ r$.

Solution

Given

T=--40° C =--- 40 +273 =233° K

Z =Height of plane =20km

M. No =1.8 R=287 J/kg K

Speed of the plane =?

Velocity of sound =$\sqrt{KRT}$= $\sqrt{1.4\ x287\ x233}$

$=\sqrt{93619.4}$

= 305.97 m/s

$$\text{M.no}=\frac{V}{C} \text{ OR } 1.8=\frac{V}{305.97}$$

V =1.8 x 305.97 = 550.7m/s

=550.7x60x60/1000 kl/hr

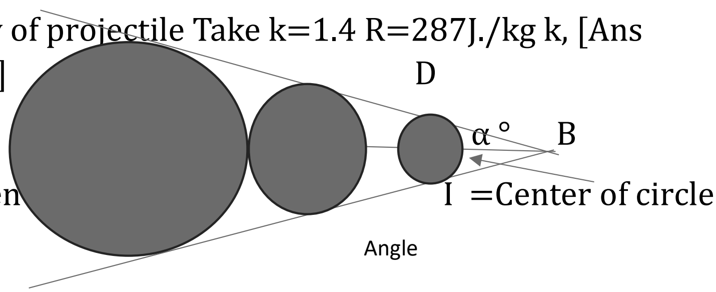
=1982.52km/hr.= speed of plane..

Q6

A projectile is travelling in air having pressure and temperature 8.829N/cm2 and (--- 5°C) If angle is 30° find ,the velocity of projectile Take k=1.4 R=287J./kg k, [Ans 656.30m/s]

=Solution

Given



Mack angle $\sin\alpha=\frac{C}{V}=\frac{1}{\frac{V}{C}}=\frac{1}{M}=\frac{V\ of\ sound}{V\ of\ projectile}$,, $\text{SIN } 30° =\frac{1}{2}$

Air pressure =8.829N/cm2 =8.829x 10^4N/m2

T =--5 +273=268°K

Angle IDB =α °=Mach angle =30°

K=1.4, R=287J/kg K

$C=\sqrt{KRT} =\sqrt{1.4x287x268}$ =328.15

Now $\sin 30=\frac{C}{V}$ =1/2

V= 328.15 X 2=656.3 m/s= velocity of projectile.

Q 7

A projectiles travels in air of pressure8.829 N/cm2 at ---10°C

At a speed of 1200km/hr. Find the Mach number and Mach angle. Take k-1.4 R=287 J/kg K [1.025, θ=77.2°]

Solution

Given

Air pressure =8.829N/cm2= 8.829 x 10^4N/m2

T=--10 +273=263° k(abs), projectile speed V=1200km/h r

V = 333.33m/s K=1.4 R=287

Speed of sound= C=$\sqrt{kRT}$ =$\sqrt{1.4x287x263}$

=325.07m/s

Mach no=$\frac{V}{C}$ =$\frac{333.33}{325.07}$ =1.025

Mach angle =sinα =$\frac{C}{V}$ =$\frac{325.07}{333.33}$ =.975°

„α =77.16° OR 1.347 radian.

Q8

Find the Mach number when the aero plane is flying at 900km/h r through still air having pressure of 8.0N/cm2 and temp --- 15° C Take K=1.4 R=287J/kg k. Calculate pressure, temperature and density of air at stagnant point on the nose of the plane. {.776,11.9N/N/cm2, 16.00° c , ,1434kg/m3]

Solution

K=1.4. R=287J/kg K, T=--15+273= 258° k

V=Speed of plane =900km/hr=900x900/60x60=250m/s

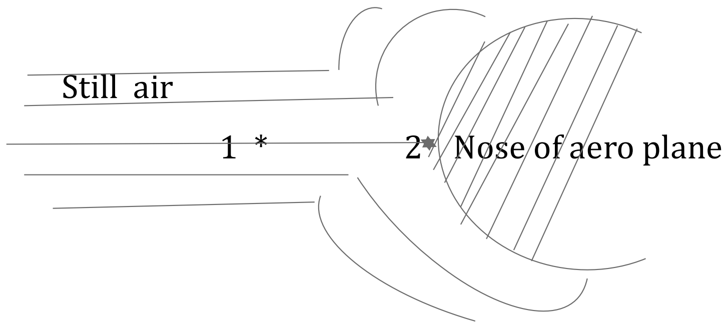
P=pressure =8N/cm2=8 x $1o^4$ N/m2

C=Speed of sound $=\sqrt{kRT}$ $=\sqrt{1.4x\ 287x\ 258}$

$=\sqrt{103664.4}$ =321.97 m/s

I) Mach no$=\frac{V}{C}=\frac{250}{321.97}$ =0.776

II) Pressure stagnation point (P_s=nose point)(Eq 15.21)



$P_S=P_1[1+\frac{k-1}{k}M_1^2]^{\frac{K}{K-1}}$

$=8 \text{ x } 10^4[1+\frac{1.4-1}{2}X(.776)^2]^{\frac{1.4}{1.4-1}}$

$=8 \text{ x } 10^4[1+.120]^{3.5}=11.84 \text{ x}10^4$ N/m2

=11.84 N/cm2

III) T_S =Stagnant temperature $= T_1[1+(\frac{k-1}{2})M_1^2]$

$=258[1+(\frac{1.4-1}{2})(.776)^2]$ =258X1.120

=288.96 ° K

,t =288.96 ----273=15.96°C

IV) Density of air at stagnant point $\rho_S=\frac{P_S}{RT_S}$

$=\frac{11.84X10^4}{287X288.96}=\frac{118400}{82931.52}$

$=1.427$ kg/m3

Q9

Find the velocity of air flowing at the outlet of nozzle, filled to a large vessel which contains air at a pressure of 294.3 N/cm2(abs) and a temp. of 30°c ..The pressure at the outlet of the nozzle is 137.34N/cm2(abs). Take k-1.4 R =287J/Kg K [Ans 242.98m/s]

Solution

Given

Pressure inside vessel P_1=294.3 N/cm2=294.3 x $10^4 N/m2$

Temperature t_1=30° c ,T_1=30 +273 =303°K

Pressure at outlet of nozzle $=P_2$= 137.34 X10^4 N/m2

K=1.4 R=287J/kg K

V_2=Velocity at outlet of nozzle (e q u. 15.25)

$$=\sqrt{[\frac{2k}{k-1}\frac{p_1}{\rho_1}\{1-(\frac{P_2}{P_1})^{\frac{k-1}{k}}\}]}$$

$$=\sqrt{(\frac{2X1.4}{1.4-1})(\frac{294.3X10^4}{\rho_1})[1-\frac{137.34X1\ \ 4}{294.3X10^4}]^{\frac{1.4-1}{1.4}}}$$

$$=\sqrt{\frac{2.8}{.4}}\ X\sqrt{\frac{294.3X10^4}{\rho_1}}\ \sqrt{.196}\quad \text{-------}\quad 1$$

But $\frac{P_1}{\rho_1}=RT_1$ So $\rho_1=\frac{P_1}{RT_1}=\frac{294.3X10^4}{287X303}$=33.84 kg/m3

Putting the value of ρ_1 in 1 we get

$$V_2 = \sqrt{\frac{7X294.3X10^4X.196}{33.84}} = \sqrt{119320.21}$$

$=345.42$ m/s=velocity of air at the outlet.

Q10

A nozzle of diameter 20mm is fitted to a large tank which contains air at 20° C. The air flows from the tank into the atmosphere. For adiabatic flow find the mass rate of flow of air through the nozzle when pressure of air in tank i) 5.886N/cm2(gauge) and ii) 29.43N/cm2(gauge). Take k=1.4 R=287J/kg K atmospheric pressure= 9.81 N/cm2.

[Ans .114kg/s, .291 kg/s]

Solution

Given

Temp in tank = T_1=20 +273=293 K

Diameter of nozzle =D_2=20mm=.02m ,K=1.4

Area $A_2 = \frac{\pi}{4} x, 02^2 = .000314$ m2 g=9.81 N/cm2, R=287J/kg K

I to find mass rate of flow

a) when pressure of air tank 5.886N/cm2 (gauge)

=5.886 +9.81 =15.696 x10^4 N/m2

$=P_1[$

P_2 = Atmospheric pressure. = 9.81x10^4kg/m2

$\therefore$ pressure ratio $=\frac{p_2}{p_1} = \frac{9.81}{15.696} = .625 = n$

Since pressure ratio .625 > .528 hence as per equation no (15.28) and (equation no 15.26);

$$\text{Mass rate of flow} = m = A_2\sqrt{\frac{2K}{K-1}P_1\rho_1[n^{\frac{2}{k}} - n^{\frac{k+1}{k}}]} \quad ----------1$$

$$\frac{P_1}{\rho_1} = RT_1 \quad \therefore \quad \rho_1 = \frac{15.696X\quad {}^4}{287X\ 293} = 1.866 \text{ kg/m3}, \ n = .625$$

$$\frac{P_1}{\rho_1} = RT_1 \quad \text{we get } \rho_1 = \frac{15.696x10^4}{287\ x293} = 1.866\text{kg/m3}$$

Putting respective values in equation in 1 above we get

Mass rate of flow m

=.000314 x362.799 =.1139 kg/s

II

Mass rate of flow of air when tank pressure is

29.43 N/cm2 (gauge pr)= m=29.43 +9.81 =39.24N/cm2

=39.24x10^4N/m2(abs)

$$\text{Pressure ratio} = n = \frac{P_2}{P_1} = \frac{9.81}{39.24} = .25$$

In this case n that is pr. ratio is .25 which is less than .528

Hence in this case we have to use value of x^2n =.528 and not .25

$$\text{We have find the value of } \rho_1 = \frac{P_1}{T_1R} = \frac{39..24\quad {}^4}{293x287} = 4.666 \text{ kg/m3}$$

$$\text{Mass rate of flow of air} = m = A_2\sqrt{\frac{2K}{K-1}\ P_1\rho_1\ [n^{2/k} - n^{\frac{k+1}{k}}]}$$

Putting values m=.000314 $\sqrt{7x392400x4.666x(.4016 - .3346}$

$$= .000314\sqrt{858710.1} \quad = .000314x926.66$$

=.2907 kg/s

Type equation here.

Q11

Find the mass rate of flow of air through a venturi meter having inlet diameter 400mm and throat diameter 200mm. The pressure at the inlet of the venturi meter is 27.468 N/cm2(abs) and temperature of air at inlet is 20° c The pressure at the throat is 25.506 N/cm2. Take k=1.4

R=287 J/kg K [11.13kg/s]

Solution

Given

D_1 =Inlet diameter =400mm=.4m

Area =A_1=π/4 X$D_1{}^2$=π/4 x$(.4)^2$=.1248m2

Area2=A_2=π/4 x D_2^2 =π/4 x $(.2)^2$ = .0312 m2

Pressure of air at inlet =p_1=27.468 N/cm2=27.468 x10^4N/m2

Do- outlet =p_2=25.506x10^4 N/m2

K=1.4 R=287J/kg K

Pressure ratio=n = $\frac{p_2}{p_1} = \frac{25.506x10^4}{27.468x\quad 4}$ =.9285

Density of gas at inlet is obtained from e q u of state ;

$$\frac{p_1}{\rho_1} = R\,T_1 \;\therefore\; \rho_1 = \frac{P_1}{RT_1} = \frac{27.468X10^4}{287X29\ 3} = 3.2664 \text{ kg/ m3}$$

For adiabatic process we have

$$\frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k} \quad \text{or} \quad (\frac{\rho_2}{\rho_1})^k = \frac{P_2}{P_1} \quad ;\rho_2 = \rho_1(\frac{P_2}{P_1})^{\frac{1}{K}} = 3.26(.9285)^{\frac{1}{1.4}}$$

$=3.26 \text{ x } (.9285)^{.714}$

$=3.26$ x .9484

ρ_2 =3.09 kg/m3

Mass rate of flow through venturi meter

$$= \ m \ = \ \rho_2 x A_2 \frac{\sqrt{\frac{2K}{K-1}(\frac{P_1)}{\rho_1)}[1-(\frac{P_2}{P_1} \ )^{\frac{K-1}{K}}]}}{\sqrt{1-(\frac{A_2^2}{A_1^2})(\frac{P_2}{P_1} \ )^{\frac{2}{K}}}}$$

Putting the values we get finally

$$„ \ m = 3.09 \text{ x } .0312 \sqrt{\frac{7x84092(1-\cdot 979)}{1-.0625 \quad .8994}}$$

$$= ,0964 \sqrt{13097.58} = .0964\text{x}114.44$$

$$= 11.032 \text{ kg/s}$$

Q 12

Calculate the numerical factor by which the actual pressure difference by the gauge of a pilot tube should be multiplied to allow for compressibility when value of Mach no is .7. Take k=1.4.

[Ans 1.1285]

Solution

Given

Mach no =.7 k=1.4 $=M_1$

Using equation (15.33) compressibility correction factor

$$(\text{C.C.F}) = [1+\frac{M_1^2}{4}+\frac{2-K}{24}M_1^4 + \ ----]$$

$$= [1 \ + \ \frac{.7^2}{4} + \frac{2-1.4}{24}(.7)^4 + \text{------}]$$

$$= [1 + .49/4 + \frac{.6}{24} \text{X } .2401 + \text{-------}]$$

Type equation here.

$= [\ 1 + .1225 + .006O025 + \text{------}]$

$= 1.1285\ .$

Q13

Find the Mach no when an aero plane is flying at 1000km/hr through still air having pressure at 7 N/cm2 and --5°c. Take R =287J/kg K. Calculate the pressure and temp of air at stagnant point. Take k=1.4.

Solution

Given

Velocity =1000 km/hr=$\frac{1000x1000}{60x60}$ =277.77m/s

p_1=7N/cm2 , k=1.4 , R=287.14 J/k s K

T_1=--5+273=268° K

Speed of sound $C_1 = \sqrt{KRT_1} = \sqrt{1.4x287.14x268}$

=328.2m/s

So the Mach no=M=$M_1 = \frac{V_1}{C_1} = \frac{277.77}{328.2}$=0.846

We have to calculate now stagnant pressure point p_S and stagnant temperature T_S:--------------

$$P_S = P_1[1 + \frac{K-1}{2} M_1^2]^{\frac{K}{K-1}}$$

$$=7x10^4[1 + \frac{1.4-1}{2}(.846)^2]^{\frac{1.4}{1.4-1}}$$

$$=7x\ 10^4[1 + 0.2x(.846)^2]^{3.5}$$

$$=7x\ 10^4[1 + .143143]^{3.5}$$

$=7x10^4x\ (1.143143)^{3.5}$

$=\ \ 7\ x\ 10^4 x\ 1.597 = 11.179\ x10^4 = P_S$

Now to find $T_S = T_1[1 + \frac{K-1}{K}\ M_1^2] = 268[1 + \frac{1.4-1}{2} x(.846)^2]$

$=306.38°k = (306.38—273) = 33.38°C$

END OF CHAPTER 15

Fluid Mechanics and Hydraulic Machines

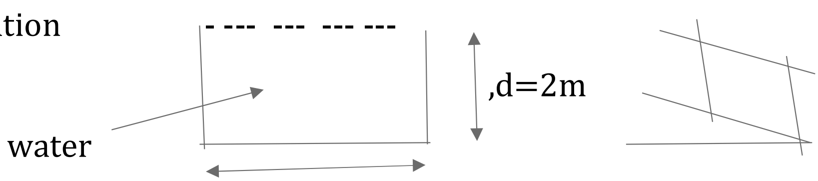
Flow In Open Channels

Chapter 16

Q1

Find the velocity of flow and rate of flow of water through a rectangular channel of 5 m wide 2m deep, when it is running full. The channel is having bed slope of 1 in 3000. Take Crazy's constant = C=50 [Ans 0.962 ii 9.62m3/s]

Solution



, b=5m bed slope=$\frac{1}{3000}$= i°

Cross section area= b x d=5x2 =10m2

Crazy's constant = C=50

Perimeter of channel =p =b +2xd=5+2x2=9m

Hydraulic mean depth = m =$\frac{area}{perimeter}$=$\frac{10m2}{9m}$=1.11m

i) velocity of flow =(equ.16.4)=V=C$\sqrt{m\ i}$ =50$\sqrt{1.11x\frac{1}{3000}}$

=50x$\sqrt{.00037}$ =50x.019235 =.9617 m/s

ii) rate of flow = Vx area of channel

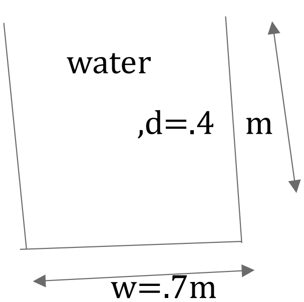
=.9617 x 10m2

=9.617 m3/s

Q 2

A flow of water of 150 lit/s flow down in a rectangular flume of width 70 cm and having adjustable bottom slope . If Crazy's constant is 60 ,find the bottom slope necessary for uniform flow with a depth of flow of 40 cm. Also find the conveyance k of the flume. [Ans 1in 2341.5, 7.258]

Flume means inclined gravity chute.



Solution

Depth of channel =d=40cm =.4m ,d=.4

Width of channel =b=70cm=.7m

P =perimeter =.7 +2x .4=1.5m

A =Area of flow =.7m x.4 m=.28 m2

Crazy's constant =C=60 ,Q=150L/S =.15m3/s

To find i) to find bottom slope =?

ii) k =?

i) hydraulic mean depth =m $=\frac{A}{P}=\frac{.28}{1.5}$ =.1866 m

using equation 16.5 , Q =A C$\sqrt{mi}$ =

$.15 = .28 \times 60\sqrt{.1866\, i}$

.1866 i=$(.0089)^2$ =.00007921 ,

Therefore i $=\frac{.00007921}{.1866} = .0004244 = \frac{1}{\frac{1}{.0004244}}$

$= \frac{1}{2356.2}$ =bottom slope.

ii)conveyance K of flume

$= AC\sqrt{m}$ =.28 x 60 $\sqrt{.1866}$

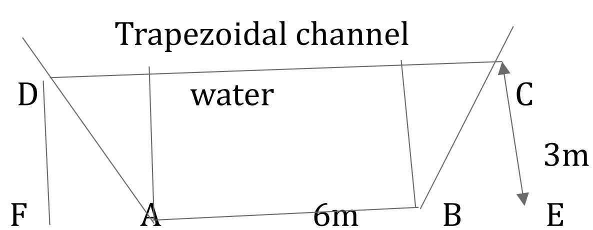
=7.224m3/s

Q3

Find the discharge through a trapezoidal channel of width 6m and side slope 1 is to 3 (1 horizontal 3 vertical). The depth of flow

Of water is 3m and Chezy's constant C=60. The slope of bed of the channel is 1 in 5000.[Ans 23.26 m3/s]

Solution



Side slope =1 hor. 3 vertical

FA=BE=3X1/3=1m , C=60 Bed slope=$\frac{1}{5000}$

DC=AB+BE+AF=6+1+1=8 m

$BC=\sqrt{1^2+3^2}\ = 3.162m$

P =wetted perimeter DA+ AB+ BC=3.162+6+3.162

=12.324 m

Hydraulic mean depth $=\frac{A}{m}=\frac{21}{12.324}$=1.7 m

Discharge =Q =A C $\sqrt{mi}$ = 21 x60 $\sqrt{1.7x\frac{1}{5000}}$

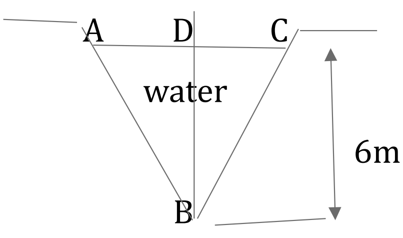
=1260x.01844

=23.234 m3/s

Q4

Find the rate of flow of water through a V shaped channel having total angle between the sides as 60°.Take value of C=60.

And shape of the bed 1in 1500 . The depth of flow 6m.[Ans 32.864m3/s]



angle ABC= 60°

angle ABD=30°

Solution V Shaped channel

C=50. I = 1/1500 , tan30 $=\frac{AD}{BD}=\frac{AD}{6}$

AD= 6x .5773=3.462m

AB $=\sqrt{AD^2+BD^2}\ =\sqrt{3.462^2+6^2}$ =6.927m

Wetted perimeter=P =AB+BC=6.927 +6.927=13.854 m

Hydraulic mean depth=m $=\frac{A}{P}\ =\frac{\frac{1}{2}X\ ACXBD}{13.854}=\frac{\frac{1}{2}X2XADXBD}{13.854}$

$$=\frac{ADXBD.}{13{,}854}=\frac{3.462X6}{13.854}=\frac{20.772}{13.854}=1.5 \text{ m}$$

Rate of discharge =Q=A C $\sqrt{m\ i}$

$$= 20.772 \text{ x } 50\sqrt{1.5\ x\ \frac{1}{1500}}$$

$$=1038.6\ \sqrt{.001}$$

$$=1038.6 \text{ x } .0316$$

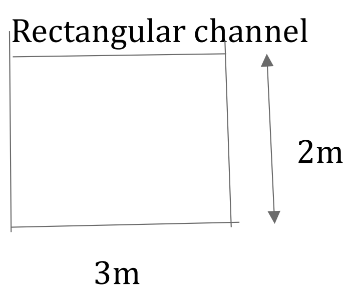
$$=32.819 \text{ m3/s}$$

=Q =Rate of discharge.

Q 5

Find the discharge through a rectangular channel 3m wide .having depth of water 2m and bed slope as 1 in 1500 . Take k=2.36 in Bazin's formula [Ans 5.184m3]

Solution



Width of channel =3m, depth of channel =2m

Bed slope =1/1500 , area of channel =3m x2m =6m2

Wetted perimeter= P = 2+3+2 =7m

Hydraulic mean depth= $\frac{A}{P}=\frac{6}{7}$ =.,857m

Bed slope = 1/1500 , value of k=2.36.

Now we will use Begins 'formula (equation 16.6)

$$C = \frac{157.6}{1.81+\frac{K}{\sqrt{m}}} = \frac{157{,}6}{1.81 \quad \frac{2.36}{\sqrt{.857}}} = 36.15$$

Discharge =Q = A C $\sqrt{m\, i}$

$$=6 \text{ x} 36.15 \sqrt{m \quad i}$$

$$=216.9 \sqrt{.857 \, x \, \frac{1}{1500}}$$

$$=216.9 \text{ x} \sqrt{.00057}$$

$$=216.9 \text{x} .02387$$

$$=5.177 \text{m}3/\text{s}$$

Q 6

Find the discharge through the rectangular channel given in the above question no5taking the value N=0.012 in Manning formula.

