

**SOLUTIONS FOR
UNSOLVED PROBLEMS**
— OF —
**HEAT AND
MASS TRANSFER**
(R.K.Rajput).

ANUP KUMAR SEN



BlueRose ONE^{llc}
S t o r i e s M a t t e r
New Delhi • London

BLUEROSE PUBLISHERS
India | U.K.

Copyright © Anup Kumar Sen 2026

All rights reserved by author. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the author. Although every precaution has been taken to verify the accuracy of the information contained herein, the publisher assumes no responsibility for any errors or omissions. No liability is assumed for damages that may result from the use of information contained within.

BlueRose Publishers takes no responsibility for any damages, losses, or liabilities that may arise from the use or misuse of the information, products, or services provided in this publication.



BlueRoseONE
Stories Matter
New Delhi • London

For permissions requests or inquiries regarding this publication,
please contact:

BLUEROSE PUBLISHERS
www.BlueRoseONE.com
info@bluerosepublishers.com
+91 8882 898 898
+4407342408967

ISBN: 978-81-6744-569-8

First Edition: September 2026

Preface to the First Edition

I am happy to present my 8 Th book “Solutions for Unsolved Problems of Heat and Mass Transfer”. The original book is “Heat and Mass Transfer” by Er R K Rajput.

My book will be helpful for Engineering students as well as AMIE and UPSC students. I will suggest it is better they also consult the original book time to time to understand the theory. Sums are basically equations based which again based on theories developed by Scientists at times.

The book has 13 Chapters out of which 2 chapters have no unsolved problems So effectively has 11 Chapters. I tried my best to make sums error free. Most sums are lengthy. If student find any correction is must, I will be happy to rectify in future edition.

With sincere hope that common students will be benefited from my book I wish them all good luck.

Anup Kumar Sen.

Contents

Chapter 1

Basic Concepts	1
----------------------	---

Chapter 2

Heat Transfer by Conduction.....	6
----------------------------------	---

Chapter 3

Conduction – Steady state two and three dimensions	51
--	----

Chapter 4

Conduction – Unsteady – State (Transient).....	59
---	----

Chapter 5

Introduction to Hydrodynamics.....	85
------------------------------------	----

Chapter 6

Dimensional Analysis	86
----------------------------	----

Chapter 7

Forced Convection LAMINAR FLOW	87
--------------------------------------	----

Chapter 8

Free Convection.....	133
----------------------	-----

Chapter 9

Boiling and Condensation	147
--------------------------------	-----

Chapter 10

Heat Exchangers	163
-----------------------	-----

Chapter 11

Thermal Radiation – Basic Relations	178
---	-----

Chapter 12

Radiation Exchange Between Surfaces	182
---	-----

Chapter 13

Mass Transfer	194
---------------------	-----

Chapter 1

Basic Concepts

Q 1. The inner surface of a plane brick wall is at 40°C and other surface is at 20°C . Calculate the rate of heat transfer per m square of surface area of the wall which is 250mm thick. The thermal conductivity of the brick is $0.52\text{ W/m}^{\circ}\text{C}$. [Ans 41.6 W/m^2]

Solution 1

Given, $t_1 = 40^{\circ}\text{C}$ $t_2 = 20^{\circ}\text{C}$
thickness $= L = 250\text{ mm} = .250\text{ m}$
thermal conductivity $= 0.52\text{ W/m}^2$
 $A = 1\text{ m square.}$

$$\begin{aligned} Q &= \text{Heat transfer / m square} = \frac{K A (t_1 - t_2)}{L} \\ &= \frac{.52 \times 1 \times (40 - 20)}{.250} \\ &= 41.6\text{ W/m}^2 \end{aligned}$$

Q2 A plane wall (thermal conductivity $= 10.2\text{ W/m}^{\circ}\text{C}$ of 100 mm thickness and area 3 m^2 has steady surface temperature 170°C and 100°C . Determine

- i) rate of heat flow across the plane wall;
- ii) temperature gradient in the flow direction

Solution:

Given, thermal conductivity $= 10.2\text{ W/m}^{\circ}\text{C}$

$$\text{thickness} = L = 100\text{mm} = .1 \text{ m}$$

$$\text{Area} = 3 \text{ m}^2$$

$$\text{Surface temps. } t_1 = 170^\circ \quad t_2 = 100^\circ\text{C}$$

$$\text{Temp. Difference} = 170 - 100 = 70^\circ\text{C}$$

$$\text{Thermal conductivity} = k = 10.2 \text{ W/m}^\circ\text{C}$$

i) rate of heat flow across the plain wall as per Fourier law

$$\begin{aligned} Q &= -KA \frac{dt}{dx} = -kA \frac{t_2 - t_1}{L} \\ &= -10.2 \times 3 \times \frac{170 - 100}{.1} \\ &= -\frac{102 \times 210}{.1 \times 1000} \text{ kW} \\ &= 21.42 \text{ kW} \quad \text{ANS.} \end{aligned}$$

$$\text{II) Temperature gradient} = \frac{dt}{df} = \frac{-Q}{KA} = \frac{21420}{10.2 \times 3} = -700^\circ\text{C} \quad \text{ANS.}$$

Q 3, Determine the heat transfer by convection over a surface $.75 \text{ m}^2$ if the surface is 200°C and fluid is 80°C . The value of convective heat transfer is $25 \text{ W/m}^2\text{C}$. [Ans 2.25kW]

Solution :

Given

$$\text{Surface area} = .75 \text{ m}^2$$

$$\text{Surface temp.} = 200^\circ\text{C}$$

$$\text{Fluid temp} = 80^\circ\text{C}$$

$$\text{Convective heat transfer} = 25 \text{ W/m}^2\text{C} = h$$

$$\text{Rate of heat transfer} = Q = h A (t_s - t_f)$$

$$= 25 \times .75 \times (200 - 80) \text{ w}$$

$$= \frac{25 \times .75 \times 120}{1000} \text{ kw}$$

$$= 2.25 \text{ kw ans.}$$

Q 4 A surface area 3 m^2 and at 200°C exchanges heat with another surface at 30°C by radiation. If the value of factor due to geometric location and emissivity is .69, determine

i) rate of heat transfer

ii) value of thermal resistance. iii) Equivalent convection coefficient.

$$[\text{Ans 1. } 4885.6 \text{ w ii) } .0348^\circ\text{C/w iii) } 9.58 \text{ w/m}^2]$$

Solution given; surface area $= 3 \text{ m}^2$

$$\text{Surface temp } t_1 = 200^\circ\text{C} = 200 + 273 = 473 \text{ K}$$

$$t_2 = 30^\circ\text{C} = 30 + 273 = 303 \text{ K}$$

$$\text{Factor} = .69$$

$$\text{Emissivity (st Boltzmann constant)} = 5.67 \times 10^{-8} \text{ w/m}^2\text{K}^4$$

1. heat lost/rate of heat transfer

$$= F\sigma A(T^4 - T^4)$$

$$= .69 \times 5.67 \times 10^{-8} \times (473^4 - 303^4)$$

$$= .69 \times 5.67 \left\{ \left(\frac{473}{100} \right)^4 - \left(\frac{303}{100} \right)^4 \right\}$$

$$= .69 \times 5.67 \{ (4.73)^4 - (3.03)^4 \}$$

$$= 3.9 \times 416.22 = 4869$$

II) Value of thermal resistance (R_h) radiation

$$Q = \frac{T_1 - T_2}{R_h \text{ radian}}$$

$$4869.7 = \frac{473 - 303}{R_h}$$

$$R_h = \frac{170}{4869.7} = .0349 \text{ C/W ANS}$$

III) Equivalent convective coefficient = h_r

$$Q = h_r X A X (t_1 - t_2), \quad h_r = \frac{Q}{A (t_1 - t_2)} = \frac{4869.7}{3 (200 - 30)}$$

$$= 9.45 \text{ w/m}^2\text{C ANS}$$

Q5 A surface at 200°C exposed to surrounding at 60°C convects and radiates heat to the surroundings. The convection coefficient and radiation factor are 80 w/m²°C and unity respectively. If the heat is conducted to the surface through a solid of conductivity 15 w/m°C what is the temperature gradient at the surface? [Ans -889 .4°C]