[Ans 11.64 m3/s]

Solution

Using Manning formula (16.8)

$$C = \frac{1}{N} m^{\frac{1}{6}}$$

$$= \frac{1}{0.012} .857^{.166}$$

$$=.7264/0.012$$

$$= 81.225$$

Discharge Q= A C $\sqrt{m\, i}$

$$=6 \text{ x } 81.225 \sqrt{.857x\frac{1}{1500}}$$

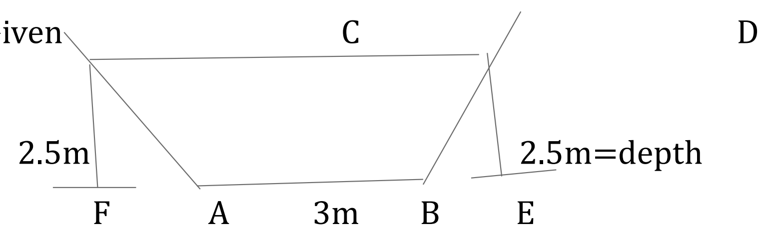
$$=6\text{x}81.225\text{x}\sqrt{.00057}$$

$$=6 \text{ x}81.225 \text{ x}.02387=11.633 \text{ m3/s}$$

Q7

Find the bed slope of trapezoidal channel of bed width 3m, depth of water 2.5m and side slope 2 horizontal and 3 vertical, when discharge through the channel is 10m3/s. Taking the value of N=0.03 in manning's formula C=$\frac{1}{N}\sqrt{m\ i}$ [Ans 1 in 1803]

Solution Given



Trapezoidal channel

Q=10m3/s, DE=CF=2.5m , slope ; horizontal =2

Vertical 3 , so 2/3 x2.5=1.66 =FA=BE

CD=3 +1.66+1.66 =6.32

Area of trapezoidal section-=$\frac{AB+C}{2}$X DEPTH

$$=\frac{3+6.32}{2}X2,5=11.65\text{m2}$$

Wetted perimeter =CA+AB+BD

$$=\sqrt{CE^2+FA^2}+3+\sqrt{BE^2+CE^2}$$

$$=\sqrt{2,5^2+1.66^2}+3+\sqrt{2.5^2+1.66^2}$$

$$=6.016+3=9.016=\text{P}$$

Hydraulic depth =m=$\frac{A}{P}=\frac{11.65}{9.016}$=1.29

$$C=\frac{1}{N}Xm^{\frac{1}{6}}=\frac{1}{.03}1.29^{.166}=34.77$$

$$Q = A\,C\sqrt{m\,i} = 11.65x\,34.77\sqrt{1.29xi} = 10 \text{ m3/s}$$

$$\sqrt{i} = \frac{10}{405.07x1.135} = .0217$$

$$\text{So} \quad i = (.0217)^2$$

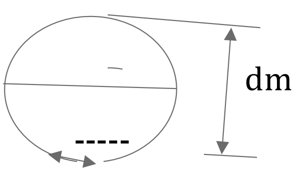
$$= .0004709 = \frac{1}{\frac{1}{.0004709}} = \frac{1}{2123} \quad \text{Ans.}$$

Q8

Find the diameter of a sewer pipe which is laid at a slope of 1 in 10000 and carries a discharge of 1000 lit/s when flowing half full. Take the value of Manning's N=0.02 [Ans 2.6m

Solution

Given



Half full in pipe

$$\text{Slope } i = \frac{1}{10000}$$

Discharge =Q = 1000 lit/s= 1000/1000 m3/s

= 1m3/s

Mannig's N=0.02 , Dia . of pipe = d m

$$\text{Area of cross section (half full)}=\frac{1\pi}{2x4}d^2=\frac{\pi d^2}{8}=A$$

$$\text{Wetted perimeter} = \frac{2\pi d}{2x2}=\frac{\pi d}{2}$$

$$\text{Hydraulic depth} = m = \frac{A}{P} = \frac{\frac{\pi d^2}{8}}{\frac{\pi d}{2}} = \frac{d}{4}$$

$$\text{Manning formula } C = \frac{1}{N} m^{\frac{1}{6}} \text{ ------------ } 1$$

$$Q = A\,C\,\sqrt{m\,i} \qquad \text{(putting values)}$$

$$1\text{m/s} = \frac{\pi d^2}{8} \text{x} \frac{1}{N} m^{\frac{1}{6}} \sqrt{m\,i}$$

$$= \frac{\pi d^2}{8} \text{x} \frac{1}{.02} \text{x } m^{\frac{1}{6}} m^{\frac{1}{2}} \sqrt{i}$$

$$= .39 d^2 \text{x} 50 \text{x } m^{\frac{2}{3}} \text{x} \sqrt{\frac{1}{\frac{1}{0000}}}$$

$$= .39 \text{x} 50 \text{ x} d^2 (d/4)^{\frac{2}{3}} \sqrt{.0001}$$

$$= .39 \text{x} 50 \text{ x} \frac{1}{4^{\frac{2}{3}}} \text{x } .01 \text{x } d^2 x d^{\frac{2}{3}}$$

$$= ,39 \text{x} 50 \text{x} .01 \text{ x} \frac{1}{2.496} \text{x } d^{\frac{8}{3}}$$

$$1\text{m3/s} \quad = .39 \text{ x} 50 \text{ x} .004 \; d^{\frac{8}{3}} = .078 \; d^{\frac{8}{3}}$$

$$d^{\frac{8}{3}} = d^{2.66} \quad = \frac{1}{.078} \quad = 12.8$$

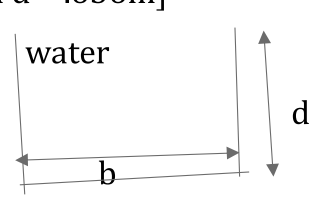
$$\text{S o} \quad d \; = (12.8)^{\frac{1}{2.66}} = 2.6\text{m} = \text{diameter of sewer pipe}$$

Q9

A rectangular channel carries water at the rate of 500lit/s when bed slope is 1 in 3000 Find most economical dimension of the channel if C=60 [b =1.272m d= .636m]

Solution

Rectangular channel



To find most economical dimension =?

Q=Discharge =500lit/s=500/1000 m3/s=1/2 m3/s

Slope = i = 1/3000 C-60

Using equation 16.9 and equation 16.10 :

16.9) when b=2d

Hydraulic depth=A/P $=\frac{bxd}{2d+b} = \frac{2dxd}{2d+2d} = \frac{2}{4d}d^2$ =d/2

Q =A.C $\sqrt{m\,i}$ ---------- 1

For most economical dimensions

As per above equations 16.9 and 16.10

Either m= d/2 and b =2d

Accordingly putting values on 1

$$.5 = (b \times d) \times 60 \sqrt{\frac{d}{2} x 1/3000}$$

$$=2d \times d \times 60 \sqrt{\frac{d}{6000}}$$

$$=2d^{2+1/2} X \frac{60}{77.45}$$

$$d^{2.5} = \frac{.5X77.45}{120}$$

∴ d $=(.322875)^{\frac{1}{2.5}}$=.636m =depth

And b=2d =2x .636 =1.272m

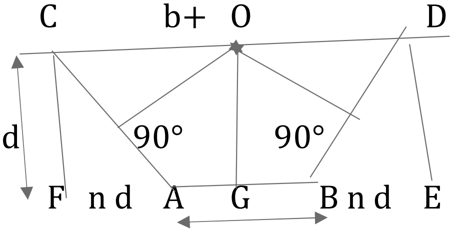
So most economical dimensions b=1.272m, d=.636m .

Q10

A trapezoidal channel has a slide slope of 1 horizontal and 2 vertical and slope of bed 1 in 2000. The area of cross section is 42 m2. Find the dimension of the section if it is most economical. Determine the discharge of the most economical section if C=60.

[Ans d=4.918m b=6.08m Q=88.36M3/S]

Solution



Condition for most economical trapezoidal sections are

i) $\frac{b+2nd}{2}$ =d $\sqrt{n^2+1}$ =BD

ii)m=2(b+ n d)

iii) three sides of trapezoidal section of most economical are tangential to the semicircle described on the water line.

Slope of bed =1/2000=i

„ n=1/2 =horizontal/vertical (given)

A=42m2 , C=60

Calculation:---

i)

equation -=16.11

.. $\frac{b+2nd}{2} = d\sqrt{n^2+1}$

Or $\frac{b+2x1/2d}{2} = d\sqrt{1/2^2+1}$

$\frac{b+d}{2}$=d$\sqrt{\frac{5}{4}}$ or b=1.236 d ----- 1

But area of trapezoidal section

A=$\frac{b+(b+2nd)}{2}$xd=(b+nd)d=(1.236d+1/2 xd)d =1.736 d^2

Or 42m2=1.736 d^2

., d=$\sqrt{24.193}$ =4.918m

„ b=1.236d=1.236 x4.918

„b=6.08m

ii

discharge for most economical section

hydraulic mean depth=m =d/2= 4.918m/2=2.459m

Therefore Discharge Q=A C $\sqrt{m\ i}$

= 42 x60 x $\sqrt{2.459x\frac{1}{2000}}$

=42x60 $\sqrt{.00123}$

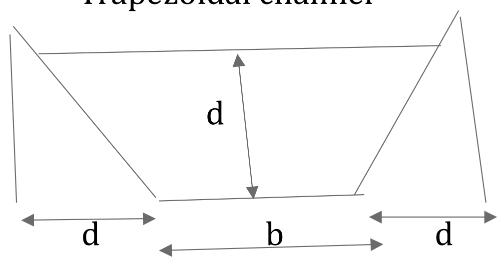
=42 x60 x.0350

=88.2m3

Q11

A trapezoidal channel with side slopes of 1 to 1 has to be designed to convey 9m3/s at a velocity of 1.5 m/s so that the amount of concrete lining for the bed and the sides is the minimum Calculate the area of the lining required for I m length of canal. [Ans 6.62m2]

Trapezoidal channel



side slope =n= 1/1=1

Q=Discharge 9m3/s

V=velocity =1.5 m/s

Area of flow = Q/V=9/1.5 =6 m2

, b =width d= depth

The amount of concrete lining for the bed and side to be minimum section should be most economical . For that condition should follow equation 16.11;--

Half of the top width - =one of sloping side

$\frac{b+2nd}{2} = d\sqrt{1+n^2}$ (here n=1)

„ b +2d =d√2 =1.414d

„ b+2d=2.828d

„ b =.828d

A = (b+ n d) d =1.828d^2=A=6

„d^2 =6/1.828 =3.282

„ d =1.811 m=depth

„b =.828 d=.828 x1.811=1.499m

Area of lining required for 1 m length of canal

= wetted perimeter x length of canal

=P m x 1m =Pm2

P=b+2d$\sqrt{1+n^2}$ =1.499+2x1.811$\sqrt{2}$

=1.499+2x1.811 x1.414

=6.6205 m

Hence area of lining =P x 1 = 6.62 x 1 =6.62 m2

Q12

A power canal of trapezoidal section has to be excavated through hard clay at least cost. Determine the dimension of the canal given discharge equal to 15m3/s bed slope 1;2000 and Manning's N=0.020. [Ans b=2.956m d=2.56m]

Solution

Given

Q=15m3/s. slope =1;2000 N=.020

For excavation of canal at the least cost , the trapezoidal section should be most economical section (value of N) is given by equation (16.14) as N=.020.

„ b=width of , d=depth of flow'

For most economical section

Half of top width =length of one side of slope

Or $\frac{b+2nd}{2} = d\sqrt{n^2+1}$ [n=1/√3 ,best slope of the side of channel.(16.14 equation) of trapezoidal section.

Accordingly value of n is considered .]

$$\frac{b+2xdx\frac{1}{\sqrt{3}}}{2} = d\sqrt{(\frac{1}{\sqrt{3}}+1)} = \frac{2d}{\sqrt{3}}$$

„ b $=\frac{2x2d}{\sqrt{3}}-\frac{2d}{\sqrt{3}}=\frac{2d}{\sqrt{3}}$

Area of trapezoidal section =A = (b+ n d) d

$$=(\frac{2d}{\sqrt{3}}+\frac{1d}{\sqrt{3}})d=\sqrt{3}\ d^2$$

Hydraulic depth for most economical section

= m=d/2

Discharge = Q =A C $\sqrt{m\ i}$ when C=$\frac{1}{N}m^{\frac{1}{6}}$

$$15=\sqrt{3}d^2x(\frac{1}{N}\,\mathrm{x}m^{\frac{1}{6}})\sqrt{mx\,\frac{1}{2000}}$$

$$=\sqrt{3}\mathrm{x}\frac{1}{.020}\sqrt{\frac{1}{2000}}\,d^2m^{\frac{1}{6}+\frac{1}{2}}$$

$$=1.732\ \mathrm{x}50\mathrm{x}.02236\mathrm{x}d^2m^{\frac{2}{3}}$$

$$=1.936\mathrm{x}d^2(\frac{d}{2})^{2/3}$$

$$15\quad=\frac{1.963}{2^{.66}}xd^{\frac{8}{3}}\ =1.2253\ d^{2.66}$$

$$„\ \mathrm{d}=(\frac{15}{1.2253})^{\frac{1}{2.66}}\ =(12.24)^{.376}=2.56\mathrm{m}$$

$$„\quad \mathrm{b}=\frac{2d}{\sqrt{3}}=\frac{2x2.56}{\sqrt{3}}=2.956\ \mathrm{m}$$

Most economical dimension for the section

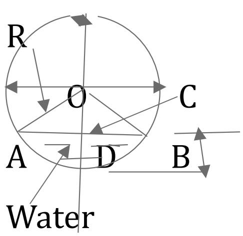
Are b=2.956m and 2,56m.

Q13

Find the discharge through a circular pipe of diameter 4.0 m , if the depth of water in the pipe is 1.33m and pipe is laid at a slope of 1 in 1500 . Take the value of Chezy's constant as 60

[Ans 4.89m3/s]

Flow through circular channel (eq, 16.5.4)



water depth =CD=1.33m

O is the center of pipe

angle AOD=Angle BOD= θ, Angle AOB=2θ

R=D/2=4m/2=2m=OA=OB=OD

OC =OD—CD= R—CD=2m –1.33m

OC =0.67m

Wetted perimeter =P = $\frac{2\pi R}{2\pi}$ X2θ =2Rϑ m

Wetted area =ADB CA = Area ADBOA --- ΔAOB

$=\frac{\pi R^2}{2\pi}$ X2θ ----1/2 XAB X OC

$=R^2\theta - \frac{2RSin\theta\ RCos\theta}{2}$

A $=R^2(\theta - \frac{sin}{2})$

Hydraulic mean depth $=\frac{A}{P}=\frac{R}{2\theta}(\theta\text{--}\frac{Sin\theta}{2})$

,,I =slope =1/1500   C =60

Cos θ=OC/OA=.67/2=.335

Θ=70.43° OR 1.23 radians

P -=Wetted perimeter=2Rθ=2 X2 X1.23

=4.92m

A=Wetted area=$R^2(\theta-\frac{sin2\theta}{2})$

$= 2^2(1.23\text{--}\frac{Sin2x\quad .43}{2})$

$=4[(1.23\text{---}(\frac{\sin(180-140.86)}{2}]$

$=4\ (1.23\text{---}\frac{sin39.14}{2})$

$=4(1.23\text{---}\frac{.6312}{2})$

=4 x(1.23---.3156)

=4 x.9144

A=3.6576 m2

Hyd. mean depth =m=$\frac{A}{P}=\frac{3.6576}{4.92}$ =.7434

Q =A C $\sqrt{m\ i}$

$= 3.6576\text{x } 60\text{x } \sqrt{.7434x\ \frac{1}{1500}}$

=3.65x60 x$\sqrt{.0004956}$

=3.6576 x60 x.02226

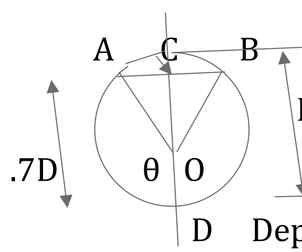
=4.885 m3/s

Q14

Water is flowing through a circular channel at the rate of 500 lit/s. The depth of water in channel is 0.7 times the diameter and slope od the bed of channel is 1 in 8000. Find the diameter of the circular channel if the value of Manning's N=0.015[Ans1.425m]

Solution

Given



D=Dia. Of circular channel

angle AOC=φ, LAOD=θ

Depth of water=.7D m =d m

Q=500lit/s=500/1000 =.5m3/s

OC=DC—DO=.7D--.5D=.2D m

Sin AOC=Sin(180—θ)=AC/AO=AC/D/2

Sin θ=2AC/D ∴AC=Sin θ/2

Cos φ=cos AOC=OC/OA=.2D/.5D=2/5=.4

„φ=23°, θ=157°=$\frac{157X\pi}{180}$ =2.74 radians

Wetted P=2 x D/2 x 2.74 =2.74 D ------------- 1

Wetted A-=$R^2(\theta - \frac{SIN2\theta}{2})$=$(D/2)^2(2.74 - Sin2x157/2)$

$=\frac{D^2}{4}(2.74$ --$\frac{Sin\ (180+1340)}{2}$$=D^2/4\{2.74$—(-- Sin46/2)}

$=\frac{D^2}{4}$ (2.74--.72/2)=.77D^2

Hydraulic depth =$\frac{A}{P}$ =$\frac{.77\quad 2}{2.74}$ = .281D m

By Manning formula

Q=Discharge =1/N x A x $m^{2/3}$x $i^{1/2}$

$$.5 \quad =\frac{1}{.015}x.77D^2x(.281D)^{.66}x(\frac{1}{8000})^{\frac{1}{2}}$$

$$=66.66\text{x } .77\ D^2\ \text{ x } .4326D^{.66}x(.000125)^{.5}$$

$$=66.66 \text{ x } .77\text{x}.4326 \text{ x}.011\ D^{2.66}$$

$$D^{2.66} = \frac{.5}{66.66\quad .77X.4326X.011} = 2.05$$

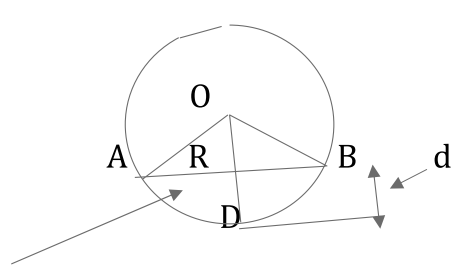
$$\text{D}=(2.05)^{\frac{1}{2.66}} =(2.05)^{.3759}=1.309 \text{ m}$$

Diameter of circular channel =1.309m

Note :--- Students may verify the calculation please as there is little difference in answer.

Q15

The rate of flow of water through a circular channel of diameter .8m is 200 lit/s. Find slope of the bed of the channel for maximum velocity. Take C=50



Water

O Center of circle, R= Radius=.8/2=.4

D=Dia .=.8m

The velocity of flow through a circular channel will be maximum when the hydraulic mean depth m or A/P is maximum for a given value of C and (i)

Variable factor θ only and its value as calculate in equation (16.18) it comes to by hit and trial 257°30'=2θ so θ= 128° 45'

The depth of flow d comes to =R(1+.62) =.81D

=.81 x .8=.648 m

,θ=value in radian=128° 45'x π/180=2.247 rad

As per equation 16.19 hyd. Mean depth m =.3D=.3 X.8=.24 m

Wetted perimeter (16.16) P =2Rθ=D θ=.8 X 2.247

= 1.7976m

A=m x P= .24 X 1.7976=.43424 m2

Q =Discharge=A C $\sqrt{m\ i}$

$.2 \;\; = .4342 \text{ x}50 \text{ x}\sqrt{.24i} \;\; = 21.71\ \sqrt{.24\ i}$

$.24\text{i} = \frac{.2}{21.71} = (.00921)^2$

$\text{So (i} = \frac{.0000848}{.24} = .0003526 = \frac{1}{\frac{1}{.0003526}} = \frac{1}{2836}$

=1 : 2836=slope of the bed

Q 16

Determine the maximum discharge of water through a circular channel of diameter 2.0m when the bed slope of channel is 1 in 1500. Take C=50 [Ans 3.02 m3/s]

Diameter of pipe=2.0m R=Radius=1.0m

Bed slope =1/1500 C=50

Max discharge angle=θ =154°=$\frac{154\ x\pi}{180}$ =2.6878 radians

(16.16) P =Wetted perimeter

=2Rθ=2 x1x2.6878=5.3756m

(16.17) A =Wetted area =R^2(θ---$\frac{SIN2}{2}$)

A = 1X1(2.6878--$\frac{sin2\ x154}{2}$)

= (2.6878 ---sin308/2}

= (2.6878---sin (360—52)/2)

=2.6878 –sin52°/2)

=2.6878 +.778/2

A =3.05 m2

Maximum discharge=Q = A .C . $\sqrt{m\ i}$

= 3.05 x 50 x $\sqrt{.5673\ x\ \frac{1}{1500}}$

=3.05 x50 x$\sqrt{.0003782}$

=3.05 x 50 x.0194473

=2.9657 m3/s

Type equation here.

Q 17

The discharge of water through a rectangular channel of width 6 m is 18m3/s when depth of flow of water is 2m.

Calculate i) specific energy of flowing water ii) critical depth and critical velocity. Iii) value of minimum specific energy.'

[1=2. 115m 2, .971m , 3. 087 iii 1.457m]

Solution

Given

Q =Discharge=18m3/s, width =b=6m,depth=h=2m

Area =bx h= 6x2=12m2

Discharge /unit width=18/6= 3 m2/s

V=Velocity of flow=Q/A=18/12=1.5 m/s

1 specific energy of flowing water (eq no 16.21)=E

$$=\text{h} + V^2/2g = 2 + \frac{1.5^2}{2x9.81} = 2 + .1146 = 2.1146\text{m}$$

2 a) critical depth (16.23)

$$= h_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{3^2}{9.81}\right)^{\frac{1}{3}} = \left(\frac{9}{9.81}\right)^{.333} = (.917)^{.333} = .971\text{m}$$

b) critical velocity (eq , 16.24)

$$V_C = \sqrt{g\ xh_c} = \sqrt{9.81x.971} = \sqrt{9.5255} = 3.0863\text{ m/s}$$

3 minimum specific energy (16.25)

$$= \frac{3h_c}{2} = \frac{3x.971}{2} = 1.456\text{ m}$$

Type equation here.

Q18

The specific energy for a six meter wide rectangular channel to be 5kg-m/kg. If the rate of flow of water through channel is 24m3/s, determine alternate depth of flow. [4.831m , 0.169 m]

Solution

Given

Width =6 m =b, E=Specific energy =5kh-m/kg =5m

Q= Discharge =24m3/s

Equation (16.21) specific energy=E =h +$\frac{V^2}{2g}$

[V=Q/A=24/bx h=24/6xh h=alternate depth]

S o E= h +(h/4)^2/2g

$$5\text{-}=h+\frac{8}{gh^2}=h+\frac{8}{9.81\quad{}^2}=h+\frac{.8155}{h^2}$$

$$5h^2-h^3-.8155=0$$

$$h^3-5h^2+.8155=0 \quad \text{-----}1$$

The above equation 1 is a cubic equation. By solving by trial and error method we will get h=4.831m and 0.169m.

Q 19

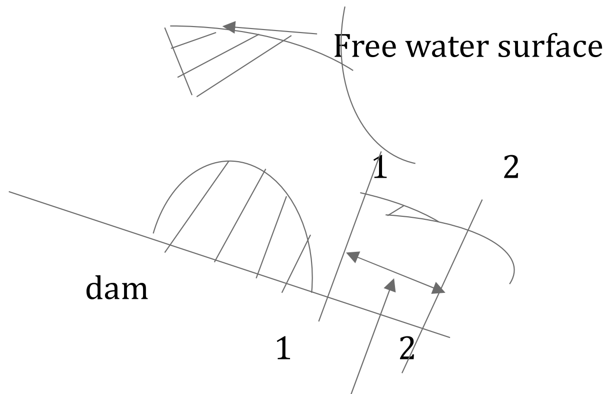
The depth of flow of water at a section of 5 m wide is .6 m .The discharge through the channel is 15m3/s. If hydraulic jump takes place on the down stream ,find the depth after the jump.[Ans 1.474 m]

Solution

Given width of channel=5m=b

Depth of flow before jump=di =.6m

Discharge of flow before jump =15m3/s



,discharge /unit width=15/5=3m3/s

Let d_2 be depth of flow after jump .

Depth of flow after jump is given by equation (16.27)

$$d_2\text{=di/2} + \sqrt{(\frac{d_1^2}{4} + \frac{q^2}{gd_1})}$$

$$= \frac{.6}{2} + \sqrt{.6^2/4 + 2\frac{3^2}{9.81x.6}}$$

$$= .3 + \sqrt{.09 + \frac{18}{5.886}} = .3 + 1.774$$

$$=1.474\text{m}$$

Q20

For question19,find the loss of energy/kg of water due to hydraulic jump. [Ans 0’188 m]

Solution

Depth of flow=.6m=di

Width of channel=b=5m

As found out in (Q 19) di =depth of flow after the hydraulic jump

=1,474m

As per equation (16.30)loss of energy h_L after jump

$$= \frac{(d_2-d_1)^3}{4d_1d_2}$$

$$=\frac{(1.47 \quad .6)^3}{4\,x1.474x.6}$$

$$=\frac{.874^3}{3.5376}$$

$$=\frac{.6676}{3.5376}$$

= .188m m

Q21

A sluice gate discharging water into a horizontal rectangular channel with a velocity of 8 m/s and depth of flow is .5m. The width of channel 6m. Determine whether hydraulic jump will occur and if so find the height and loss of energy per kg of water.

Also determine the horse power lost in the hydraulic jump.

[Ans yes,1.816m,1.293m ,413.76 h p]

Solution

Velocity of flow =8m/s , di =depth of flow=.5m=V_1

Width of channel =b =6m ,

1 hydraulic jump will occur =?

Discharge per unit width =Q/b=$\frac{V_1\,xd_1xb}{b}$ =V_1d_1=8 x.5 =4m3/s

Froude no on the upstream side $(F_{e1})=\frac{V_1}{\sqrt{gd_1}}=\frac{8}{\sqrt{9.81\ .5}}$

$$=\frac{8}{\sqrt{4.905}}=\frac{8}{2.214}=3.6133$$

As upstream Froude number is more than 1, the flow will shoot on the upstream side Shooting flow is unstable flow and it will convert itself into streaming flow by raising its height and hence hydraulic jump will take place.

2

Height of hydraulic jump $(d_2 - d_1)$(e q u. 16.31) =?

$$d_2 = \frac{d_1}{2}(\sqrt{1+8F_1^2}\ \text{---}\ 1$$

$$=\frac{.5}{2}(\sqrt{1+8(3.6133)^2}\ \text{--}1$$

=.5/2 X10.266 –1=1.566 m

height of hydraulic jump=$d_2 - d_1$ =1565-- .5=1.066m

3. $h_L=\frac{(d_2-d_1)^3}{4d_2d_1}=\frac{(1.066)^2}{4x.5x1.566}=\frac{1.211}{3.132}$ =.3866 kg -m/kg

Power loss in kw =$\frac{\rho gxQ\ Xh_L}{1000}=\frac{1000X\ \ .81\ X(8X.5X6)X.3866}{1000}$

= 9.81X24 X.3866

=91.0 KW=91.0 X1.34 h.p

Q22

Find the rate of change of depth of water in a rectangular channel of 12 m wide and 2 m deep , when water is flowing with a velocity of 1.5m/s. The flow of water through the channel of bed slope 1 in 300 is regulated in such a way that the energy line is having a slope 1 in 8000. [Ans ‘000235]

Solution

Width =12m and depth =2m=h

Velocity V of water=1.5 m/s

Bed slope = i_b=1/300 , energy slope =i_e=1/8000

=0.00333 =0.000125

Let rate of change of depth of water=dh/dx

Using equation 16.3 2

$$Dh/dx=\frac{i_b-i_e}{1-\frac{V^2}{gh}} = \frac{.0033 \quad .000125}{1-\frac{1.5x1.5}{2x9.81}}$$

$$=\frac{.003205}{0.88533}=0.00362011$$

No mistake either in process or calculation but answer not matching.

Q23

Find the slope of free water surface in a rectangular channel of width 15m , having depth of flow 4m . The discharge through the channel is 40m3/s. The bed of channel is having a slope of 1 in 4000. Take the value of Crazy' constant C=50. [Ans 0.000184]

Solution

Given

Width of channel =b =15m

Depth of flow=h=4m

Discharge =Q =40m3/s

Bed slope =i_b=1/4000=.00025

Crazy's constant =50. P=Perimeter =b+2h=15+2x4=23m

Type equation here.

Discharge Q= V X Area=C$\sqrt{mi}$ X A=50X 15x4 $\sqrt{m\, i}$

,,m=hydraulic mean =A/P=60/23

From Crazy's formula

$$Q=C\sqrt{mi} x A$$

$$40=50\sqrt{(\frac{60}{23} i_e x 60)}$$

$$\sqrt{i} = \frac{1}{75\sqrt{\frac{60}{23}}} = \frac{1}{75\sqrt{2.60}} = \frac{1}{75x1.6124} = .008269$$

$$i_e = .0000683 = \text{energy slope.}$$

Let slope of water surface= dh/dx As per equation

(16.32) $dh/dx=\frac{i_b-i_e}{1-\frac{V^2}{gh}} = \frac{0.00025-0.0000683}{1-\frac{V^2}{9.81X4}}$ -------- 1

[V=Q/A=40/60=;66m/s]

Putting the value of V

We get dh/dx from --- 1 (above)

$$= \frac{.0001817}{1-\frac{..66^2}{39.24}} = \frac{'0001817}{1-.0111}$$

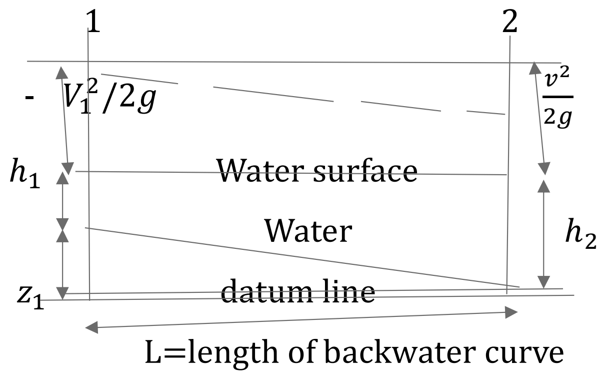
$$=\frac{.0001817}{.9889}$$

$$=.00018374 \text{ ans.}$$

Q24

Determine the length of the back water curve caused by an afflux of 1.5 m in a rectangular channel of width 50 m depth 2m. The slope of the bed is given as 1 in 2ooo . take Mannings;s N=0.03 , [Ans 4566m]

Solution



$i_e = energy\ slope i_b = bed\ slope = 1/2000$ =.0005

V_1 $= Velocity\ at$ se V_2=Velocity at sec,2

L=Length of back water. h_L=energy loss

Width of channel=50m=b,

Afflux=$h_2 - h_1$=1.5m ,depth of channel=h_1

$h_2 =$ 1.5 +h_1=1.5+2=3.5 m

N=.03

Area of flow at sec 1=Ai =b x hi= 50 x 2=100m2

P_1=Wetted parameter =b +2hi=50+2x2 =54m

Hyd. mean dept. =mi =Ai/Pi=100/54=1.85m

Using Manning's formula= V=$\frac{1}{N} X m_1^{2/3} i_b^{1/2}$

$=\frac{1}{.03} X (1.85)^{\frac{2}{3}} x$ (.0005)$^{..5}$

=33.33 x1.5 x.02236

V_1 =1.1178 m/s

Specific energy at section 1

$$E_1=\frac{V_1^2}{2g}+h_1 \quad =\frac{(1.1178)^2}{2x9.81}+2 = .063+2=2.063\text{m}$$

From continuity equation velocity V_2 at sec.2

$$V_2=\frac{V_1A_1}{A_2}=\frac{1,1178X100}{bxh_2}=\frac{111.78}{50x3.5}=.6387\text{m/s}$$

$$A_2=175\text{m2}$$

Wetted perimeter at sec 2 $=P_2$ =b $+2h_2$=50+2x3.5

=57m

$$P_1 \quad =54$$

$$m_2=\frac{A_2}{P_2} \quad =\frac{175}{57} \quad =3.07\text{m}$$

Specific energy at sec 2 -2

$$E_2=h_2+\frac{V^2}{2g} \quad =3.5+\frac{.6385^2}{2X9.81} \quad =3.52$$

We have to find average velocity Vav

$$h_{av}= \quad \frac{h_1+h_2}{2}=\frac{2+3.5}{2}=2.75\text{ m/s}$$

$$\text{Vav}=\frac{V_1A_1}{Aav} \quad =\frac{1.1178Xbxh_1}{bxvhav} \quad =\frac{1.1178}{2.75}=.813\text{m/s}$$

$$,, \ m_{av}=\frac{m_1+m_2}{2} \quad =\frac{1.85+3.07}{2}=2.46\text{m}$$

We have to find the i_e =energy slope basing Manning's formula

$$\text{Vav}=\frac{1}{N}m_{av}^{2/3}i^{.5}=\frac{1}{.03}2.46^{.66}i_e{}^{.5}=.813\text{ m/s}$$

$$\sqrt{i} \quad =\frac{.813}{33.33x1.81} \quad =.01347$$

$$i_e \quad =(.01347)^2=.000181$$

The length of the backwater is obtained from equation (16.34)

$$L=\frac{E_2-E_1}{I_b-i_e} = \frac{3.52-2.063}{.0005-.000181} = \frac{1.457}{.000319} = 4567.398\text{m ans.}$$

END OF CHAPTER 16

Fluid Mechanics and Hydraulic Machine

Impacts of Jets and Jet Propulsion

Chapter 17

Q1

Find the force exerted by jet of water of diameter 100mm on a stationary flat plate ,when the jet strikes the plate normally with a velocity of 30m/s [Ans 7068.6N]

Solution

Given

V=30m/s . A=Area of cross section of jet

$=\pi/4 \text{ x} d^2 = \pi/4 \text{ x } .01 = .007853\text{m2}$

Force $=\rho \text{ A } .V^2 = 1000 \text{ X } .007853 \text{ X } 30^2$

$=900\text{X}1000 \text{ X}.007853$

$=7067.7\text{N}$ ANS.

Q 2

A jet of water of diameter 50mm moving with a velocity 20m/s strikes a fixed plate in such a way that angle between plate and jet is 60°. Find the force exerted by the jet on the plate i) with direction normal to the plate and ii) in the direction of jet.

[Ans 680.13 N ,589N]

Solution

Given dia. Of jet=50mm=.05m, V=velocity of jet=20m/s

i) the force exerted by the jet of water in the direction normal to the plate is given by (17.2)

$$F_n = \rho V^2 Axsin\theta = 1000 \text{ x } (20)^2 \frac{\pi}{4} x(.05)^2 \sin 60°$$

$$= 1000\text{x } 400 \text{ x}.001963 \text{ x } \frac{\sqrt{3}}{2}$$

$$=1.963 \times 200 \times 1.732$$

$$=679.98 \text{ N}$$

ii) force in the direction of jet (equation 17.3)

$$F_X = \rho \, A \, V^2 \sin^2\theta$$

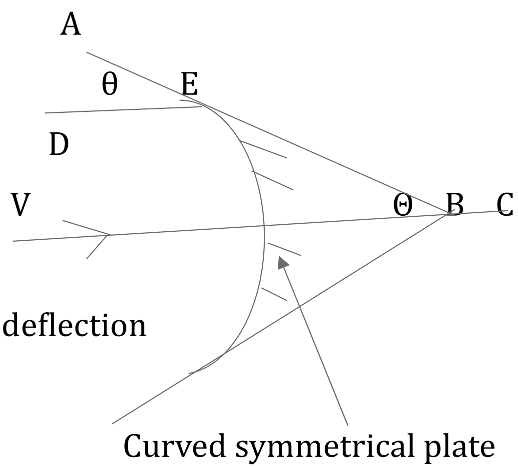
$$=1000 \times .001963 \times (20)^2 \sin^2 60°$$

$$=1.963 \times 400 \times 3/4$$

$$=1.963 \times 300 = 588.9 \text{ N}$$

Q3

A jet of water of diameter 100mm moving with a velocity of 30m/s strikes a curved fixed symmetrical plate at the center. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate. [Ans 10602.7N]



Angle ABC=180—θ=angle of deflection

Angle AED =θ

Solution dia. Of jet=100mm=.1m, A=$\frac{\pi}{4} X (,1)^2$=.007853m2

Velocity=30m/s, angle of deflection=120°=180—θ.

So 180—θ=120 ∴ θ=60°

Force exerted by jet on the curved plate in the direction of jet is given by equation (17.5)=F_X=ρ A $V^2(1 + Cos\theta)$

=1000x .007853 x$(30)^2$(1+cos60°)

=7.853x900x(1+1/2)

=7.853 x 900 x 1.5

=10601.55 N

Q4

A jet of water of diameter 100mm moving with velocity of 20m/s strikes a curved fixed plate tangentially at one end at an angle of 30°to the horizon. The jet leaves the plate at an angle of 20°to the horizon Find the force exerted by the jet on the plate in the horizontal and vertical direction [5672.34N, 496.3N]

Solution

Given

„ d =jet dia.=100mm=.1m

Area of cross section of jet=.007853m2

Velocity of jet=20m/s

Angle made by jet at inlet tip with horizon=θ=30°

Angle made by jet at outlet tip with horizontal=φ=20°

The force exerted by jet water in direction of x-axis is given by equation (17 .8) and in the direction of y axis by eq.(17.9) :--- F_X=ρ x A x V^2[cos θ +cos φ] .

=1000 x .007853x $(20)^2$[cos30°+cos20°]

$=7.853 \times 400 \, [\frac{\sqrt{3}}{2} + .93969]$

$=785.3 \times 4 \times \{.86603+.93969]$

$=785.3 \times 4 \times 1.80572$

$=5672.12N$

$F_Y = \rho \, AV^2(Sin\theta - sin\varphi)$

$=1000 \times .007853 \times (20)^2(\sin30° -- \sin20°)$

$= 7.853 \times 400 \times [.5--.342]$

$=7.853 \times 4 \times 15.8$

$=496.3N$

Q5

A jet of water 30mm diameter moving with a velocity of 15m/s, strike a hinged square plate of weight 245.25 N at the center of the plate. The plate is of uniform thickness Find the angle through which the plate will swing. [θ =40° 25' 6"]

Solution

Given

Diameter of jet=30mm =.03m

V=Velocity of jet =15m/s

W =weight of plate =245.25 N

A =Area of jet c/s=$\frac{\pi}{4}.03^2$=.0007067m2

The angle through which the plate will swing is given by equation (17.10) Sin θ=$\frac{\rho AV^2}{W} = \frac{1000\ x.0007067x(15)^2}{245.25}$ =.64834

Sin θ = .64834 θ=40° 40'

Q6

A plate is acted upon at its center by a jet of water of diameter 20mm with a velocity of 20m/s . The plate is hinged and is deflected through an angle of 15°.Find the weight of plate If the plate is not allowed to swing what will be the force required at the lower edge of the plate to keep the plate in vertical position [485.5 N,62.8 N]

Solution

Given

Dia of jet =20mm=.02m

A=Area $=\frac{\pi}{4}\ x(.02)^2 = .000314$m2

Velocity =V=20m/s

„ θ =15°=angle of swing.

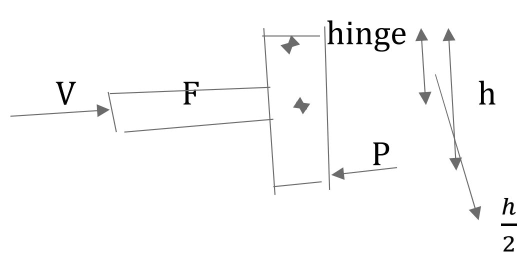
I) Using equation 17.10 of angle of swing

$$\text{Sin } \theta = \frac{\rho A V^2}{W}$$

$$\text{Sin } 15° = \frac{1000\ x.000314x(20)^2}{W}$$

$$W = \frac{125.6}{.25882} = 485.28 \text{ N}$$

11) If the plate is not allowed to swing a force P will be required at the lower edge as shown in fig below .The weight of the plate is acting vertically downward through the C.G of the plate.



Let F force is exerted by jet of water'

„ h =height of plate

=distance at which force P is applied.

The jet strikes at the center of the plate and hence and hence distance of center the jet from hinge = $\frac{h}{2}$.

Taking moment about hinge O

P x h = F x h/2

$\therefore$ P =$\frac{Fx\,h}{h\,x\,2}$ =$\frac{F}{2}$ = $\frac{V^2\rho A}{2}$ =$\frac{1000\ X\ .000314X20^2}{2}$

=$\frac{.314X\ 20^2}{2}$ =$\frac{.314x\ 400}{2}$ =31.4 x 2

= 62.8 N

Q7 A jet of water of diameter 150mm strikes a flat plate normally with a velocity of 12 m/s. The plate is moving with a velocity of 6m/s in the direction of the jet and away from the jet. Find force exerted by the jet on the plate 2..work done by the jet on the plate /s . 3.power of the jet and 4. Efficiency of the jet .

[636.3 N 2 3817.6Nm/s,3.82kw ,25%]

Solution

Given

Jet d I a.= 150mm=.15m

Area of c/s of jet=π/4 $(.15)^2$=.01767m2

Velocity of jet=12m/s

U =velocity of plate in jet direction=6m/s

i. force exerted by jet on the plate= (17.11)

$$=F_X \quad =\rho \text{ x A x } (V\text{--}U)^2 = 1000 \text{ x } .01767\text{x } (12\text{--}6)^2$$

$$=17.67\text{x}36=636.12 \text{ N}$$

Ii) work done by jet on the plate /s

$$=F_X \text{ x U}=636.12 \text{ x } 6 = 3816.72\text{Nm/s}$$

Iii)power of jet in kw= $\frac{work\frac{done}{s}\ by\ jet}{1000} = \frac{3816.72}{1000}$ =3.8167kw

iv) efficiency of jet = $\frac{output\ of\ the\ jet/s}{input\ of\ the\ jet\ /s} = \frac{work\ done\ jet\ per\ sec.}{kinetic\ energy\ /s}$

$$=\frac{power\ of\ the\ jet}{-(\rho AV)V^2} = \frac{3.816KW}{\frac{1}{2}X1 \quad X.01767\ X(12)^3}$$

$$=\frac{3816.72Nm/s}{15266.88Nm/s} = 25\%$$

Q 8

In the problem seven the jet strike the plate in such way that the normal on the plate makes an angle 30° to the axis of the jet, find i)the normal force exerted on the plate ii) power iii) efficiency of the jet. [551ii) 2.86 kw iii) 18.74%]

Solution

Given diameter of jet =150mm=.15 m

Area of c/s jet -=.01767,vel. Of jet V =12m/s

Vel. of plate =U=6m/s, angle between jet and plate

=90--30=60°

i) when the plate moving with a velocity of 6m/s the normal force on the plate is given by (equ.17.13)

F_n=ρ A (V--U)$^2 sin\theta$ =1000 x .01767(12--6)$^2 sin60$

=17.67 x (6x6)x .86603

=550.89 N

Ii) power of the jet

Work done by the jet=force in the direction of jet x distance moved by the plate in the direction of jet/sec

$= F_X xU{=}F_n\ sin60x\ U$=550.89X.86603Xu

=550.89 x .866o3 x6=2928.6 Nm/s

Power of jet in KW $=\frac{work\ done\ by\ jet}{1000}=\frac{2928.6}{1000}$

=2.93 KW.

„ iii) efficiency of the jet= $\frac{output}{input}=\frac{work\ done\ /s}{kinetic\ energy\ of\ jet}$

$=\frac{2928.6Nm/s}{\frac{1}{2}x(\rho xAXV)V^2}=\frac{2928.6}{\frac{1}{2}X\ 12^3X1000X.01767}$

$=\frac{2928.6}{15266.88}$

=.1918

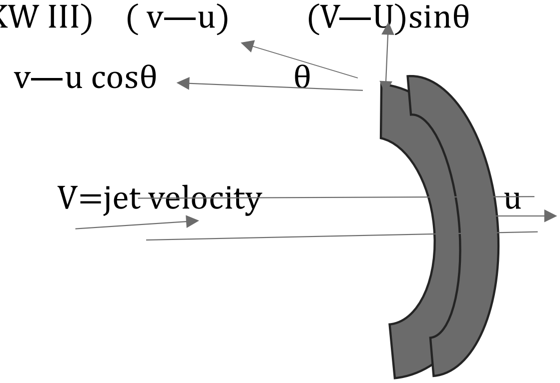
=19.18%

Q 9

A jet of water diameter 100mm strikes a curved plate at its center with velocity of 15m/s. The curved plate is moving with a velocity of 7 m/s in the direction of jet. The jet is deflected through an angle 150°.Assuming the is smooth fixed i) forced exerted on the plate in the direction of the jet ii) power of the jet iii)efficiency.[i. 938N II)6.56KW III) (v—u) (V—U)sinθ

Solution

Given



Moving curved plate

U=7m/s

„ d =100mm=.1m

A= $\pi/4(.1)^2$=.007853m2

Angle of deflection= 150°

Angle made by relative velocity at the outlet of the plate

=180 –150 =30° =θ°

1.equation (17.17) force exerted on the plate in the direction of jet $F_X = \rho A (V—U)^2(1 + cos\theta)$

$= 1000 \text{ x}.007853\text{x}(15\text{--}7)^2(1+\cos30)$

$= 7.853\text{X}64 \text{ X } (1+.86603)$

=937.8 N

2power of the jet $= \frac{work\ done\ byxjet}{1000} = \frac{F_X x\ U}{1000} = \frac{937.8\ x7}{1000}$

$=\frac{6564.6}{1000}$ =6.564 kw

3.effeciency of the jet $=\frac{output}{input}=\frac{work\ done\ by\ jet/s}{kinetic\ energy\ /s}$

$=\frac{F_X x U}{\frac{1}{2}\rho A\,V(V^2)}=\frac{937.8\ X7}{.5(1000)X.007853X(15)^3}$

$=\frac{6564.6}{13251.93}$

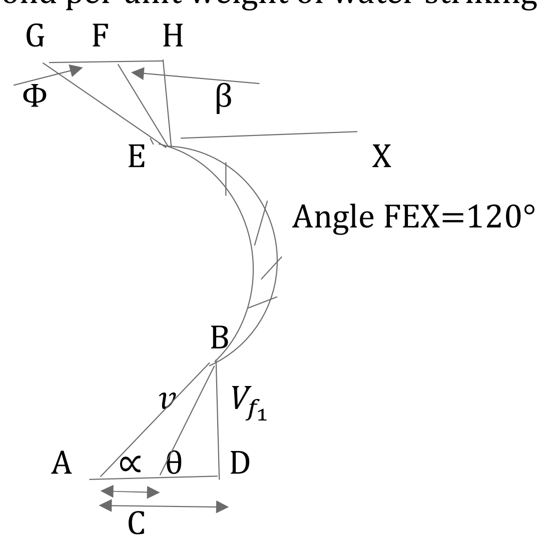
$=.4953$

$=49.53\ \%$

Q 10

A jet of water having velocity of 30m/s strikes a curved vane which is moving with a velocity of 15m/s , The jets makes an angle of 30° with direction of motion of vane at inlet and leaves at an angle 120° to the direction of motion of vane at outlet.