Convective coefficient = $h = 80 \text{ w/m}^2\text{C}$ $k = \text{conductivity} = 15 \text{ w/m}^2\text{C}$

To find Temp gradient $\frac{dt}{dx}$:

Heat conducted through plate = convection heat losses + radiation heat losses.

$$Q_{con} = Q_{conv} + Q_{rad}$$

$$-K A \frac{dt}{dx} = hA (t_s - t_{surr}) + F\sigma A (T_s^4 - T_{sur}^4)$$

$$-15x \frac{dt}{dx} = 80 (200 - 60) + 1 \times 5.67 \times 10^{-8} \{ (200 + 273)^4 - (60 + 273)^4 \}$$

$$-- \frac{dt}{dx} = 80 \times 140 + 1 \times 5.67 \times \left\{ \left(\frac{473}{100} \right)^4 - (3.33)^4 \right\}$$

$$-- \frac{dt}{dx} = \frac{80 \times 140}{15} + 1 \times 5.67 \{(4.73)^4 - (3.33)^4\}$$

$$-- \frac{dt}{dx} = 746 + 142.74$$

$$= -889.4 \text{ } ^\circ\text{C ans.}$$

Chapter 2

Heat Transfer by Conduction

Q 1. The inner surface of plane brick wall is at 50°C and the outer surface is at 30°C . Calculate the rate of heat transfer per m^2 of surface area of the wall, which 250mm thick. The thermal conductivity of the brick is $.52 \text{ W/m}^{\circ}\text{C}$. [Ans; 41.6 W/m^2]

Solution: Given

Inner surface temperature $t_i = 50^{\circ}\text{C}$

Outer surface temperature $t_o = 30^{\circ}\text{C}$

Thermal conductivity $= k = .52 \text{ W/m}^{\circ}\text{C}$

$Y = \text{thickness of wall} = 250 \text{ mm} = \frac{250}{1000} = .25 \text{ m}$

To find rate of heat transfer $= Q/A$

EQN-2.1

$$Q = -K A \frac{dt}{dx}$$

$$Q/A = -K \cdot \frac{t_i - t_o}{Y}$$

$$= -.52 \times \frac{50 - 30}{.25}$$

$$= 41.6 \text{ W/m}^2 \text{ Ans.}$$

Q 2. Temperature of inner and outer surfaces of a boiler wall made of 20mm thick steel and covered with an insulating material of 5mm thickness are 300°C and 50°C respectively. If the thermal conductivity of steel and insulating material are $58 \text{ W/m}^{\circ}\text{C}$ and $.116 \text{ W/m}^{\circ}\text{C}$ respectively. Determine

The rate of heat flow through the boiler wall. [Ans; 5.8 kw/m²]

Solution given

$$t_i = 300^\circ\text{C}$$

$$t_o = 50^\circ\text{C}$$

$$K (\text{steel}) = 58 \text{ W/m}^\circ\text{C}$$

$$K (\text{ins. mat}) = .116 \text{ W/m}^\circ\text{C}$$

8

$$\text{Thickness of insulation} = 5\text{mm} = .005\text{m} = L_{\text{ins}}$$

$$\text{Thickness of steel} = 20\text{mm} = .02\text{m} = L_{\text{st}}$$

$$\begin{aligned} Q/A &= \frac{t_s - t_{in}}{\frac{L_I}{KI} + \frac{L_S}{KS}} \\ &= \frac{300 - 50}{\frac{.005}{.116} + \frac{.02}{58}} = \frac{250}{.00034 + .0431} \\ &= 250/.0434 \\ &= 5760 \text{ W/m}^2 \\ &= 5.76 \text{ kW/m}^2 \text{ Ans.} \end{aligned}$$

Q 3

A mild steel tank of wall thickness 10mm contains water at 90°C. The thermal conductivity Of mild steel is 50 W/m °C. and heat transfer coefficient for the inside and outside of tank are 2800 and 11 W/m² respectively. If the atmospheric temperature 20°C, calculate;

- i) rate of heat loss /m² of the tank surface area.
- ii) temperature of outside surface of the tank.