Calculate 1. vane angle if the water enters and leaves vane without shock 2. work done per second per unit weight of water striking the vane. [53 47.7 ,15 41,



Velocity of jet $=V_1$=30m/s, Velocity of vanes=U_1=15m/s

Angle made by the jet at inlet in the direction of motion of vane $=\alpha = 30°$(angle BAD)

Angle made by jet leaving vane in the direction moti∴on =120°

=angle EFH=180—120 =60°

In problem $u_1 = u_2 = 15$

$v_{r_1} = v_{r_2}$

I) vane angle =mean angle made by the relative velocities at inlet and outlet θ and ∅From figure above Δ ABD we

V_{f_1}=$v_1 sin\alpha$=30 sin30 =30 x .5=15m/s

$v_{w_1} = v_1 cos\alpha$=30cos30 =30 x.86603=25.9809

Tan θ =$\frac{v_{f_1}}{v_{w_1}-u_1}$=$\frac{30sin30}{30cos30-15}$ =$\frac{15}{25.98-1}$ =1,366

a) θ= 53° 40’ ans.

From Δ ABC Sin θ =$\frac{v_{f_1}}{V_{r_1}}$ OR $V_{r_1} = \frac{v_{f_1}}{sin\theta}$=$\frac{15}{.8}$ =18.75m/s =V_{r_2} ---II}

From Δ EFG applying sign rule

$$\frac{V_{r_2}}{\sin{(180-B)}} = \frac{u_2}{\sin{(\beta-\varphi)}}$$

18.75/sin60=15/sin(60—φ)

Sin(60—φ)=15 xsin60/18.75=15 x.86603/18.75

=12.99/18.75 =.6928

So 60—φ= 43° 50’ ∴ φ =60 –43° 50’ =16° 10’ ans.

11) work done /s/unit weight of water striking the vane/sec is given by equation (17.21)

=1/g (V w i + Vw2) Xu Nm/N

Vw2=GM—GF=Vr2cosφ—U2

=18.75 cos16 10’—15

=18.75 x.95715—15

=17.946—15 =2.946 m/s=Vw2

$U = U_i = U_2 = 15$

Work done /unit weight of water

$= 1/9.81\ [V_{w1} + V_{w2}] \times 15$

$= 1/9.81\ [25.98 + 2.946] \times 15$

$= 1/9.81 \times 28.92 \times 15 = 433.8/9.81$

$= 44.22$ Nm/N ans.

Q11

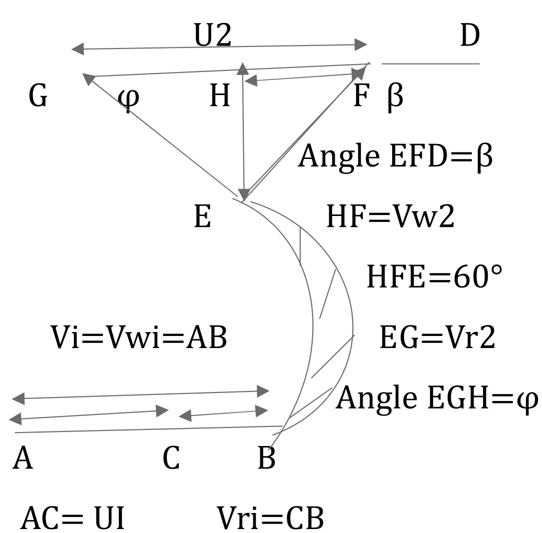
A jet of water 50mm having velocity of 30m/s strikes a curved vane which is moving with a velocity of 15m/s in the direction of the jet . The jet leaves the vane at an angle of 60° to the direction of motion of vane at outlet .

Determine 1. Force exerted by by the jet on the vane in the direction of motion.2.work done/s by the jet.[662.5N II 9937.5Nm/s]

Solution

Diameter of jet -=d=50mm Area=$\pi/4 \times (.05 \times .05) = .001963 m^2$

Velocity of jet=V_i=30m/s, velocity of vane =U_i=15m/s



As jet and vane are moving in same direction $\alpha=0$.

Angle made by water leaving jet with direction of motion=60°

„ $\beta=180---60=120°$

For this problem we have UI=U2=U=15 m

V r i =Vr2

Now in Δ EFG EG =Vr2=15m/s, GF=U2=15m/s

Angle GEF= {180 –(60+φ)} = (120 –φ)

From sine rule --

$$EG/\sin 60 = GF/\sin(120—\varphi)$$

$$15/.86603 = 15/\sin(120—\varphi)$$

$$\sin(120—\varphi)=.88603=\sin 60$$

$$„ \ \varphi = 60°$$

$$Vw2=HF=GF—GH=U2---Vr2\cos\varphi$$

$$=15---15\cos 60=15—15 \times .5=7.5 \text{ m} =Vw2$$

1. Force exerted by jet on vane in direction on motion is given by equation (17.19)

$$= F x=\rho A \ Vri \ (Vwi—Vw2)$$

$$=1000 \times .001963 \times 15(30---7.5)$$

$$=1.963 \times 15 \times 22.5$$

$$=662.5 \text{ N}$$

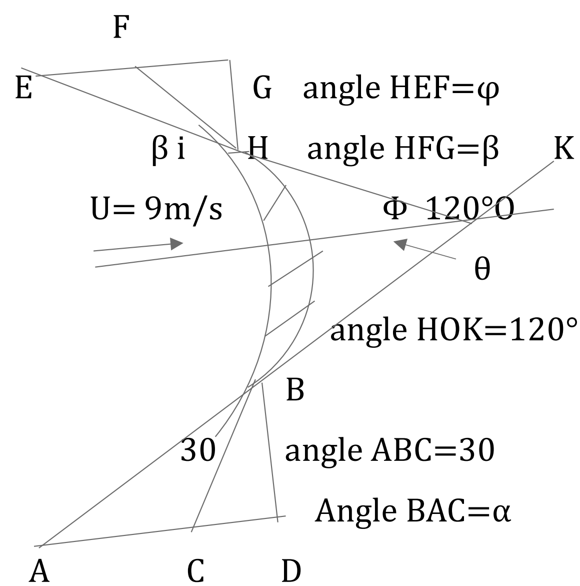
2. Work done by the jet= Fx x U

$$=662.5 \times 15$$

$$=9937.68 \text{ Nm/s}$$

Q 12

A jet of water having a velocity of 20m/s strike a curved vane which is moving with a velocity 9m/s. The vane is symmetrical and so shaped that the jet is deflected through 120°.Find the i) angle of the jet at inlet of the vane so that there is no shock ii) what is the absolute velocity jet at outlet in magnitude and direction and iii) work done/sec/unit weight of water striking? [17° , 5.95m/s ,β=79 6’, 18.57Nm/N]



AB=Vi AC=Ui, AD= Vwi, BD=Vfi ,BC=Vri, angleBCD=φ

EH=Vr2,FH=V2, HG=Vf2,EF=U2 FG=Vw2

Assume vane is smooth.

Velocity of jet=Vi=20m/s, velocity of vane=Ui=9m/s

As vane is symmetrical so(θ=φ), angle of deflection=120

$$=180---(\theta+ \varphi)$$

$$(\theta+ \varphi) =180—120=60°$$

$\Theta=\varphi=30°$

i) let angle of jet at inlet be $=\alpha$

ii) absolute velocity of jet outlet be=V2

angle made by V2 at outlet with direction of motion of vane $=\beta$

as vane is smooth here Ui=U2=U=9m/s and

Vri=Vr2

Applying the sine rule AB/sin(180—φ)=AC/sin(30—α)

$$Vi/\sin(180—30)=Ui/\sin(30—\alpha)$$

Or, Vi/sin(180—30)=Ui/sin(30—α)

20/sin30=9/sin(30—α)

So, sin(30—α)=(9Xsin30)/20=(9 x 1)/20x2

Sin(30—α)=.225=sin13°

30—α =13

„ α =17°

ii) V2=Absolute velocity.

From sine rule

AB/sin(180—30)=CB/sinα

Vi/sin30=Vri/sin17

V r i =Vi x .29237/1/2=20 x2 x.29237=11.6948=Vr2

From velocity triangle HEG at outlet ------

Vr2cosφ=EG=U2+Vw2

11.6948 cos30 =9 +Vw2 , Vw2=11.6948x .86603—9

Vw2=10.13—9=1.13m/s

Also we have Vr2 sin φ=Vf2 =11.6948 xsin30=11.6948x1/2

$$Vf2 = 5.847 m/s$$

In triangle HFG $V2 = \sqrt{(Vf2)^2 + (Vw2)^2}$

$$= \sqrt{5.847^2 + 1.13^2}$$

$$= 5.955 m/s$$

Tanβ =Vf2/Vw2 =5.847/1.13=5.174

„ β =79° 6'

∴ angle made at outlet with direction of motion β I

=180—β =180—79° 6'=100°54'

iii) work done /unit weight (17.21) of water

=1/g [Vw1+Vw2] x U Nm

=1/g [Vi cosα +1.13]x 9

=1/9.81 x[20xcos17+1.13] x9

= 1/9.81(20 x.95630+1.13]

=20.250 x 9/9.81

= 18.27Nm/N

Q 13.

Ajet of water having a velocity of 15m/s strikes a curved vane which is moving with a velocity of 6 m/s in the same direction as that of jet at inlet. The vane is so shaped that the jet is deflected through 135°.The diameter of the jet is 150mm. Assuming the vane to be smooth find i) the force executed by the jet on the vane in the direction of motion ii) power of the vane iii) efficiency of the vane [2443.5N II 14.65 KNIII 49.16%]

Solution

EF=U2,FG=Vr2,DF=V2,Velocity of jet=Vi=15m/s,

Velocity of vane=U=Ui=U2=6m/s .

Inlet and vane same direction so $\alpha=0$ .di a. of jet=150mm

Area of jet=$\pi/4(.15x.15)=.0177m2$, angle of deflection=135°

$=(180—\varphi)\ \therefore \varphi=(180—135) = 45°$

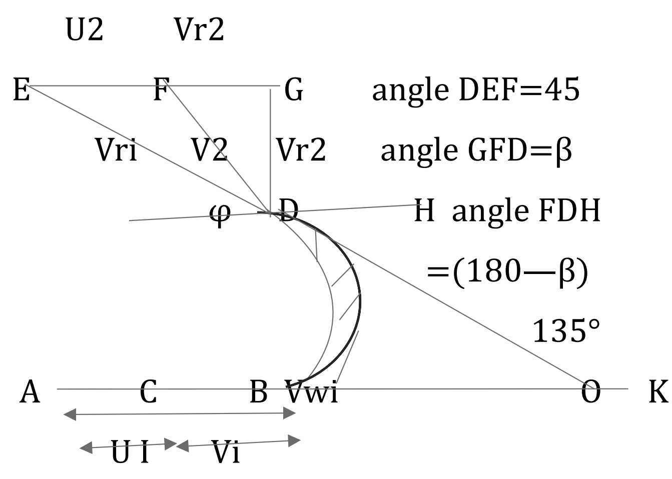
As vane is smooth Vri=Vr2. From inlet velocity which is straight line ,we have Vri=Vi—Ui=15—6==9m/s.

Vwi=Vi=15m/s.

From the outlet velocity triangle DEG we have Vr2=Vri=9m/s

Vr2cosφ=U2+Vw2 , or, 9cos45=6+Vw2 or, 9x.707=6+Vw2,

Vw2= 6.363—6=.363m/s



1. Force exerted by the jet on the vane in direction of motion(17.19) = Fx=ρ A Vri [Vwi+Vw2]

=1000 x .0177x 9(15+.363)

=2447.32N

2. Power of the vane=Fx X u Nm/s =2447.32 x6=14683.92 w

=14683.92/1000 =14.68 kw

3 Efficiency of vane =work done /s on vane/ kinetic energy supplied by jet/

= (Fx x U)/(1/2 x(ρ x A x V)(V x V))

= (14683.92x2)/ (.5 x1000x.0177 x 15) x (15x15)

=14683.92 x2/ 59737

=.4916

=49.16%

Q 14

If in the above problem the jet of water instead of striking a single plate strikes a series of curved vanes find

1. force exerted by the jet on the vanes on the direction of motion
2 power of the vanes.

3.efficiency of the vanes.[4072.5N ii 24.43kw III 81.9%]

Solution

In continuation with Q 13

Vi=velocity of jet =15m/s

U=Ui=U2=velocity of vanes= 6m/s

„ α =0 area of jet= 0.0177m2

„ φ =45° , Vw i =15m/s ,Vw2=.363m/s

For the series of vanes mass of water striking the vanes /s

=ρ A Vi =1000 x .0177 x15 =17.7 x15 =265.5m3/s

1. force exerted by the jet on vanes in direction of motion

   =Fx=(ρ A Vi)(Vwi+Vw2)=265.5 (15+.363)=4078.87

2. Power of the vanes in kw

   =work done/s in kw/1000

   =Fx x U/1000=4078.87 X6/1000

   =24472.8/1000

   =24.47KW

3. Efficiency =work done /s /(.5 x mass of water /s) x Vix Vi

   =4078.8 x 6/.5x(265.5)(15x15)

=.8193 =81.93%

Q 15.

Ajet of water having velocity of 30m/s strikes a series of radial curved vanes mounted on a wheel which is rotating at 300 rpm. The jet makes an angle of 30° with the tangent to wheel at inlet and leaves the wheel with a velocity 4m/s at an angle of 120° to the tangent to the wheel at outlet. .Water is flowing from the outward in a radial direction . The outer and inner radii of the wheel are 0,6, .3m respectively. Determine 1. Vane angle at inlet and outlet ii work done /sec/kg of water iii efficiency of the wheel.[42 10.7',27 17.8' ii 52.92 Nm/N iii56.5%]

Solution

Velocity of jet $=V_i=30m/s$, velocity of vane $=U_i=U_2$

Speed of wheel $=300rpm$, angular speed $=2\pi x\ 300/60$

„ $\omega = 31.41$ radians,

let $\alpha =30°$ angle made by jet at inlet.

V_2=velocity of jet at outlet 4m/s

Angle made by jet at outlet =120°, angle $\beta=60=(180—120)$

Outer radius of wheel$=R_i=.6m$

Inner radius of wheel $=R_2=.3m$

$U_i=\omega \times R_i =31.4 \times .6=18.84$ m/s

$U_2=\omega R_2=31.4 \times .3 =9.42$ m/s, cos 30=.866

Sin30 =.5 cos 60=.5

1, vane angles at inlet and outlet that is angles made by V_{r1} and V_{r2} I,e. θ and φ,

$\Delta ABD\ V_{wi}=V_i \cos \alpha =30 \times \cos30=25.98$

$V_{fi}=V_i \sin\alpha=30\sin30 =15m/s$

In Δ =CBD Tan $\theta=BD/CD=V_{fi}/AD—AC=15/25.98—U_i$

$=15/25.98---18.84$

$=15/7.14=2.1$

$=2.1$

.. $\theta =64° 35'$=vane angle

Type equation here.

From outlet velocity $V_{w2}=V_2\cos\beta = 4\cos 60 = 4x.5 = 2$ m/s

$V_{f2}=V_2\sin\beta = 4\sin 60 = 4$ x.866=3.464 m/s

In Δ EFH Tan φ =Vf2/U2+Vw2=3.464/9.42+2=.303

=tan16°50'

Vane angle =φ =16° 50'

2. work done /s/kg of water (17.26)

=ρa Vi(Vwi Ui +Vw2 U2)/weight of water/s

=ρ A Vi(25.98x18.84+2x 9.42)/g xρ A Vi

= 508.3/9.81 =51.81 Nm/N

3. Efficiency (1728) =2(Vwi.Ui±Vw2 U2)/V_1^2

=2(25.98 x18.84±2x 9.42)/30x30

= 2x508/900 , 2x470/900

= both are more than 1 means efficiency is more than 100% which seems to erroneous.

Another equation for efficiency (17.4.6 para of original book)

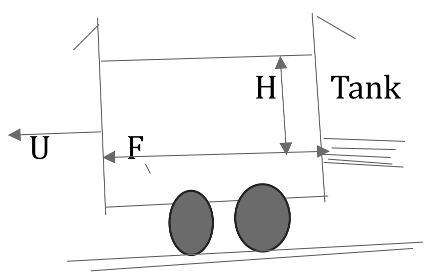
$$=\frac{V_1^2-V_2^2}{V_1^2} = \frac{30^2-4^2}{30^2} = 884/900 = 98.2\%$$

This formula also giving higher efficiency but within 100% and hence acceptable subject correctness in calculation which students may please check at their ends.

Q16

The head of water from the center of the orifice fitted to a tank is maintained at 6m of water . The diameter of the orifice is 150mm. The tank is fitted with frictionless wheels at the bottom and tank is moving with a velocity of 4m/s due to reaction of jet carrying out from the orifice. Determine i)propelling force on the tank ii) work done per second iii)Efficiency of propulsion

[i)2847.3 NII)11389v



H=6m=head of water

„ d =diameter =150mm =.15m

Area =area of orifice = π/4 (.15x.15)=.0177m2

U=Velocity of tank=4m/s

Let us assume Cv=coefficient of velocity=1(say)

(Since it is not provided.)

Velocity of jet =Cv x $\sqrt{2gH}$ =1 x $\sqrt{2x9.81x6}$

=10.85m/s

Force exerted on the tank as per equation (17.29)

F=ρ A V^2= 1000 X .0177 (10.85 X10.85) =1960.45

1.Propelling forces on the tank (eq.17,30)

Type equation here.

F x=ρ A(V+U) x V =1000 x .0177(10.85+4) x10.85

=2851.8 N

2 work done /sec=Fx U=2851.8 X 4=11407.2 Nm/s

3 efficiency (eq.17.31=2V U/(V+U) $=\frac{2VU}{(V+U)^2} = \frac{2\,X 10.85 X 4}{(10.85+4)^2}$

=868/220.5255= .3936

= 39.39%

Q17

The water in a jet propelled boat is drawn midship and discharged at the back with absolute velocity of 30 m/s. The cross section area of the jet at the back is .04m2 and the boat is moving in sea water with a speed 30km/hr. Determine i) propelling force of the boat ii)power iii) efficiency of the jet propulsion .

[45995.6N, II 383.14KW III)34.02 %]

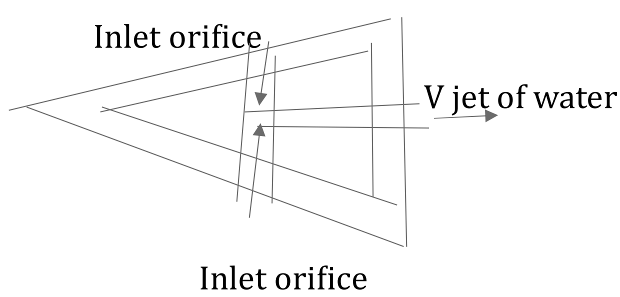
Given

V=Absolute velocity of jet=30m/s

Area of jet =.04m2

Speed of boat=30km/hr=30x1000/60x60

=8.33m/s



1. propelling force of jet (17.33) F=mass of water issue/s

X change of velocity.

=ρ A (V+U)V

= 1000x .04 x(30+8.33)x30

= 1200x 38.33

= 45996N

2 Power required=work done by jet/second

=F x U =Propelled force x speed of boat

=45996 x8.33 watt

=45.996 x 8.33 kw

=383.14 kw

3 efficiency of propulsion (eq.17.36)

Work done /s by jet/energy required by jet

$$=\frac{\rho\ A(V+U)VU}{\frac{1}{2}X\rho\ A(V+U)[(V-U)^2-U^2]}=\frac{2U}{V+2U}=\frac{2X8.33}{30+2X8.33}$$

= 16.66/46.66 =35.7%

Q18

Water in a jet propelled boat is drawn through inlet opening facing the direction of motion of the ship. The boat is moving in sea water with a speed of 40km/hr . The absolute velocity of jet of water discharged at the back is 40 m/s and area

Type equation here.

of the jet is .04 m2. Find the propulsion force and efficiency of propulsion .[81775.3N, 35.71 %]

Solution

Given

Speed of boat $=U=40$ km/hr$=40x1000/60x60$

$=11.11$m/s

Area of jet $=.04m2$

1. propelling force(17.33)

$F=\rho A (V+U) V=1000X.04 (40+11.11) 40 =81776$ N

2. Efficiency of propulsion (17.36) $=2U/V+2U$

$=2 \times 11.11/40+2x11.11$

$=22.22/62.22$

$=.3570$

$=35.70\%$

END OF CHAPTER 17

Fluid Mechanics and Hydraulic Machines

Hydraulic Machines—Turbines

Chapter 18

Type equation here.