[Ans I, 820 w/m^2 ii, 89.6°c]

Solution

$$\text{Given } h_{hf} = 2800 \text{ w/m}^2 \text{ } ^\circ \text{c} \quad h_{cf} = 11 \text{ w/m}^2 \text{ } ^\circ \text{c}$$

$$L = 10 \text{ mm} = .01 \text{ m} \quad k = \text{not provided}$$

9

$$t_{hf} = 90^\circ \text{c} \quad t_{cf} = 20^\circ \text{c}$$

i) rate of heat loss = $q = UA (t_{hf} - t_{cf})$ where U is overall coefficient of heat transfer,

$$\begin{aligned} \text{where } \frac{1}{U} &= \frac{1}{h_{tf}} + \frac{1}{h_{cf}} \\ &= \frac{1}{2800} + \frac{1}{11} \\ &= .000357 + .090909 \\ &= .091266 \end{aligned}$$

$$\therefore U = \frac{1}{.091266} = 10.957 \text{ w/m}^2 \text{ } ^\circ \text{c}$$

$$\text{So, } q = \text{rate of heat loss} = UX (t_{hf} - t_{cf})$$

$$= 10.957(90 - 20)$$

$$= 10.957 \times 70$$

$$= 767 \text{ w/m}^2$$

ii) temperature of outside surface of steel tank

let t_2 be the outside temperature of steel tank

$$\text{then } q = h_{cf} \times (t_2 - t_{cf})$$

$$= 11 \times (t_2 - 20)$$

$$t_2 = \frac{767}{11} + 20$$

$$= 69.72 + 20 = 89.72^\circ \text{C (Ans)}$$

10

Q 4

Calculate the rate of heat flow through the wall of a refrigerated van of 1.5 mm steel sheet at outer surface, 100mm plywood at the inner surface and 2cm of glass wool in between, if the temperature of inside and outside surface are -15°C and 24°C respectively. Take thermal conductivities of glass-wool, steel and plywood are 0.014, 23.2 and $0.052 \text{ W/m}^\circ\text{C}$ respectively. [Ans 6kW]

Solution

Given $L = \text{Steel thickness} = 1.5 \text{ mm} = 0.0015 \text{ m}$

$L = \text{glass wool} = 20 \text{ mm} = 0.02 \text{ m}$

$L = \text{plywood} = 100 \text{ mm} = 0.1 \text{ m}$

Thermal conductivity of steel $= 23.2 \text{ W/m}^\circ\text{C}$

Do - of glass wool $= 0.014 \text{ W/m}^\circ\text{C}$

- Do - of plywood $= 0.052 \text{ W/m}^\circ\text{C}$

- $t_i = -15^\circ\text{C}$

- $t_o = 24^\circ\text{C}$

$$\text{Rate of heat flow } q = \frac{t_o - t_i}{\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C}}$$

$$\begin{aligned}
 &= \frac{24 - (-15)}{\frac{.0015}{23.2} + \frac{.02}{.014} + \frac{.1}{.052}} \\
 &= \frac{39}{.000646 + 1.428 + 1.923} \\
 &= \frac{39}{3.35} \text{ } ^\circ \\
 &= 11.63 \text{ W/m}^2
 \end{aligned}$$

Q 5

Calculate the temperatures at the contact surfaces of the wall of a furnace made up of 250 mm of the brick, 120 mm insulation

Brick and 200 mm of red brick. The inner and outer surface temperatures of the wall are 850°C and 65°C respectively. Neglect resistance of mortar joints.