Q 1

A Pelton wheel has a mean bucket speed of 35m/s with a jet of water flowing at the rate of 1m3/s under a head of 270m.The buckets deflect the jet through an angle 170°. Calculate the power delivered to the runner and the hydraulic efficiency of the turbine Assume co-efficient of velocity as .98. [2523.8kw, 95.3%]

Solution

Speed of bucket =U=U1=U2=35m/s

Discharge Q = AVi=1m3/s. Head of water =H=270m

Angle of deflection=170° so vane angle=180—170=10°=φ

Cos 10=.98481, Cv=.98, velocity of jet =Vi

$=C\ v\sqrt{2gh}=.98\sqrt{2x9.81x290}$

=71.32m/s

Vri=Vi—Ui=71.32—35=36.32 V w i=Vi=71.32m/s

From outlet velocity triangle Vr2=Vri=36.32m/s

Vw2=Vr2 cos φ—U2 =36.32cos10—35

=35.768—35=.7683m/s

Work done by jet/s on runner(eq.18.9)=

=ρ (Avi)[Vwi+Vw2]x U=1000 x1 [71.32+,7683]35

=2522800Nm/s

1. power given to turbine =work done by jet/sec/1000 kw

=2522800/1000

=2522.8kw

2 hydraulic efficiency

$$=\frac{2[Vwi+V\quad)]Ui}{V_1^2}=\frac{2[71.32\quad .7683]x\ 35}{71.32^2}$$

$=5045.6/5086.5424$

$=99\%$

Q2

A Pelton wheel is to be designed for the following specification,

1 Power=735.75kw, 2shaft power head=200m 3,speed=800rpm

4 Efficiency=86% 5,jet diameter = not to exeed 1/10th of wheel diameter.

Determine i) wheel diameter ii) no of jet required iii)diameter of jet. Take Cv=.98 speed ratio=0.45 [wheel dπia, D=.673m

No of jet=2,dia of jet=67.3mm]

Solution

Given shaft power=735.75 kw

H=Head=200m, N=SPEED=800rpm

Over all efficiency=86%

Jet dia /wheel dia =d/D=1/10. Cv=Kvi=.98

Speed ratio=Kui=0.45

Velocity of jet =Cv$\sqrt{2gh}$ =.98 $\sqrt{2x\ 9.81\ x200}$

Type equation here.

=.98 x 62.64=61.39 m/s

Velocity of wheel =U=Vi=U2=speed ratio x√(2gh)

=.45 x 62.64

=28.18 m/s

1) U=Πdn/60 so D =Ux 60/ΠN=28.18 x60/πx800

=1690.8/2513.27=.6727m

2) d=diameter of jet.=1/10 x D=1/10 x .6727

=.067m=.067m1000mm=67.3mm

3.discharge of one jet=q

=area of one jet x velocity of jet

=π/4 d^2x Vi=π/4 x (.067x.067)x 61.39

= .21644m3/s

4. efficiency of turbine =shaft power/water power

$$=735.78/\frac{\rho gHXQ}{1000}$$

$$.86 \quad =\frac{735.78X1000}{1000X9.81X20 \quad .Q}$$

$$Q=\frac{735.75X1000}{1000X9.81X200X.86}=735750/1687320$$

=.4360 m3/s

$$5.\ \text{number of jets}=\frac{total\ discharge}{discharge\ by\ one\ jet}$$

=.4360/.21644

= 2 jets.

Type equation here.

Q3

A Pelton wheel is having a mean bucket diameter of .8m and is running at 1000rpm .The net head on the Pelton wheel is 400m . If the side clearance angle is 15° and discharge through is 150 lit/sec. find 1. Power available at the nozzle 2, hydraulic efficiency of turbine.[i= 588.6kw ii=98%]

Solution

Given

D=Dia. Of wheel=.8m

Speed of wheel=N=1000rpm

Tangential velocity =U=Πdn/60=π x.8 x1000/60

=2513.274/60

= U=41.888m/s

„ Φ =side clearance angle=15°, cos φ =cos 15°=.96593

Net head=400m

Discharge =Q=150 lit/s= .15m3/s

Velocity of jet at inlet =Vi=Cv x√(2g H)= 1 X√(2 X9.81 X400

=√(8X981)=88.58m/s=Vi

1. power available at nozzle(18.3)

$$= \quad W .p=w x \ H/1000 = \frac{\rho g Q H}{1000} = \frac{1000x9.81\ x.15\ x400}{1000}$$

=15 x4 x9.81

=588.6 kw

$$2.\ \text{hydraulic efficiency} = \frac{2(Vi-U)(1XCOS\varphi)U}{V_1^2}$$

$$= \frac{2(88.58-4\ \ .88)(1+COS15)41.88}{88.58X8\ \ .58}$$

$$=\frac{7690.19}{7846.41}$$

=.98

=98%

Q 4

Two jets strikes at brackets of a Pelton wheel ,which is having a shaft power (s.p) as14715 kw. The diameter of each jet is 150mm. If the net head on the turbine is 500m ,find overall efficiency of the turbine. Take Cv=1. [Ans 85.7%]

Solution :

Number of jets=2 , shaft power (s.p)14715kw.

Dia. of each jet =150mm=d. =.15m

Area of jet $=\pi/4(.15)^2=.0176m2$

Net head=H=500 m , Cv=1.0,

velocity of each jet=Vi= $\sqrt{(2gh)}$ x Cv

$=\sqrt{(2x9.81\ x500)}$ x 1=99.04 m/s

Discharge of each jet=q=A x Vi= .0176 x99.04 =1.743m3/s

Q=total discharge=2x 1.743=3.486 m3/s

1. power of inlet of turbine=w.p=ρ g Q H/1000KW

=1000 x9.81 x3.486 x500/1000

=166o8. 33 kw

2. efficiency overall=S .P/W.P=14715/16608.33=88.6%

Type equation here.

Q 5

Following data are related to Pelton wheel

H=Head at base nozzle =110m

Diameter of jet=d=7.5cm

Discharge =Q=200LIT/S=200/1000m3/s =.2m3/s

Shaft power=191.295 kw

Power absorbed in mechanical resistance =3.675kw

Determine i) power lost in nozzle ii)power lost due to hydraulic resistance in the number. [Ans i).874 ii) 9.97kw]

Solution

Discharge Q=Area of jet x velocity of jet

$.2 = \pi/4(.075)^2 = .004398Vi$

Vi=.2/.004398=45.47m/s

Shaft power=191.295kw

Power absorbed in mechanical resistance=3.675kw

Power at the base of nozzle =ρ g Q H/100

=1000 x9.81 x .2 x110/1000

=215.82kw

Power corresponds to K.E OF jet in KW

$= 1/2 \text{ x } \rho \text{ x } Q \text{ x } Vi^2/1000$

=1/2 x .2x (45.47x 45,47)

=206.75kw

Type equation here.

1. power at the base of nozzle

=power of jet +power lost in nozzles

215.82 = 206.75 +power lost in nozzles

Power lost in nozzles =215.82—206.75=9.07 kw

2. also power at the base of nozzles= power at shaft +power lost in nozzles +power lost in runner +power lost due to mechanical resistance .

So power lost in runner =215 .82—(191.295 +9'07 +3.675)

=215.82 –204.04=11.78 kw

Q6

Design a Pelton wheel a head 80m and speed 300rpm. The Pelton wheel develops 103 kw shaft power. Take Cv=.98 ,speed ratio (Kui)=0.45 and efficiency overall =.8 [D=1.135M d=72.6mm bucket size=36.3 x8.7 , no of bucket =23]

Solution

Given

S .p (shaft power)=103 kw

H=80m , N=300rpm ,speed ratio=d/D=.45

Cv=.98 , efficiency =.80 , Vi= Cv x √(2GH)

=.98 √(2 x9,81 x 80)

=38.8 m/s

U=Ui=U2= speed ratio √(2GH)=.45√(2 x 9.81 x80)

=17.468 m=U=Ui=U2

i) U=ΠDN/60=17.468 , D=17.468 X60/π X300

=17.468/15.70=1.12 m=D

ii) D/d=m= jet ratio ,m=12, D=1.12 m

„ $d = D/m = 1.12/12 = 93mm$

iii) width and depth of bucket

width of bucket $= 5 \times d = 5 \times 93 = 465$ mm.

$= .465$ m

Depth of bucket $= 1.2\ d = 1.2 \times .465m = ,1116m$

iv) no of bucket $= Z =$ (eq .18.17)

$= (15 + D/2d) = (15 + 1120/2x930)$

$= 15 + 6.02) = 21.02 = 22$ number of buckets.

Q 7

An inward flow Reaction turbine has internal and external .6m and 1.2 m respectively. The velocity of flowthrough the runner is constant and equal to 1.8m/s. Determine i) discharge through runner ii) width at outlet if the width at inlet is200mm.

[i)1.357m3/sii)400mm]

Solution

Given external dia. Of turbine=Di=1.2m

Internal dia .of turbine =D2=.6m

.Vel . of flow at inlet Vfi=Vf2 =1.8m.

ii) Width of turbine at inlet Bi=200mm=.2 m

Width of turbine at outlet =B2 =?

Using equation (18.21) for discharge

i) $Q = \pi D_i B_i V_{fi} = \pi \times 1.2 \times .2 \times 1.8 = 1.357\ m^3/s$ Ans,

since $V_{fi} = V_{f2}$ so $\rightarrow \pi B_i D_i V_{fi} = \pi B_2 D_2 V_{f2}$

ii) so $B2=\frac{DiBi}{D2}$ =1.2 x.2/.6 =.24/,6 =,4m

so width of turbine at out let =400mm Ans.

Q8

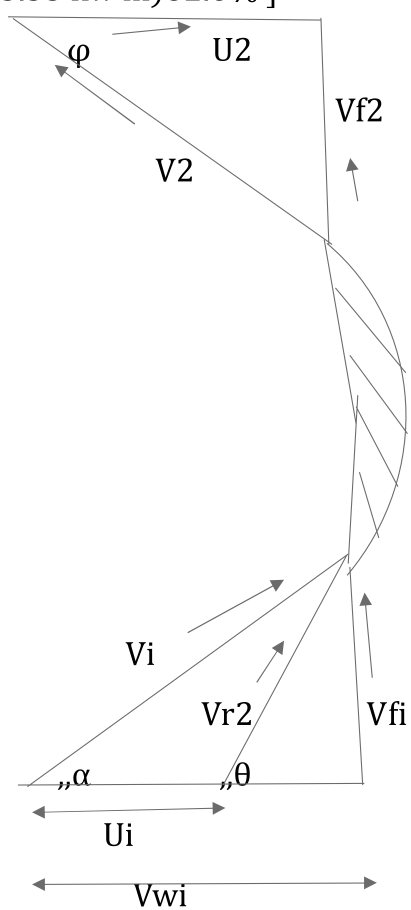
A reaction turbine wors at 500 rpm under a head of 100 m. The diameter of turbine at inlet is 100cm and flow area is .35m2. The angles made by absolute and relative velocities at inlet is 15° and 60°respectively with tangential velocity. Determine i) The volume flow rate ii) power developed iii) efficiency.

Assume whirl at outlet to be 0.

[i)2.905m3/s ii) 2355.35 kw iii)82.6%]

Solution

Given



Speed of turbine =500rpm , Head=H=100m,Di (inlet) = 1 m

Flow area =πDi xBi=.35m2, angle made by absolute velocity=α=15°,angle made by relative velocity =θ =60°

Whirl at outlet =Vw2=0

Tangential velocity at inlet =Ui=πDiN/60 =π x 1x 500/60

=1570/60=26.18m/s

From inlet velocity triangle;

Tanα =Vfi/Vwi=tan15=.26795

∴ Vfi=Vwi x .268=30.97 x .268=8.3 m/s

Tanθ=Vfi/(Vw1---Ui)=tan60°=1.732

Vfi= 1.732(V w i—Ui) =1.732 Vwi—1.732Ui =Vwi x.268

Or 1.732 Vwi--.268Vwi=1.732 x 26.18

1.464Vwi=45.34

Vwi=45.34/1.464 =30.97 m/s

Vfi=8.3m/s

i) volume flow rate (eq.18.21)

Q=(π Di Bi) x Vfi=(.35)x 8.3=2.905 m3/s

ii) work done /s on the turbine (18.18)=ρ x Q Vwi x Ui

=1000 x 2.905 x 30.97 x26.18 =2355358.313 Nm/s

Power delivered in kw=work dne Nm/s/1000 kw

=2355358/1000 kw =2355.35 kw

Iii)Hydraulic efficiency (18.21B)=Vwi Ui/Gh=30.97 x26.28/9.81x100=810.79/981=.8264 =82.64%

Q9

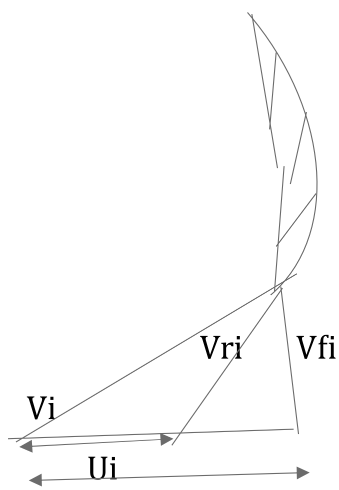
An inward flow reaction turbine has an extended diameter of 1 m and its breadth at inlet is 200mm. If the velocity of flow at inlet is 1.5m/s find the mass of water passing through the turbine /s. Assume 15% of area of flow is blocked by blade thickness .If the speed of the runner is 100 rpm and guide blades make angle of 15° to the wheel tangent, draw the inlet velocity triangle and find

i)runner vane angle ii) velocity of wheel at inlet .iii)absolute velocity of water leaving the guide vanes iv)the relative velocity of water entering the runner blade.

[i.602.214 ii, 76.19iii)5023m/s iv)3.087m/s]

Solution

Given



Vwi ,(Inlet velocity Triangle)

Exit diameter or external diameter =Di=1m

Breadth at inlet = Bi=200mm=.2m, velocity of flow at inlet

=Vfi =1.5m/s , area blocked by vanes =15%,speed=N=100rpm

Guide blade angle =α=15°,tangential velocity at inlet= Ui

$U_i = \Pi D_i N/60 = \pi \times 1 \times 100/60 = 5.23 m/s$

Type equation here.

Area blocked by the blade thickness=15%=15/100 xπ x Di x Bi

= .15 xπ x1 x .2

Actual area flow takes place= .85 x π x 1 x.2=.534m2

Mass of water passing /sec=ρ x area x Vfi

=1000 x.5341.5=801kg/s

1) Runner vane angle θ

From inlet velocity angle tan α=Vfi/Vwi=1.5/Vwi =tan15°

Vwi=1.5/tan15=1.5/.26795=5.598m/s

Tan θ=Vfi/Vwi—Ui =1.5/5.598—5.23=4.07

„ θ =76° 10'

II) Velocity of wheel at inlet Ui=5.23m/s (rpm considered as100 to match the design)

III)Absolute velocity Vi of water leaving the guide vanes =Vfi ÷ sinα =1.5÷sin15°= 1.5÷.26795 =5.59m/s

IV) The relative velocity of water entering the runner blade (V r i)

Sin θ =V f i /V r i

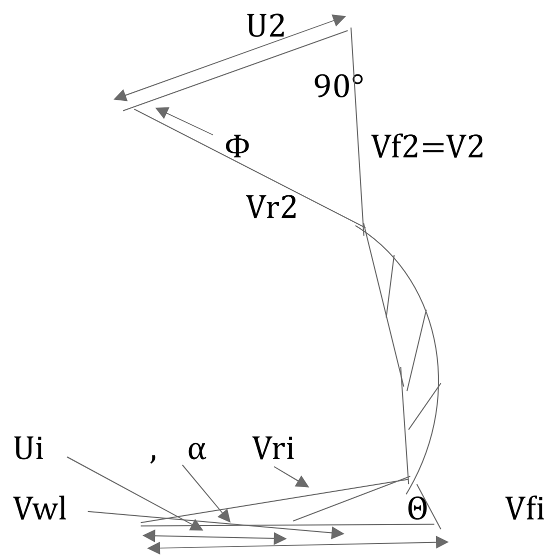
V r i =V fi/sin θ=1.5/sin 76° 10'=1.5/.97=1.546 m/s Ans

Q10

An outward flow reaction turbine has internal and external diameter of runner as 0.5m and 1.0m respectively. The guide blade angle 15° and velocity of flow through the runner is constant and equal to3m/s. If the speed of turbine is 250 rpm , head of turbine is 10 m and discharge at outlet is radial , determine i) runner vane angle at inlet and outlet ii) work done by water on the runner /s /unit weight of water striking /s iii) hydraulic efficiency, [Ans i)32° 48' 12° 55' ii)7.47m iii)7.47%]

Solution

Runner Vane angle diagram.



internal diameter D_i=.5m, external diameter D_2=1.0m

,,α =guide blade angle=15°,velocity of flow =V_{fi}=V_{f2}=3m/s

Speed N=250rpm H=head=10m discharge at outlet=radial.

$\therefore V_{w2}=0 . V_{f2}=V_2$.

Tangential velocity at inlet and outlet are

$U_i=\Pi DN/60 =\pi .5 \times 250/60 =6.04$m/s

$U2 = \pi$ X1 X250/60 =13.08 m/s

From the inlet velocity triangle tan α=V fi/V w i, , V w i=V f i/tanα

V w I =V f i/tan15° =3/.26795=11.196

I Runner vane angle and outlet velocity angles θ and φ

Tan θ= V f i/V w i—Ui =3/11.196---6.04 =.582

„ $\theta = tan^{-1}.582$ =30° 10'=runner vane angle

From outlet velocity angle

Tan φ =Vf2/U2 =3/13.08=.22935

„ φ =12° 55' runner outlet vane angle

II) work done by water /s/unit weight/s=1/g x V w ix V i

=1/9.81 x 11.196 x6.04=6.893Nm/

III) Hydraulic efficiency=V w i x Ui/g H= 11.196X 6.04/9.81 X 10

= .689 =68.9%

IV) Velocity of flow is constant through runner (V f i—V f 2) and discharge is radial at outlet(β=90° V w 2= 0), the degree of Reach{R) is (18.20)

$$R=1 -- \frac{COT\alpha}{2(COT\alpha - COT\theta} = 1--3.732/2(3.7321—1.67530)$$

=1—3.721/2X2.05 =1 ---.907

=.093=9.3%

Q 11

A Fransis Turbine with an overall efficiency 70% is required to produce 147.15 kw. It is working under a head of 8m .The peripheral velocity =0.30 $\sqrt{2gh}$ and the radial velocity of flow at inlet is $0.96\sqrt{2gh}$.The wheel runs at 200 rpm and the hydraulic losses in the turbine are 20% of the available energy. Assume radial discharge determine i)guide blade angle ii) wheel vane angle at inlet iii) diameter of wheel at inlet iv) width of the wheel at inlet .[35°45',ii)42° 54' iii)35.9 cm iv)9.75cm]

Solution

Given overall efficiency =70%

Shaft power produced =147.15 kw, H=Head =8m

Peripheral velocity = Ui=.3√(2gH)=.3√(2x 9.81 x8) =3.684m/s

Velocity at inlet =Vfi=.96√(2gH) =.96√(2x 9.81x8)=11.788m/s

Speed =N=200rpm,hydraulic losses =20%of available energy

Discharge at outlet =radial. Vw2=V2.

Hydraulic efficiency = (total head at inlet---hydraulic loss) /head at inlet

=(H--.2H)/H=.8H/H= 80%

However , efficiency =(Vwi x Ui)/ gH= .80

Vwi=.8 xgH/Ui=(.8 x9.81 x8)/3.684

=62.78/3.684 =17.04 m/s

1 Guide blade angles

„α=inlet velocity angle, tanα =Vfi/Vwi=11.788/17.04

=.6917

∴ α =guide blade angle =34° 40'

b) wheel vane angle =θ

tan θ =Vfi/(Vwi—Ui)=11.788/(17.04—3.684)

= '88289

$\therefore \theta = 41°20' =$ wheel vane angle

2. diameter of wheel at inlet (Di)

$U_i = \pi D_i N/60$

$D_i = U_i x\, 60/\pi \times N = 3.684 \times 60/\pi \times 200$

$= 221.04/628.318 = .3518 = D_i$

3. width of the wheel at inlet (Bi)

Efficiency = S.P/W.P

W.P = W.H/1000 = (ρ g Q) H/1000

= 1000 x 9.81 x Q x 8/1000

=9.81 x Q x 8

Efficiency =S.P/WP=147.15/W P=.70

W.P=147.15/.70

Putting value of W.P

9.81 x 8 x Q= 147.15 /.70

Q =147.15/9.81 x 8 .70 =2.6785 m3/s

Using equation (18.21) to find Bi=width of the wheel

$Q = \pi D_i B_i V_{fi}$

$B_i = Q/\pi \times D_i \times V_{fi} = 2.6785/\pi \times .3518m \times 11.788$

=2.6785/13.02 =.205 m=20.5 cm Ans.

Q12

The following data is given for Francis turbine ;net head=70m

Speed =600rpm,S.P=367.875KW, overall Efficiency =85%,hydraulic efficiency =95% ,flow ratio =0.25 . breadth ratio =0.1 , outer diameter of runner=2 x inner diameter of runner.. The thickness of the vane occupy 1o% of circumference area of runner. Velocity of flow is constant at inlet and outlet and discharge is radial at outlet,

Deter mine i)guide blade angle ii) runner vane angles at inlet and outlet iii)diameter of runner at inlet and out let. Iv)width of wheel at inlet(Bi)

[i)12° 20’ ii) 18° 57 50° 17’ ’iii)49.245m iv)49mm]

Solution

Given flow ratio $=Vfi/\sqrt{(2gh)}=.25$

$=.25 \times \sqrt{(2 \times 9.81 \times 70)}=9.265m/s$

Breadth ratio $=Bi/Di=.1$

$Bi=.1 \times Di$

Outer diameter Di=2 x inner diameter =2x D2

Velocity of flow = Vfi= V f 2=9.265m/s

Thickness of the vanes =10% of circumferential area of runner.

Actual area of flow $=90\% \times (\pi Di\ Bi)=.9\ \pi\ Di\ Bi$

Discharge outlet =radial

$\therefore$ Vw2=0, Vf2=V2

Using relation overall efficiency =S.P/W.P

Or $.85=367.875/W.P$

W.P =432.79 KW’

But $w.p =w.H /1000=1000 \times 9.81 \times Q \times 70/1000$

$Q = W P / 9.81 X70=432.79/9.81 x 70$

$=.63\ m3/s$

Using equation (18.21)

$Q= .9 x \pi Di Bi Vf2$

$.63 =.9 x\pi x Di x .1 x Di\ \ x\ \ 9.265$

$.63 =2.61Di^2$

$Di=.49m$ so $D2 =Di/2=.245m$

Width of wheel $BI=.1 Di= .1 x .45 = .049m =49mm$

I. Guide blade angle (α)

$Tan\ \alpha =V f i/V w i=9.265/42.38=.2186$

„ $\alpha =12° 20'$

II. Runner vane angles (θ and φ)

$Tan\ \theta=V f i/Vwi—Ui=9.265/(42.38—15.39)$

$=.3432$, so $\theta =18 °50'$

$Tan\ \varphi =Vf i/U2=9.265/ U2$

Also $U2 =\Pi D2N/60=\pi x .245 x 600/60 =7.69m/s$

$Tan\ \varphi =9.265/7.69= 1.2$

$\therefore\ \varphi =50° 20'$ Ans.

III. Diameter of runners at inlet Di and outlet D2

$Di =.49m\ \ D2= .245m$

IV. Width of wheel = 49mm

Q13

A Kaplan turbine working under a head of 15 m develops 7357.5 kw shaft power. The outer diameter of runner is 4m and hub diameter is 2 m, The hydraulic and overall efficiency of turbine are 90% and 85% . The guide blade angle at extreme edge of the runner 30°, which is 0 at outlet.

Determine i)runner vane angles at inner and outer extreme edge of runner ii)speed of turbine [103° 10', 26° 58' 5''ii 58.5]

Solution

Given head =15m, S.P=7357.6 KW

Outer diameter of runner =Do=4m

Hub diameter =Db=2m, guide blade angle =α =30°

H y d, efficiency =90% overall efficiency =85%

Velocity of whirl at outlet=0,

Efficiency overall = S.P/W.P, .85= 7357.5/W.P

W.P=7357.5/.85=8655.88 KW. =ρ g Q H/1000

OR 1000 x 9.81 x Q x15/1000 =8655.88

Or , Q=8655.88/ 9.81 X 15 =58.82 m3/s

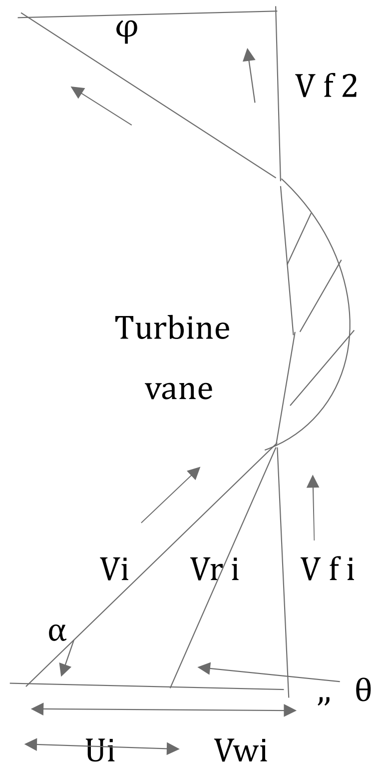
Since hub diameter Db = 2 m brings Ui more than Vwi and hence in tan θ ,calculation (Vwi –Ui) comes negative and hence Db is considered 2.5m to make (Vwi—Ui) positive.

Using eq(18.25) Q=π/4 $(Do^2 - Db^2)xVfi$

$$58.82 = \pi/4(4^2 - 2.5^2)Vfi$$

Vf i =7.68 m/s

U2 outlet



Inlet angles

I. $\tan 30° = V_{fi}/V_{wi} = .57736$

$V_{wi} = 7.68/,5773 = 13.30 m/s$

Now hydraulic efficiency=.9 =Vwi x Ui/g H=13,.3 XUi/9.81 x15

$U_i = .9 \times 9.81 \times 15/13.3 = 9.95$ m/s

$\tan\theta = V_{fi}/V_{wi} — U_i = 7.68/13.30 --- 9.95 = 7.68/3.35$

$= 2.29 \quad \therefore \quad \theta = 66\ 20'$

For Kaplan turbine $U_i = U_2 = 9.95$ m/s and $V_{fi} = V_{f2} = 7.68$ m/s

$\tan\varphi = V_{f2}/U_2 = 7.68/9.95 = .771$

„$\varphi = 37°\ 40'$

II Speed of turbine

Ui=U2=π Do N/60 = 9.95

N =9.95 x 60/4π =47.53 =48 rpm

Q 14

A Kaplan turbine runner is to be designed to develop 7357.5 kw shaft power (s.p). The net available head is 10 m. Assume that the speed ratio is 1.8 and flow ratio is 0.6. If the overall efficiency is 70% and diameter of boss is ,4 times more than diameter of runner, find the diameter of runner, its speed and specific speed.

[4.39m,109.63 rpm,528.82]

Solution

Given S.P=7357.5 KW, Head =10m. speed ratio =1.8

Flow ratio =.6, overall efficiency =70%= 0.7

Diameter of boss =.4 x diameter of runner, Db=.4 Do

Speed ratio =Ui/√(2 g H)=1.8

Ui=1.8 √2 x9.81 x 10) =1.8 x 14=25.2 m/s

Flow ratio =Vfi/√(2g H)=.6

Vfi=.6 x √ (2x 9.81 x10)=,6 x 14=8.4 m/s

Overall efficiency =shaft power/ρ g Q H/1000

Q =S.P x 1000/1000 x9.81 x 10 x .70

=7357.5/68.67=107.14 m3/s

Q through Kaplan turbine=π/4 $[Do^2 - Db^2]Vfi$

107.14=π/4$[\ D_O^2 - .4D_O^2]8.4$

$=\pi/4D_O^2[1 - .16].8.4$

$=D_O^2$x 22.167/4

1. D_O^2 =107.14/5,54 =19.339

D = 4.39 m=diameter of runner.

11. speed of turbine =60x Ui/π x Do=60 x 25.2/π x 4.39

=1512/13.79 =109.64 rpm.

111. specific of turbine=$\frac{N\sqrt{S.P}}{H^{\frac{5}{4}}}$ =$\frac{109.64}{10^{1.25}}$x $\sqrt{7357.5}$

=9403.8/17.78

= 528.8 rpm

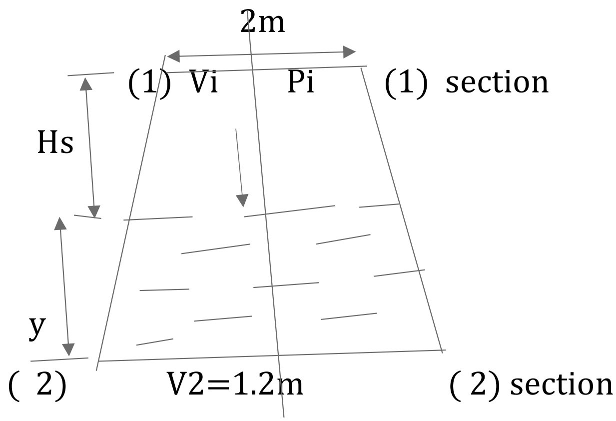
Q 15

A conical draft tube having inlet and outlet diameter .8m and 1.2 m discharges water at outlet with a velocity of 3m/s. The total length of the draft tube is 8m and 2 m of length of draft tube is immersed in water. If the atmospheric pressure head is 10,3m of water and loss of head due to friction in the draft tube is equal to o.25 times the velocity head at outlet of the tube;

Find 1pressure head at inlet.

2. efficiency of draft tube

[1.2.551m 75.3%]



Conical draft tube

Diameter of inlet =Di=.8m

Diameter of outlet =D2=1.2m

Velocity at outlet =V2=3m/s

Total length of tube=Hs+y=8m , y = height in water=2m

Hs= 8—2 =6 m

Atmospheric pressure = Pa/ρ g =10.3 mπ

Loss of head due to friction hf=.25x velocity head

$$=.25 \text{ x } \frac{V2^2}{2g} = (.25 \text{ x } \frac{3^2}{2\ x9.81}) \quad =2.25/19.62$$

Discharge through tube=A2 V2=π/4(1.2 X1.2) X3

= 3.3696m2

Velocity at inlet Vi=Q/Ai=3.3696/π/4 x D_1^2

= 3.3696/π/4 x(,8x.8)

=3.3696/.5026

=6.7m/s

1. pressure head at inlet

Type equation here.

Pi/ρ g=Pa/ρ g--- Hs---$(\frac{V_I^2}{2\,g}$ ---$\frac{V_2^2}{2g} - hf)$

$=10.3 - 6 - [(\frac{6.7^2}{2x9.81}$--$\frac{3^2}{2x9.81} - (9x.25)/19.6]$

$=4.3 - [\frac{44.89--9--2.25}{19.62}]$

=4.3 –1714

=2.586 m(abs)

11 , efficiency of draft tube(eq.18.29)

$=[(\frac{V_1^2}{2g} - \frac{V^2}{2g})$ –hf]/$\frac{V_1^2}{2g}=[(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - .25\frac{V_2^2}{2g})/\frac{V_1^2}{2g}$

$=(V_1^2 - V_2^2 -.25V_2^2)/V_1^2=(1—1.25X\frac{V_2^2}{V_1^2})$

=[1 --- 1.25 X$(3/6.7)^2$] =1 ---1.25 x.2

=1--.25 =.75 =75%

Q16

A is to operate under a head of 30m at 300rpm. The discharge is 10m3/s . if the efficiency is 90% determine 1specific speed of the machine 11power generated 111. Type of turbine.

[1 -219.9 11- 2648.7 kw 111-Fransis]

Solution

Given head =H=30m, speed =N=300rpm

Discharge =Q =10m3/s , efficiency overall =90%

Now using relations

Overall efficiency = power developed/water power

.9 =P/ρ g Q H=P/1000 x 9.81 x10 x30

1. P= .9 x10 x30 x9.81 x1000 w

= 9 x 30 x9.81x1000/1000 kw

=2648.7 kw = power generated

11. specific speed (18.28) =Ns=N$\frac{\sqrt{P}}{H^{5/4}}$

$= 300\frac{\sqrt{2648.7}}{30^{5/4}}$

=300 X 51.46/70.21

=15438/70.21

=219.88 rpm

111. type of turbine

Since the above specific speed 219.88 is between 51 and 255 ,the turbine is considered to be in the category of Francis turbine.

Q 17

A turbine develops 7357.5 kw S.P. when running at 200rpm. The head in the turbine is 40 m . If the head on the turbine is reduced to 25 m , determine speed and power developed by the turbine,{ 158.11 rpm , 3635.34 kw]

Solution

Given

Power delivered =Pi=7357.5 kw

Speed N=200rpm, head Hi=40m

If head reduced =H2 =25m speed N z(two)=?

Power = P z(two) ?

Using eq.(18.32)

$\frac{Ni}{\sqrt{Hi}}=\frac{Nz}{\sqrt{Hz}}$ $\therefore$ N 2 $=\frac{Ni\sqrt{H_2}}{\sqrt{Hi}}=\frac{200\sqrt{25}}{\sqrt{40}}$ =1000/6.324

= 158.12 rpm

Also we have

Pi/(Hi)$^{3/2}$=P2/(H2)$^{3/2}$

P2=Pi (H2/Hi)$^{3/2}$

=7357.5(25/40)$^{1.5}$

=7357.5(.625)$^{1.5}$

=7357.5 x .494

=3634.6k.w =power developed at 25 m head.

Q 18

A Pelton wheel is revolving at a speed of 200rpm and developes5886 kw S.P. when working under a head of 200m with an overall efficiency of 80%. Determine unit speed unit discharge and unit power The speed ratio of turbine is given as 0.48 Find speed , discharge and power this turbine is working under a head of 150 m.[14.14,0.265m3/s, 2.08kw,173.2rpm,3.247m3/s,3823 kw]

Solution

Speed =Ni=200rpm, power Pi=5886kw, Head Hi=200m

H2=150m ,overall efficiency =80%=.8, speed ratio=.48

New head of water =H2=150m.

1. to find Nu , Pu , Qu

a) unit speed = Nu=Ni/√H =200/√200 =200/14.14=14.14rpm

[O. Efficiency=Pi/ρ g Q Hi /1000 or .8=1000 x 5886/1000x 9.81 x200 or Qi=1000 x 5886/1000 x 9.81 x 200 =5886/1962=3m3/s=Q]

b)unit discharge Qu=Qi/√Hi=3/√200 =3/14.14 =.212 m3/s

c)Unit power = $\frac{Pi}{Hi^{3/2}}$=5886/(200)$^{1.5}$ =2.08 kw

11. when the turbine is working under a new head H2=150m

a) speed --

Ni/√Hi=N2/√H2

N2=Ni√H2/√Hi

=200√150/√200

=200 x12.24/14.14 =2448/14.14=173.12rpm

b) discharge

Qi/√Hi =Q2/√H2

Q2=$\frac{Qix\sqrt{150}}{\sqrt{200}}$

=(3x12.24)/14.14

=36.74/14.14

=2.598m3/s

c) power

Pi/$Hi^{3/2}$ =P2/(H2)$^{3/2}$

P2 =$\frac{PiH_2^{3/2}}{H_1^{3/2}}$ =(5886 x 1837)/2828.4

=10812582/2828.4

=3823.4kw

Q19

A Kaplan turbine working under a head of 29m develops 1287.5 kw S.P. If the speed ratio is equal to 2.1, flow ratio is .62 , diameter of boss is 0.34 times diameter of runner and overall efficiency of the turbine 89%, find the diameter of runner and speed of turbine [0.705m,1162.3rpm]

Solution

Given

S . p =1287.5 kw

Head available =H =29m

Speed ratio=Ui/√(2g H)

Ui=2.1 x$\sqrt{2x9.81x29}$ =49.98 m/s

Flow ratio =.62 =Vfi/√2g H

Vf i=.62 $\sqrt{2x9.81x29}$ = 14.756 m/s

Db= .34 Do

Overall efficiency =89% =’89 =S.P/ Water power

= 1287.5/ρ g Q H/1000

=1287.5 x 1000/1000 x9.81 x Q x29

Q= 1287.5/9.81 x .89 x29

=1287.5/253.9 =5.085 m3/s

We know (eq 18.25) Q =$\frac{\pi}{4}(D_O^2 - D_b^2)$ xVfi

=$\frac{\pi}{4}(D_O^2 -.34D_o^2)$14.756

5.085 =π/4 x.8844 (Dox Do) X14.756

Do x Do =$\frac{5.085x\ 4}{.8844\ x1\ \ .756x\pi}$ =20.34/41=.496

Do=√(.496 = .7 m and Db= .34Do

=.34 x.7=.238m

1. Runner diameter =.7m
2. Ui=inlet velocity =Ndoπ/60

N=Ui x60/π x Do=49.98 x60/π x .7

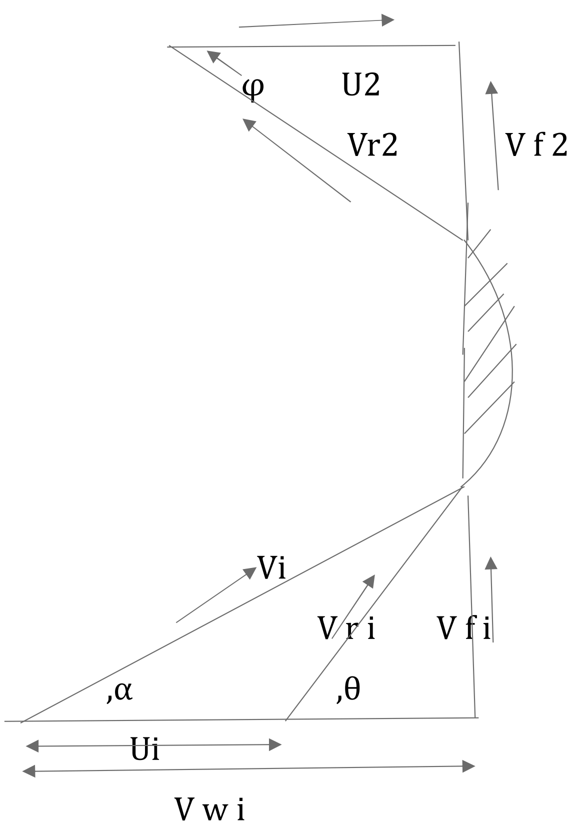
=1355.4 rpm = speed of turbine.

Q 20

A Kaplan turbine working under a head of 25m** develops 16000 kw shaft power. The outer diameter of runner is 4 m and hub diameter is 2m . The guide blade angle is 35°. The hydraulic and overall efficiency are 90 and 85° respectively. If the velocity of whirl is 0 at outlet determine runner vane angles at inlet and outlet and speed of turbine. [Ans 35° 27' ,88° 36' 9.236]

Solution

Given



**Head =H =20m, S.P=16000 KW. Outer diameter =Do=4m

Hub dia. =Db=2m ,guide blade angle =α =35°, Hyd. Eff. = ,9

Overall eff . =.85 , Velocity of whirl =0

w.p/1000 =ρ g Q H/1000 , Overall eff. =SP/WP

∗∗ note ; if we keep head 25 as given Ui is coming more V f i and in that case calculation further not possible as value of (V f i---Ui) becomes negative but if we take head as 20 then it is alright.

.85 =16000/1000 x 9.81 x Q x 20 ∗∗

Q =.16000x 1000/ 1000 x 9.81 x .85 x 20

=95.94 m3/s

Q =π/4 $(D_O^2 - D_b^2) x\, Vfi$ =95.94 (Do=outer dia. Db=boss d I a)

V f i=95.94/9.4247 =10.18 m/s

From inlet velocity triangle

Tan α =V f i/V w i ∴ V w i=V f i/tan 35° =10.18/.7=14.54 m/s

Using relation of hydraulic efficiency

.9 = V w I x Ui/g H =14.54 x Ui/9.81 x20

Ui=.9 x 9.81 x20/14.54 =176.58/14.54=12.14m/s

1. runner vane angles at inlet and outlet

Tan θ =V f i/V w i----U i=10.18/14.54---12.14

=10.18/2.4=4.24

,, θ =77° 15' inlet angle.

For Kaplan turbine

$$Ui = U2 = 12.14 \text{ m/s}$$

$$V\,f\,i = V\,f\,2 = 10.18 \text{ m/s}$$

$$\text{Tan } \varphi = V\,f\,i/U2 = 10.18/12.14 = .83855$$

$$,,\varphi = 39°5' \text{ outlet angle.}$$

11, speed of turbine is given by

$$Ui = U2 = \pi\, Do\, N/60 = \pi \text{ x } 4 \text{ x } N/60 = 12.14$$

$$N = 12.14 \text{ x } 60/\pi \text{ x } 4 = 728.4/12.566 = 57.96 \text{ rpm}$$

Q 21

This problem is solved in the original text book itself.

END OF CHAPTER 18

Fluid Mechanics and Hydraulic Machines

Centrifugal Pumps

Chapter 19

Q 1

The internal and external diameter of the impeller of a centrifugal pumps are 300mm and 600mm respectively The pump is running at 1000 rpm. The vane angles at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant Determine the work done by impeller per unit weight of water [Ans68.69Nm/N]

Solution

Given

Internal diameter of impeller $D_i = 300mm = .3m$

External diameter of impeller $D_2 = 600\ mm = .6m$

Speed = $N = 1000rpm$

Vane angle at inlet $= \theta = 20°$, vane angle at outlet $= \varphi = 30°$

Waters enters radially $\alpha = 90°$, $V_{wI} = 0$

Velocity of flow $= V_{fi} = V_{f2}$

Tangential velocity of impeller at inlet at inlet and outlet.

$U_i = \pi D_i N/60 = \pi \times .3 \times 1000/60 = 15.70 m/s$

$U_2 = \pi D_2 N/60 = \pi \times .6 \times 1000/60 = 31.416\ m/s$

From the inlet velocity triangle

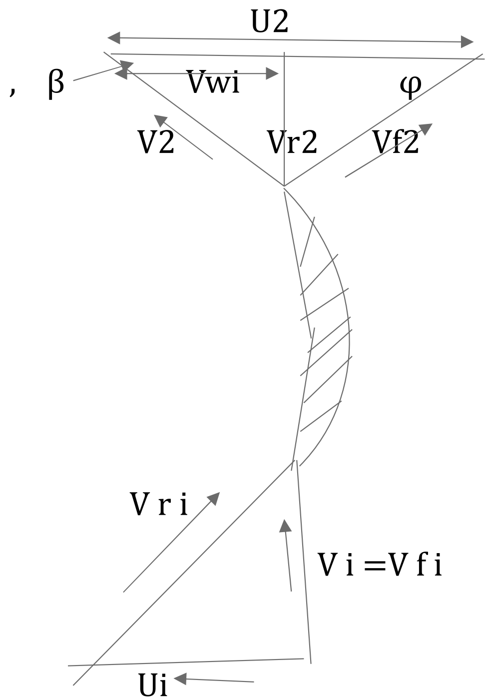
$\tan\theta = V_{fi}/U_i$, $\tan 20 = V_{fi}/15.70$

$.36397 = V_{fi}/15.70$

$V_{fI} = .36397 \times 15.70 = 5.71 m/s$

Type equation here.

V f i =V f 2= 5.71m/s



Impeller of a centrifugal pump.

From outlet velocity of triangle

Tan φ = V f i/U2 –Vw2 =5.71/31.416—Vw2

Tan 30 =5.71/31.416—Vw2

31.416 ---V w2=5.71 /.57735=9.9

V w 2 =31.416---9.9 =21.516

So work done by impeller/kg of water /s (eq ;19.1)

=1/g(Vw2 x U2)

=1/9.81 (21.516 x 31.416

Type equation here.

$=675/9.81$

$=68.9 Nm/N$

Q 2

A centrifugal pump having outer diameter equal to 2 times the inner diameter and running at1200rpm works against a total head of 75 m. The velocity of flow through impeller is constant and equal to .3 m /s The vanes are set back at an angle of 30° at outlet. If the outer diameter of the impeller is 600mm and width at outlet is 50mm determine a) vane angle at inlet b) work done per second by the impeller c) Manometer efficiency

[a) 9° 2' b)346.37 kw c) 60°]

Solution

Given

Speed =N =1200 rpm. Hm =75 m .Vf i=V f i=3m/s

Outer diameter D_2=2Di .= 600 mm , φ = outlet angle =30°

Di=300mm.=.3m , B2=Width at outlet =50mm=.05 m

Tangential velocity = Ui= πDi Xn/60 =π x .3 x 1 200/60

=18.84m/s

U2= 37.7m/s

Discharge =Q =π x D2 x B2 X V f 2 =π x.6 x.05 x3=.2827 m3/s

1. Vane angles

Tan θ=Vfi/Ui=3/18.84 =.15923

„ θ =9°2' =inlet angle

2 work done by impeller (19.2)

=w/g x Vw2 Xu2= ρ x g x Q x Vw2 x U2

= 1000x .2827 xVw2 x37.7=

Hm= manometric head (head against which pump has to act.

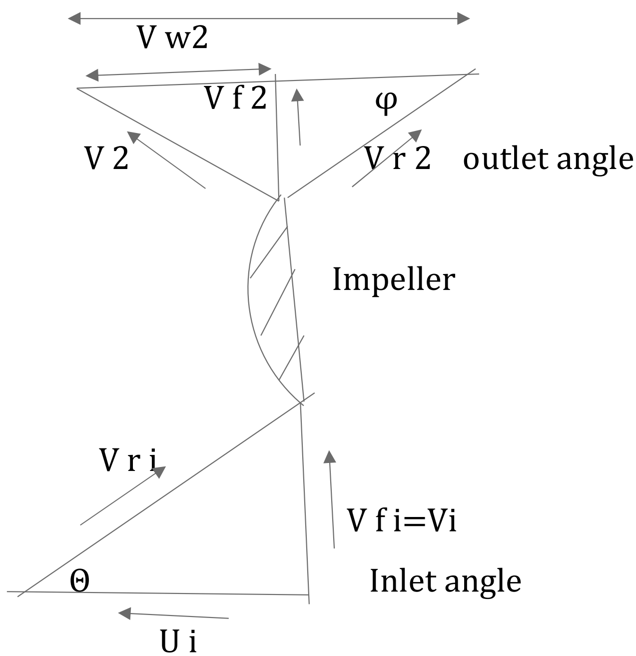
Tan φ =Vf2/U2—Vw2=3/37.7—Vw2

Vw2= 37.7---3/tan φ=37.7 –3/.57735

= 37.7 –5.196=32.5 m/s

∴ work done by impeller =1000 x.2827x 37.7 x 32.5

U2 =346378.1w=346.37 kw.



3. manometric efficiency (19.8)=g x Hm/Vw2 x U2

=9.81 x 75/32.5 x37.7

=735.75/1225.25

=.6

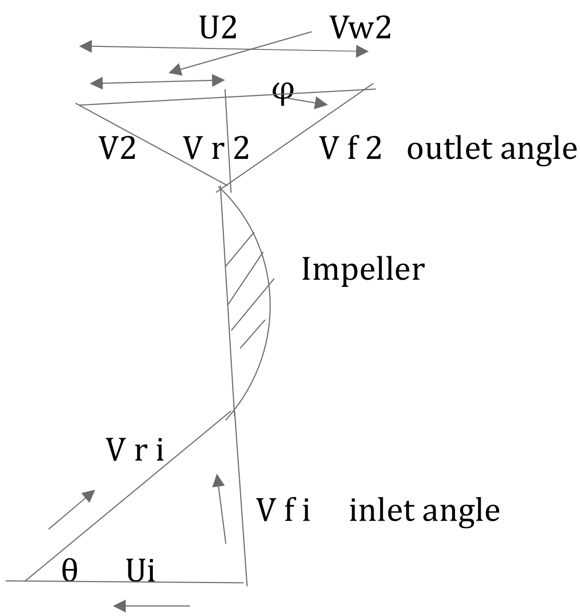
=60%

Q 3

A centrifugal pump is running at 1000 rpm. The outer vane angle of impeller is 30° and velocity of flow at outlet is 3m/s.

The pump is working against a total head od 30 m and discharge through the pump is .3m3/s .If the manometer efficiency of pump is 75%, determine i) diameter of impeller ii) width of impeller

[I 43.1 cm ii 7.4 cm]



given

speed N=1000 rpm, angle φ =30°

velocity of flow =Vf2 =3m/s, , Q=DISCHARGE =.3m3/s

Type equation here.

manometric head =Hm =30m, mano. Effi .=75%=.75

from outlet velocity triangle tan φ =$\frac{V_{f_2}}{U_2 - V_{w_2}}$ =tan 30°

3/.57735 =5.19 =Vw2 ---U2 ------- I)

Using eq.(19.8)

Manometric efficiency =$\frac{gHm}{Vw2xU}$

Vw2 x U2 =$\frac{9.81X3}{.75}$ =392 .4 ----(-II)

Combining i) and ii) above

(U2—5.19) U2 =392.4

U_2^2 ---5,19U_2 ---392.4 =0

$U_2 = \frac{-5.1 \pm \sqrt{5.1^2 + 439\ \ .4c}}{2}$ =45/2 . (--34.8)

=22.5 m/s

1. diameter of impeller

U2 =π D2 N/60 =π x D2 X 1000/60 =22.5

D2 =22.5 x 60/π x1000 =135/314.16

= .437 m=43.7cm= diameter of impeller.

2. width of impeller =B2

Q -=π x D2 x B2 x Vf2 = .3 m3/s

So B2 = $\frac{.3}{.437\ X \pi X}$ =.3/4.11 =.07299 m

=7.39 cm =width of impeller.

Type equation here.

Q 4

Find the power required to drive a centrifugal pump which delivers 0.02 m3/s of water to a height of 30 m through a10 cm diameter pipe and 90 m long. The overall efficiency of pump is 70% and coefficient f=.009 in the formulae

Hf= 4f lV^2/d x2g [Ans 11.5kw]

Solution

Given

Discharge Q=.02m3/s

Height =Hs=hs +hd=30 m.

dia, of pipe =Ds=Dd=10cm =.1m

Length = L =Ls +Ld=90m

,,hs=suction pipe hd =deliver pipe

Overall efficiency =70%, coefficient of friction=f=.009.

Velocity of water = Vs =Vd=V=Q/A=.02/π/4 x.01 x.01

=2.54 m/s

Friction and head loss in pipe ;---

.hfc +, hfd=4flV^2/dx 2g=4 x,009x 90 x2.54 x2.54/.1x2x9.81

=20.9/1.962 =10.654m

Manometric head ;----(19.7)

Hm=,(hs+ hd)+(hfs+hfd)+$\frac{V_d^2}{2g}$ =30 +10.654+2.54x2.54 /2x9.81

$$=40.654 + .3288 = 40.982\text{m}$$

Overall efficiency (19.10) $=\frac{\rho g Q Hm}{S.PX1000}$

S.P $=1000 \text{ X}9.81 \text{ X}.02 \text{ X}40.982/1000 \text{ X}.7$

$=8.04/.7 = 11.48$ KW.

Q 5

Find the rise in pressure in the impeller of a centrifugal pump through which water is flowing at the rate of 15 lit/s. The internal and external diameter of the impeller are 20 cm and 40 cm. The width of the impeller at inlet and outlet are 1.6cm and 0.8cm.

The pump is running at 1200 rpm The water enters the impeller radially at inlet and impeller vane angle at outlet is 30°(neglect losses through the impeller. [Ans 31.85 m]

Solution

Given

Q=15LT/S=.015 m3/s, Di=.2m.D2=.4m

Width =Bi =.016m ,B2=.008m, N=1200rpm, outlet $\varphi=30°$

Vfi=Q/A=.015/πx .2x .016 =1,5 m/s

Vf2 =Q/A =.015/π x.4 x .008 =1.5m/s

Tangential velocity of impeller at outlet

= U2=π D2 N/60 =π x.4 x1200/60=8π=25.13m/s

Using equation (19.12)

Pressure rise =1/2g$(V_{fi}^2 + V_{f2}^2 - U_2^2 cosec^2\varphi)$

=1/2 x 9.81$[1.5^2 + 25.13^2 - 1.5^2 cosec30^2]$

=1/19.62 x [2.25 +631.51 –2.25 x 4]

[cosec)^{2}30 =1 +cotθ^2 =1+1/tan^2θ =1 +1/.333=1+3=4]

Pressure rise =1/19.62 (2.25 +6315 –9.0) =624.5/19.62

=31.83 m

Q 6

The diameter of impeller of a centrifugal pump at inlet and outlet are 20 cm and 40 cm Determine minimum speed for starting the pump if it works against a head of 25m[1221.2rpm]

Solution

Given

Inlet dia. Di=.2 m. outlet dia.=.4m

Head Hm=25m, minimum speed = N(say)

Eq. 19.13;

Hm $=\frac{U_2^2}{2g} - \frac{U_1^2}{2g} = Hm$ -------1.

Now Ui=πDiN/60

Ui=π x.2 N/60 , U2 =π x .4 N/60

Ui =.01047N, „ U2= .02094 N

Now we put Ui and U2 above values in(1) we get;

$$\frac{.02094^2 N^2}{2x9.81} - \frac{.01047^2 N^2}{2X\ 9.81} = Hm = 25$$

$$N^2[\frac{.02094^2}{19.62} - \frac{.01047^2}{19.62} = 25$$

N^2(.00002234--.00000558) = 25

N^2 =1491646.77

N=$\sqrt{1491646.77}$ =1221.33 rpm=minimum speed

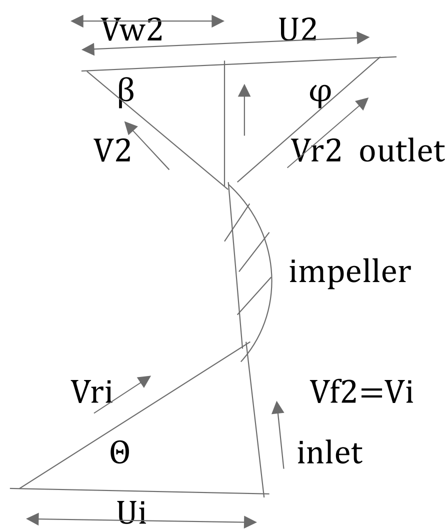
„for starting the pump.

Q7

The diameter of an impeller of a centrifugal pump at inlet and outlet are300mm and 600mm respectively. The velocity of flow at outlet is 2.5m/s and vanes are set back at an angle of 45° at outlet. Determine the minimum starting speed of the pump if manometric efficiency is 75% [159.31 rpm]

Solution

Given



U2—Vw2=Vr2/tan φ =2.5/tan 45°=2.5

So Vw2=U2---2.5

Again U2=$\frac{\pi\ D2\ N}{60}$ =π x.6 x N/60

SO Vw2 =π .6 N/60 – 2.5= (.03141 N—2.5)

Using (19.15) equ. for minimum starting speed

$$N=\frac{120xeff.xVw2\,xD2}{\pi(D_2^2-D_{1)}^2}=\frac{120X\,.75X.6\,X(.03141N-2.5)}{\pi(.6^2-.3^2)}$$

=54(.03141N—2.5)/π x .27

N=63.66(.03141 N—2.5)

1.999N---N=159.15

.999N=159.15

N=159.15/.999= 159.3 rpm. Minimum staring speed.

Q 8

A three stag centrifugal pump has impeller has 40 cm diameter and 2.5 cm wide at outlet The vanes are curved back at the outlet at 30° reduce circumferential area by 15% The manometric efficiency is 85% and overall efficiency is 75%. Determine the head generated by the pump when running at 1200rpm and discharge is .06 m3/s Find the S P. [138.75m,108.81]

Solution

Given

Q =Discharge =..06m3/s ,manometer efficiency =85%

Over all efficiency = 75%,number stages =3

D2 =.4m B2 = .025m, outlet angle =φ =30°

Reduction area = 15% =.15

Area of flow at outlet =100 –15=85%

= .85 x π Xd2 x B2= .85 X π X .4 X .025=.0267m2

Speed =1200rpm, to find 1. Head generated by pump ?

2.S.P?

1. Vf2 =Q/A=.06/.0267 =2.247 m/s

U2 =π D2 N/60 =π X .4 X1200/60 =25.13 m/s

Tan φ =Vf i/U2 –Vw 2 =2.247/25.13 --- Vw2 = tan30

. 57735 =2.247/25.3 –Vw2

Vw2 = 25.3 ---3.89 =21.41 m/s

1 Using eq 19.8

Manometric effi.= g Hm/Vw2 x U2=

. Hm= man eff. X Vw2 Xu2/g= .85 x 21.41 x 25.13/9.81

= 457.11/9.81 =46.59 m =head generated

By pump.in one stage.

Head generated I 3 stage=3 x 46.59 =139.77m

2. power out put;--

$$= \frac{weigh\ \ of\ water\ lifted\ x\ total\ head}{1000} = \frac{\rho\ g\ Q\ 139.77}{1000}$$

$$= \frac{1000\ X 9.81\ X\ .06\ X\ 139.77}{1000} = 82.268\ \text{KW}$$

Using equ. (19.10)

Overall efficiency = power output/power input to the pump

.75 =82.268/S P

So power input S .P=82.268/.75 =109 .7 KW

Type equation here.

Q 9

Find the number of pumps required to take water from a deep well under a total head of 156 m .Also the pumps are identical and running at 1000 rpm. The specific speed of each pump is given as 20 while rated capacity of each pump is 150 lit/s

[Ans =3]

Solution

Given

Total head =156 m, speed N=1000rpm

Specific speed Ns=20. Rated capacity of pump=.15m3/s

Let Hm =head develop by each pump.

Using equ.(19.18) Ns=$\frac{N\sqrt{Q}}{H^{3/4}}$ OR 20 =$\frac{1000\sqrt{.15}}{H^{3/4}}$

$$H^{3/4}=\frac{1000\sqrt{.15}}{20} = 50 \text{ X } .3873$$

H=$(19365)^{1.33}$ =51.49 m

So number of pump required = $\frac{total\ head}{head\ developed\ by\ one\ pump}$

=156/ 51.49

=3 nos.

Q10

The diameter of a centrifugal pump which is discharging 0.035m3/s of water against a total head of 25m is .05 m. The pump is running at 1200rpm. Find the head discharge, and ratio of powers of geometrically similar pump of diameter .3m when it is running at 2000 rpm, [25m .0126m3/s , 2.777]

Given

1st pump---

Qi=.035 m3/s Head =Hmi=25m, Di=.5m, speed Ni=1200rpm

2nd pump ---

$$D2 = .3m\, , N2 = 2000rpm \;\; let\ head \; = \; Hm2 \;\; discharge = Q2$$

1)

Using equ.(19.21) for discharge

$$\frac{Q_1}{D_1^3 N_1} = \frac{Q_2}{D_2^3 N_2} \quad \text{OR} \; \frac{.035}{;05^3 X1200} = \frac{Q2}{..3^3 X2000}$$

$Q2=.035 \text{ x } (.3/.5)^3 \text{x } 2000/1200=.035 \text{ x } .216 \text{ x}1.66$

=.0125 m3/s

2) equation (19.20) for head

$$(\frac{\sqrt{Hm}}{DN})_1=(\frac{\sqrt{Hm}}{DN})_2$$

$$\frac{\sqrt{25}}{.5X1200i} = \frac{\sqrt{Hm}}{.3X200}$$

$$Hm2= \left(\frac{5x3x2}{.5x120}\right)^2=(5x5\ )=25\ m$$

3) rate of power (19.22)

$$\frac{P_1}{D_1^5 N_I^3} = \frac{P_2}{D_2^5 N_2^3} \quad OR \quad \frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^5 X \left(\frac{N_1}{N_2}\right)^3 = \left(\frac{.5}{.3}\right)^5 X \left(\frac{1200}{2000}\right)^3 = 2.721 \text{ Ans.}$$

Q 11

A centrifugal pump is to discharge 0.12m3/s at a speed of 1400 rpm against a head 30m. The diameter and width of impeller at outlet are 25cm and 5 cm respectively . If the manometer efficiency is 75%, determine the vane angle at outlet.

[φ =81°]

Q=.12 m3/s speed =1400rpm. Head =30m

Diameter at outlet =D2=25cm =.25m

Width at outlet B2=5cm =.05 m

Manometer efficiency =75%, vane angle at outlet =φ =?

Tangential velocity of impeller at outlet

U2=π x D2 x N/60 =π x .25 x 1400/60 =18.326m/s

Q =π x D2 x B2 x V f 2

V f 2 = Q/ π x .25 x .05= .12/π x .25 x .05=3.07

Using eq(19.8)

Manometer efficiency =g Hm/Vw2 x U2

.75 =9.81 x 25/Vw2 x 18.326

Vw2= 9.81 x 25/.75 x18.326 =17,84 m/s

$$\text{Tan } \varphi = \frac{Vf2}{U2--Vw} = \frac{3.07}{18.321-17.84} = 3.07/.486 = 6.31$$

„φ =81°

END OF CHAPTER 19

Fluid Mechanics and Hydraulic Machines

Reciprocating pumps

Chapter 20

Q1

A single acting reciprocating pump running at a 30 rpm, delivers 0,012 m3/s of water. The diameter of piston is 25 cm and stroke

Length is 50cm Determine i) theoretical discharge of pump ii)coefficient of discharge iii) slip and % age slip of the pump.

[0.01227m3/s ii)0.978 iii).00027m3/s and 2.2%]

Solution

Given speed of pump =N=30 rpm

Actual discharge =Q=0.012m3/s

Dia of piston =D =.25 m, Area of piston $=\pi/4(.25)^2$=.04908m2

Type equation here.

Stroke length =L=50 CM =.5m

1. e q u.(20.1) Q =Theoretical discharge =A L N/60

= .04908 x .5 x 30/60 =.01227 m3/s

2. coefficient of discharge Cd =Q actual/Q theoretical

=.012 /.01227 =.978

3. slip and % age slip (20.8)

Slip = Q t h --- Q act. = .01227 ---.012 =.00027 m3/s

4, % age slip $=\frac{Q\ T\ H - Q\ AC}{Q\ T\ H} = \frac{.0122 \quad .012}{.01272-}$ =.00027/.01227=.022

=2.2%

Q 2

A double acting reciprocating pump running at 50 rpm is discharging 900 lit/m. The pump has stroke of 400mm.. The dia of piston is 250mm . The delivery and suction heads are25mm and 4mm, Find the slip of the pump and power required to drive the pump. [Ans .0027m3/s,9.3 kw]

Solution

Speed of pump =N=50rpm

Q a c t =900/60=15lt/s =15/1000 =.015m3/s

Stroke=L =400MM=.4 m, dia of piston =.25=D

AREA =π/4 D XD=π/4 X .25 X.25 =.049m2

Suction head =4m, delivery head =25m

1 slip of pump =Q t h---Q a c=2A L N/60 --.015

$=2 \text{ x } .049 \text{ x} .4 \text{ x } 50/60 \text{ -- } .015$

$=.0326 \text{ --} .015 = .0176 \text{ m3/s.}$

2. power required to drive the pump

$$P = \frac{2x\rho xg \quad L\ Nx(hs+h\ \ )}{60000} = \frac{2x100 \quad x9.81 \quad .049x.4\ x50x(4+25)}{60000}$$

$=19227.6 \text{ x } 29/60000$

$=.320 \text{ x}29 = 9.29 \text{ kw.}$

Q 5

A single acting reciprocating pump has piston diameter 15cm and stroke length 30cm The center of pump 5 m above the water level in the sump. The diameter and length of suction pipe are10 cm and 8 m respectively. The separation occur if the absolute pressure head in the cylinder during such a stroke fall below 2,5 m of water.

Calculate maximum speed at which can run without separation. Take atmospheric pr head10.3mg water. [30.45r p m]

Solution

Given

Diameter of piston=D=15cm =.15m

Area =A =π/4 (.15X .15)=.01767m2, L=Stroke length=.3m

Crank radius = r=L/2 =.3/2=.15M

Suction head = hs=5m, dia. Of suction pipe =10cm=.1m

Area of suction pipe =π/4 (.1 x ,1)=,00785 m2

Length of suction pipe =8m, suction pressure head=hsep=2.5m

Atmospheric pr. Head=10.3 m, Students are requested to ref (fig no 20.5)if interested.

From the indicator diagram (20.5),it is clear that the absolute pressure head during suction stroke is minimum at the beginning of the stroke. Thus the separation can take place at the beginning of the stroke only, In that case pressure head in the cylinder at the beginning of stroke becomes = hsep.

But pr. head in the cylinder at the beginning of suction stroke

=(hs +has)m below atmospheric pr head.

=Atmos .pr head --(hs+has)m (abs)

„hsep=10.3 ---(5 +has)(abs)

2.5 =10.3 –5 ---has.

„ has =10.3 –5 --2.5 =2.8 m ----------1.

But as per (20.14) at the beginning of suction stroke is given by relation

„has=ls/g x A/as x$\omega^2 r$,r (since θ=0, cos θ =1) ---2

Equating 1 and 2

2.8 =Ls/g x A/as x $\omega^2 r$=(8/9,81 x.1767/.00785 x ω x ω x.15)

$\omega^2 = \frac{2.8x9.81x.00785}{8x.01767x.15}$ =.21562/.02120 =1017

„ω =3.189 radians =2π N/60

N =3.189x 60/2π =30.45 rpm.

Type equation here.

Q6

A single acting reciprocating pump has a plunger of 100 mm diameter and stroke length 200mm.The center of the group is 3m above the water level in the sump and 20 m below the water level in a tank to which water is delivered by the pump The dia. And length of suction pipe are50mm and 5m while of the delivery pipe are 40mm and 30 m respectively. Determine the max speed at which pump may run without separation if separation occurs at 7.3575 N/cm2below 6the atmospheric pressure take atmospheric pressure head =10.3 m of water. [36.22rpm]

Solution

Given

Diameter of plunger D=100mm=0.1m

Stroke length =L=200mm=0.2m, crank radius =r=L/2 =.1m

Suction head =hs=3m. delivery head =hd=20m

Dia of suction pipe =ds=50mm=.05m

Length of suction pipe =ls=5m.

Dia of delivery pipe =dd=40mm=.04m

Length of delivery pipe =ld=30m

Separation pressure =Psep =7.3575 N/cm2=7.3575 $x10^4$
Separation pressure head =Psep/ρ g

$=7.3575x\frac{10^4}{1000x\ .81}$=7.5mbelow atmosphere.

1, speed of pump without separation during suction stroke

---- during suction stroke possibility of separation is only at the beginning of stroke. The pressure head in the cylinder at the beginning of suction stroke =(h s+ has) below atmos. Pr.

Hence h s e p =10.3 –(3 +h a s)

2.8 =10.3 –3 ---h a s

„ h a s=4.5m

But h a s at the beginning of suction stroke is given by

„ h a s =Ls/g x A/as x (ω x ω) r

4.5 =5/9.81 x π/4 (.1 x.1)/π/4 (.05 x.05) x (ω x ω) x .1

=.50 x4 x .1 x (ω x ω)

„ω =√ 22.5 =4.74 rad/s.

, ω =2π N/60

N=60x ω/2π =60 x4.74/6.283 =45.26 rpm

=max speed of pump without separation.

2.

Speed of pump without separation during delivery stroke ------ =during delivery stroke the possibility of separation is only at the end of delivery stroke.

Pressure head in the cylinder at the end of delivery ;----

=(Hat m +H d)—had m(abs) =10.3 +20

If separation to be avoided the pressure should equal to separation pr head.

„ h s e p=10.3 +20 – ha d

2.8 =30.3---had

„had =30.3 ---2.8 =27.5m

„„ had is given by relation ;---

„ had=Ld/g x A/ad x (ω x ω) r

Or, 27.5 = 30/9.81 x π/4(DXD)/π/4(d xd) x (ω xω) r

$=30/9.81 \text{ x } (.1/.04)^2 \text{x } \omega^2 x.1$

=3 x6.25x (ω X ω) X .1

„ ω = 3.66 =2π N/60

N=(3.66 X 60)/ 2 π =34.95 rpm

So max speed pump without separation during delivery stroke is 34.95rpm.

From the above I and 2 the max speed of pump without separation during suction and delivery is the minimum of the two that is minimum of 45,26rpm and 34..95 rpm.

∴ max speed =34.95 that is 35 rpm.

Q7

The diameter and stroke length of single acting reciprocating pump are 100mm and 200mm. The length of suction and delivery pipes are 10m 30m and their diameters are 50mm. if the pump is running at 30 rpm and suction and delivery heads are 3.5m and 20m find pressure heads in the cylinder i) at the beginning of suction and delivery stroke

ii) in the middle of the suction and delivery strokes iii) at the end of the suction and delivery stroke .At pr =10.3,m, co fric,=.009. [I 2.776m(abs)42.273 m(abs) ii) 6.22m,43.34 miii)not possible, 18.225]

Solution

Dia of cylinder =D=100mm=.1m

Stroke length=L=200mm=.2m ,crank radius =L/2=.1m

Length of suction pipe =ls=10m,

" ' delivery pipe =ld=30m

Dia. Of suction pipe =ds=50mm= .05 m

Dia of delivery pipe =dd=50mm=.05m

N =Speed of pump =30 rpm

Suction head =hs=3.5m, delivery head =hd =20m

Atmospheric pr head =H atm=10.3 m

Coefficient of friction =f=.009

Suction head due to acceleration has=Ls/g x A/as xω^2x r cos θ

=10/9.81 x (π/4 x .1 x.1) x(ω x ω)x .1 x cos θ

=.4 ω^2x cos θ =.4 x 3.1416 x3.1416 cos θ

H a s = 3.948 cos θ -------------1

Delivery head due to acceleration (20.15)

„had =ld/g XA/a x $\omega^2 r cos\ \theta$

Type equation here.

$= 30/9.81 \times (.1/.05)^2 (3.1416)^2 X\ .1 \cos\theta$

„had $= 12.06 \cos\theta$ --------------2

Loss of head due to friction

„h fs $= \frac{4fLs}{dsx\ 2g}$ (A /as x ω r sin θ $)^2$

$=\frac{4x.009x10}{.05\ x2\ x9.81} (.1/.05)^2 x3.1416\ x.1sin\theta)^2$

„h f s=.36/.981 x (1.256sin $\theta)^2$

$=(.576\ sin^2\theta)$ ------------------3

Loss of head due to friction

„„ h f d $= \frac{4\ f\ Ld}{ddx\ 2g} [(\frac{A}{ad}\omega r sin\theta)^2]$

$=\frac{4x\ .009x}{.05x2x9.81}[(\frac{.1}{.05})^2 x3.1416x\ .1xsin\theta]^2$

$=1.1\ (1.256)^2 x(sin)\theta^2$

$=1.7\ sin^2\theta$ ---------- 4

i) a)

pressure head in cylinder at the beginning of suction stroke.(20.8)

„ h a s =3.948 cos θ =3.948 , cos0=1 (θ=0°)

Pressure head in the cylinder at the beginning suction stroke

=(hs + has) below atmospheric pressure

=H atm ---(hs +h a s)= 10.3 –(3.5 +3.948)

=2.852(absolute)

i) b)pressure head in the cylinder at beginning of delivery stroke.

=(h d +had) above atmospheric pr.

= H atm +(h d +h a d)

= 10.3 +20 =12.06

=42.36 m

ii) a) pressure head in the cylinder in the middle of suction st.

θ =90°

.. ha s=0, h a d=0 h f s=.576m, h f d=1.7 $sin^2\theta$=1.7

a) Pressure head in the middle of suction stroke

=(hs +hf s) below the atmospheric pressure

=H atm ---(h s + h f s)= 10.3---(3.5 +.576)=6.224m

b) pressure head in middle of delivery stroke

=(h d+ hf d)above atmospheric pressure

= H a t m + hd + hfd =10.3 +20 +1.7 =42.0 m

iii) pressure head in the cylinder at the end of suction and delivery stroke , θ =180°

cos 180 =--- 1, sin 180 = 0

∴ has =3.948 cos 180 =--- 3.948.

And ha d=12.06 cos 180 = -- 12.06

a) pressure head in the cylinder at the end of suction stroke

=has ---3.948 = 3.5 ---3.948 = --- value

= not possible.

b) pressure head in the cylinder at the end of delivery system

.. h d --- had = 20 m –12.06m

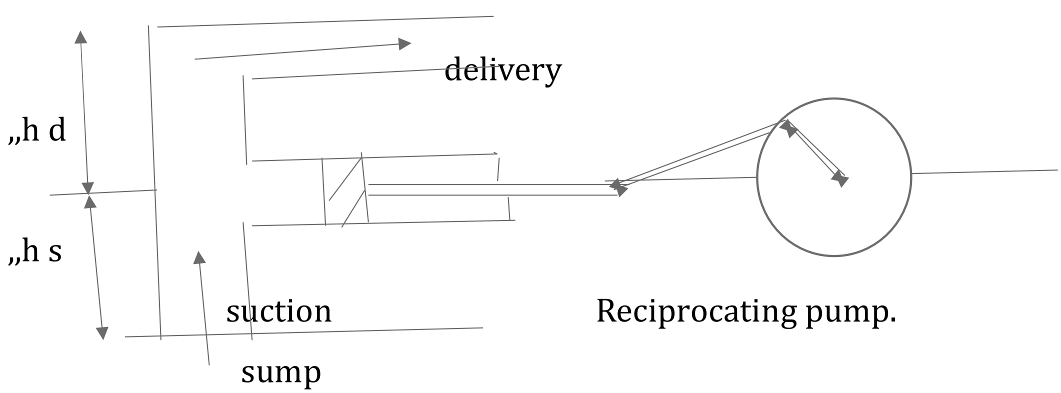
$=H\,atm + (20 - 12.06) = 18.24$ m

Q 8

For problem 7 find the power required to drive the pump if the liquid flowing through the pump is water [Ans 0.25 kw]

Solution

Given



using equation(20.22)for the work done by the pump/s

for single acting we get= $\frac{\rho g ALN}{60}$ (h s +hd+2/3hf s +2/3 h f d)

A =π/4 (.1 X.1) =.007854 m2

L=.2m, ρ =1000kg/m3, h s=3.5m , hd =20m

,, h fs=.576, h f d=1.7

4Type equation here.

Putting values

Work done /sec= $\frac{1000x\ .81\ \ .007854x.2\ x30(3.5+20+\frac{2}{3x}.576+2.3\ x1.7}{60}$

=193.4 Nm/s=193.4/1000

= .1934 =.234 kw (approx.)

Q9

The cylinder of a single acting reciprocating pump is 125mm in diameter and250mm in stroke. The pump is running at 40 rpm and discharge water to a height of 15m. The diameter and length of delivery pipe are 100mm and 30mm respectively . If a large air vessel is fitted in a delivery pipe at a distance of 1.5 m from the center of the pump, find the pressure head in the cylinder i) at the beginning of the delivery stroke ii) In the middle of the delivery stroke.

Take efficiency of friction =.0.1 [i)15.566 m ii)15.07m]`

Solution

Dia of cylinder =D=125mm=.125m

Area of cylinder =π/4 x D^2=π/4 x $(.125)^2$ = .01227m2

Stroke length L =250mm =250/1000 =.25m

Crank radius =L/2 =.25/2 =.125m

Speed of pump =N=40 rpm, angular speed ω=2Πn/60

=2 π 40/60 =4.19radians

Delivery head =h d =15m,dia of delivery pipe =dd=.1m

Ad =area of delivery pipe =π/4 x(.1x.1) =.007854m2

Length of delivery -pipe =ld=30m

Length of air vessel from center of cylinder = l'd=1.5m

Length of delivery pipe along the vessel =ld=l---l'd

=30 –1.5 =28.5m

Coefficient of friction=0.01.

1.Pressure head in the cylinder at the beginning stroke is given by equation (20.25)

$$=hd+(l'/g \text{ x } A/ad.\text{x } \omega^2 r)+\frac{4fld}{dd\,x2g}x(\frac{A}{ad}x\frac{\omega r}{\pi})^2+1/2g(\frac{A}{ad}x\frac{\omega r}{\pi})^2$$

$$=15+\frac{1.5}{9.81}x\frac{.01227}{.007854}x(4.19)^2x.125+\frac{4x.01x30}{.1\,x2x9.81}x(\frac{.01227}{.007854}x\frac{4.19\ \ .125}{\pi})^2+\frac{1}{2x9.81}(\frac{.01227}{.007854}x\frac{4.19}{\pi})^2$$

$$=15+.153\text{x } 1.56 \text{ x}(17.55 \text{ x}.125)+\frac{1.2}{1.962}x(1.56x.166)^2\text{m}$$

$$=15+.523+.61\ (1.56\text{x}.166)^2+.05(1.56x.166)^2\text{m}$$

$$=15+.523+.61\text{ x}(.259)^2+.05(.259)^2\text{m}$$

$=15+.523+.04+.0033175$ m

$=15.566$m

2. pressure head in the cylinder in the middle of the delivery(20.26)

$$=h\,d+\frac{4fld'}{dd\,x2g}(\frac{A}{ad}xar)^2+\frac{4fld'}{ddx2g}(\frac{A}{ad}x\frac{wr}{\pi})^2+\frac{1}{2g}(\frac{A}{ad}x\frac{wr}{\pi})^2$$

$$=15+\frac{4x.01\,x1.5}{.1\,x2x9.81}(\frac{.01227}{.007854}\text{x}4.19\text{x}.125)^2+\frac{4x.01x3}{.1x2x\,9.81}(\frac{.01227}{.007854}x4.19x.125/\pi)^2+\frac{1}{29.81}(\frac{.01227}{.007854}x\frac{4.19}{\pi}x.125)^2$$

$=15 +.06/1.962\ (1.56x.523)^2 + \frac{1.2}{1.962}(1.56x.166)^2 +$
$.05(1.56.1.66)^2$

$=15 +.03x.664 +.611x.067 +.05x.067 =15.05534$

=15 .06m =pressure head in cylinder in the middle.

E ND OF CHAPTER 20

Fluid Mechanics and Hydraulic Machines

Fluid System

Chapter 21

Q 1

A hydraulic press has a ram of 300 mm diameter and a plunger of 50 mm diameter . Find the weight lifted by the hydraulic press when the force applied to the plunger is 40 N.

[Ans 1440 w]

D=Ram diameter =.3 m

„d =plunger diameter =.05m

F=Force on plunger =F=40N

W=weight lifted=W N

A =Area of ram =π/4 x .3 x .3 =.07068 m2

. a =area of plunger =π/4 x .05 x.05 =⁚.0019m2

The weight lifted W is given by equ.21.11 equation

W =W/a x A =40 x .07068/.001963 =1440N

Q 2

A Hydraulic press has a ram of 150mm diameter and plunger of 30 mm. The stroke of plunger is 250mm and weight lifted 600 N . If the distance moved by the weight is 1.20m in 20 minutes. Determine a) force applied on the plunger b)power required to drive the plunger and c) number of stroke performed by the plunger. [a)24N b).0006kNc)120]

Solution

D=Ram diameter =.15m, plunger diameter =.03m

Type equation here.

Stroke =.25m, W=Weight lifted =600 N   Distance moved=1.2m

Time taken by weight=20 min.

Area of ram=$\pi/4$ x .15 x.15 =.01767 m2

Area of plunger =$\pi/4$ x .03 x .03 =.00070 m2

a)Force applied on the plunger

force applied on the plunger =F

Equation (21.1)

W=F/a x A

F= 600 x .0007/.01767 =23.77N =24N

b) power required to drive the plunger =worked per sec /1000

=weight lifted x distance travelled /time taken

=600 x 1.2 /20 x60=.6 Nm/s

Power required =work done /s / 1000 =.6/1000

=.0006 kw

c) number of stroke performed by the plunger

= total volume displaced in cylinder

/volume of liquid displaced per stroke.

1) total volume displaced in one stroke

= area of plunger x stroke length =.0007 .25=.000175m3

2) total volume of liquid displaced in cylinder

= area of ram x distance moved by weight

=.01767 x 1.2= 0.021204

So number of stroke performed by the plunger

Type equation here.

= ½ = .021204/.000175 =121 ANS.

Q 3

Water is supplied at a pressure of 1.5 N/cm2 to an accumulator having a ram of diameter 2.0 m. If the total lift of the ram is 10 m , deter mine

a) capacity of the accumulator

b) total weight placed on the ram (including weight of ram,)

[4712.4 knm, , 471240 N]

Solution

Given

Supply pr. P=15N/cm2 =15

Diameter of ram =D= 2m, Area of ram =A =π/4 x (2 x2)

= π m2

Lift of rm = L =10m

1) capacity of accumulator =P A L = 15 (10x 10x10x10)x π x 10

$$=\frac{150x10^4 x\pi}{1000}$$

=4712.388 k Nm

2) total weight (W) placed on the ram =P x A

$=15 \times 10^4 x\pi$

$=471238.89$ N. Ans

Q 4

The diameter of fixed ram pressure of water supplied to the fixed cylinder is 25 N/cm2, and fixed cylinder of an intensifier are 100 mm and 250 mm respectively. If the find the pressure of water flowing through the fixed ram? [156.25 N/cm2]

Solution

Given

Diameter of fixed ram =d=100 mm=10cm

Area of fixed ram =a =π/4 (10 x10)= 78.53 cm2

Diameter of fixed cylinder =D =250mm=25cm=.25m

Area of fixed cylinder A =π/4 x (25 x25) =490.87 cm2

P =intensity of supply pressure at the fixed cylinder =25 N/cm2

Let pressure in fixed ram = Pi

Using equation (21.8)

Pi=P x A/a= 25 x 490.87/78.53

= 156.24 N/cm2

Q5

The water is supplied at the rate of 30 lit/s from a height of 4m to hydraulic ram, which raises 3 lit/s, to a height of 18m from the ram . Determine D' Aubuisson's and Rankine's efficiencies of hydraulic ram. [45°,38.8%]

Solution given ;--discharge in pipe =Q=30 lit/s=.03m3/s

Supply head =h=4m , discharge q=3lit/s=.003m3/s

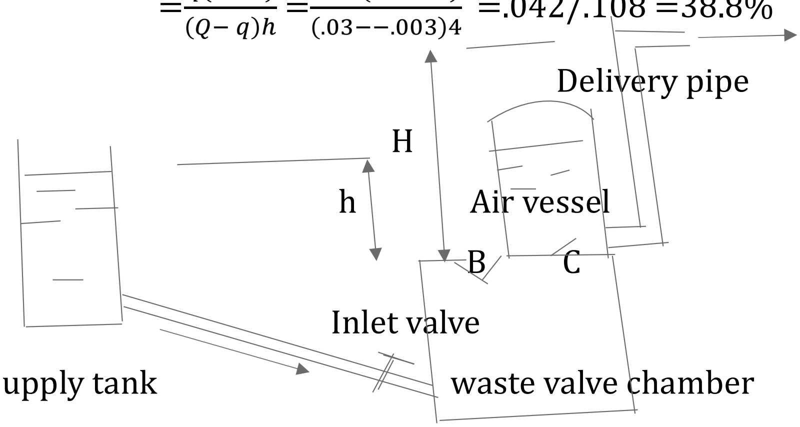
Water raise from ram =H=18m.

Using equation 21.11

1. D' Aubuisson's efficiency$=\frac{q\ xH}{QX}=\frac{.003x1}{.03x}=.45=$45%

2. Rankine's efficiency (21.12)

$$=\frac{q(H-h)}{(Q-q)h}=\frac{.003(18--4)}{(.03--.003)4}$$ =.042/.108 =38.8%



The Hydraulic Ram (B and C are valves.)

Q 6

A hydraulic lift is required to lift a load of 98.1 Kn through a height of 12 m once in every 100 seconds. The speed of lift is 600mm/s. Determine

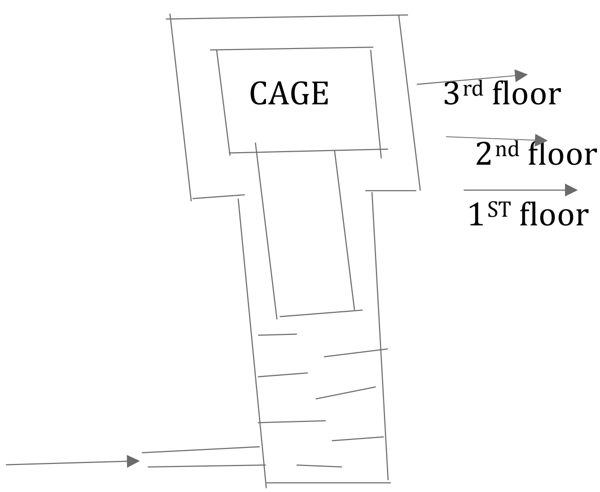
a) power required to drive the lift.

b) working period of lift in seconds.

c) idle period of lift in seconds.

[a) 11.772kw b)20 secs c)80 secs.]

Given



Hydraulic lift

Load lifted = W =98.1 KW, =98.1 x 1000=98100 kn

Height H=12m . time for one operation= t =100 sec .

Speed of lift =600mm=.6 m/s

1. work done in lifting the load in 100 secs

=W X H= 98100 X 12 =11.77200 Nm

Type equation here.

Power required = 1177200/1000 kw

= 11.772 kw.

2.working period of the lift = height of lift /velocity of lift

= 12m/.6m/s =20 seconds

3. 1dle period of lift = 100 s—20 sec =80 seconds.

Q 7

Find the efficiency of hydraulic crane which is supplied 400 lit of water under a pressure of 490.5 N/cm2 for lifting a weight 98.1 kw through a height of 10 m . [Ans 50%]

Given

Water supplied = Q =400 LIT /S= .4 m3/s

Pressure =p =490.5 N/cm2=490.5 x 10^4N/m2

W =WEIGHT LIFTED =98100 N

Height =h =10m

Output of the crane =weight lifted x height lifted.

= 98100 x 10 Nm

Input of the crane = Energy supplied by water.

=work done by water on the ram

=force on ram x distance moved by ram

=pressure x A of ram x stroke

= P x A x L=490.5 x 10^4x volume displayed

$$=490.5 \times 10^4 \times .4 = 1962000 \text{ Nm}$$

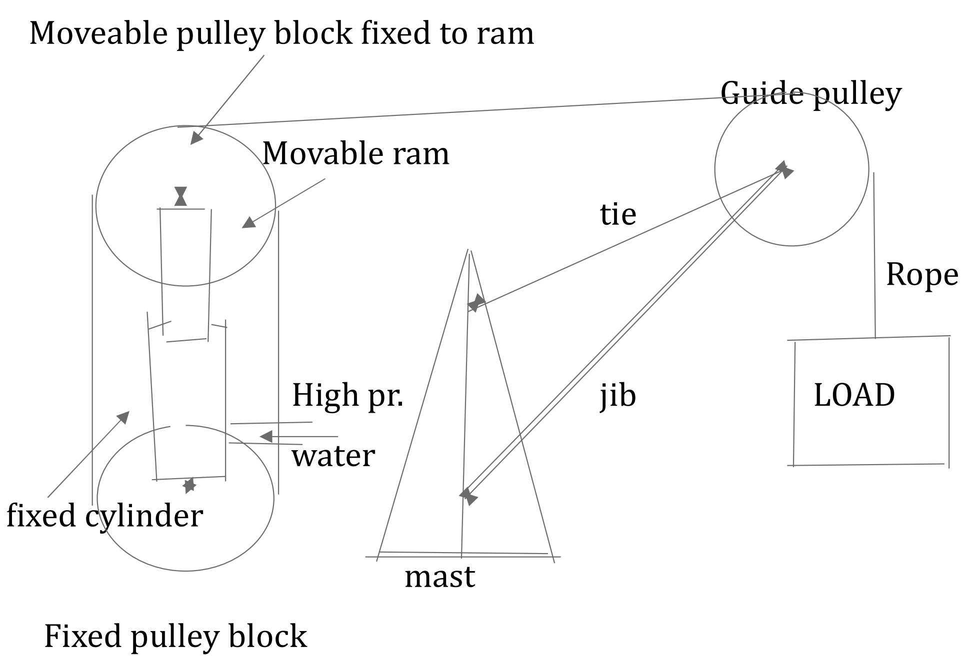
$\therefore$ efficiency =output /Input =98100 x10/1962000

$$=981/1962$$

$$= .5 = 50\% \text{ Ans.}$$

Hydraulic crane sketch given below ;--

Hydraulic crane



Fixed to cylinder jigger

Q 8

In a hydraulic coupling the speed of the driving and driven shafts are 800 rpm and 780 rpm respectively

Find 1. Efficiency of hydraulic coupling

2. The slip of the coupling [97.5%, 2.5%]

Solution

Given

Speed of driving shaft A= 800rpm= Na

Speed of driven shaft B=780rpm=Nb

Torque of shaft A=Ta, Torque at shaft B=Tb

Power available at shaft A ∝ speed of shaft A x Torque A

$\propto$ Na x Ta

a) Power available at shaft B∝ speed of shaft B x Nb x Tb

So efficiency $=\frac{NbxTb}{NaxNb}$ but Ta=Tb

$=\frac{Nb}{Na}$

$=\frac{780}{800}$

Type equation here.

$=.975$

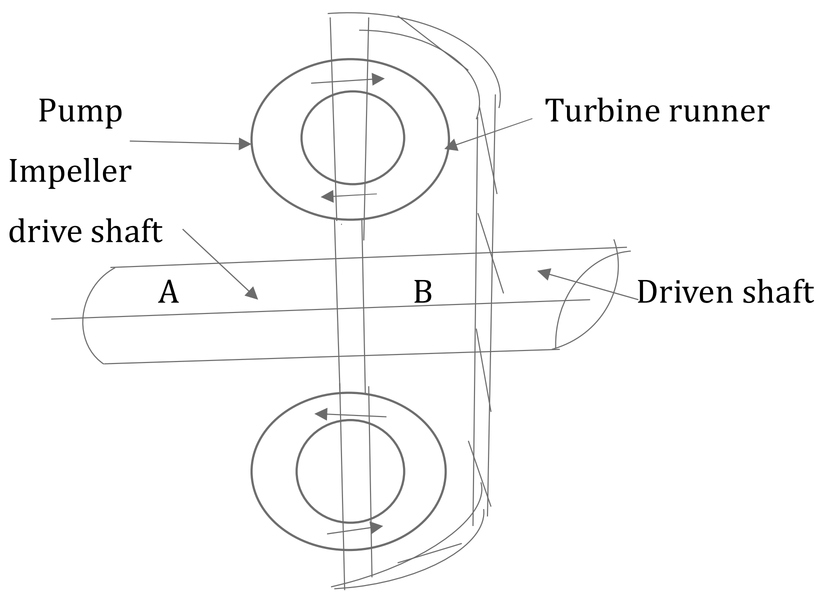
$=97.5\%$

b) Slip of fluid coupling is obtained as the ratio of difference of speed to the driving speed as stated below

$$=\frac{Na-Nb}{Na}=\frac{800-780}{800}=20/800=.025=2.5\%$$

Hydraulic coupling drawing given below ;----

Hydraulic coupling



Hydraulic coupling

Type equation here.

End of 21 Chapter

END OF ALL CHAPTER