K firebrick = 1.05 W/m°C, k red brick = 0.85 W/m°C,

K insulation brick = 1.4 W/m°C. k insulation brick = .85 W/m°C

K insulation brick = .2 W/m°C

Solution

Given A = fire brick = 250 mm = .25 m

C = Red brick = 200 mm = .2 m

B = Insulation brick = 120 mm = .12 m

R_{th} = thermal resistance = thickness / thermal conductivity

$$R_{THA} = \frac{.25}{1.05} = .238$$

$$R_{THB} = \frac{.12}{.2} = .6$$

$$R_{THC} = \frac{.2}{.85} = .235$$

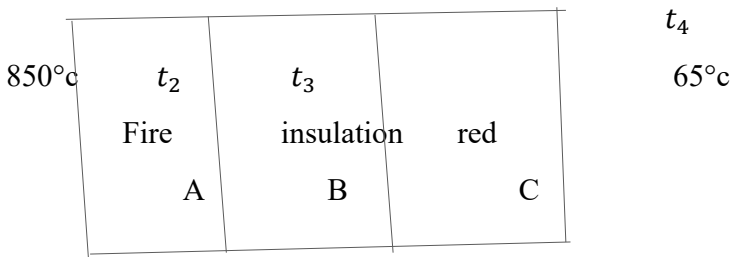
$$\text{SO } R_{Total} = .238 + .6 + .235 = 1.073$$

$$\text{So, } q = \frac{t_i - t_o}{R_{thtotal}} = \frac{850 - 65}{1.073} = 785 / 1.073 = 731.6 \text{ w/m}^2$$

$$t_3 = t_4 + qxR_{thc} = 65 + 731.6 \times .235 = 236^\circ\text{C}$$

$$t_2 = t_3 + qxR_{thB} = 236 + 731.6 \times .6 = 675^\circ\text{C}$$

Figure given below referred to above calculation.



Note; student may observe that thermal conductivity values are different for same item creates confusion and that varies the answers.

Q6 A flat wall of a furnace is made up of fire bricks, insulate

on bricks and building bricks of thickness 25cm 12,5cm 25cm respectively. The inside wall is at 600 °c and atmospheric temp is 20°. If the heat transfer coefficient for the outside surface is 10w/m²°c , calculate

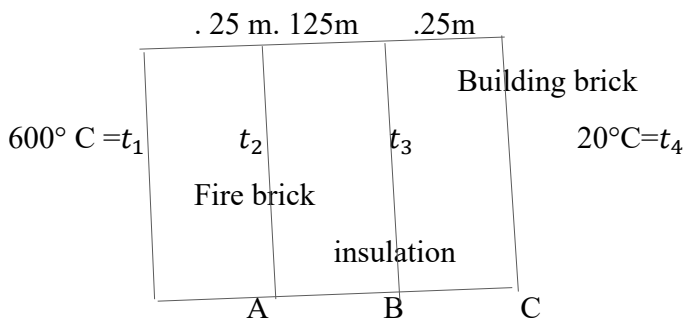
- i) heat loss per m² of wall area.
- ii) Temperature of the outside wall of furnace.

Take k firebrick = 1.4 w/m°c

K insulation brick = 0.2 w/m°C [Ans .46kw/m², 66° c]

Solution

Given



$$k_{fb} = 1.4 \quad k_{ib} = .2 \quad k_{bb} = 1$$

$$R_{thA} = \frac{LA}{K_{fb}} = \frac{.25}{1.4} = .178$$

14

$$R_{thB} = \frac{LB}{K_{ib}} = \frac{.125}{.2} = .625$$

$$R_{thC} = \frac{LC}{K_{bb}} = \frac{.25}{.5} = .5 \text{ (consider } k=.5 \text{ since nothing given.)}$$

$$R_{th \text{ total}} = .178 + .625 + .5 = 1.303$$

$$\text{i) heat loss /m}^2 = q = \frac{t_1 - t_4}{R_{th \text{ total}}}$$

$$= \frac{600 - 20}{1.303}$$

$$=4451\text{w/m}^2$$

$$= .445 \text{ kw/m}^2$$

ii) temp of outside wall

$$t_1 = t_2 + q \times R_{thA}$$

$$600 = t_2 + 445 \times .178$$

$$t_2 = 600 - 79 = 521^\circ\text{C}$$

$$t_2 = t_3 + q \times R_{thB}$$

$$t_3 = t_2 - 445 \times .625 = 521 - 278$$

$$t_3 = 243$$

$$t_3 = t_4 + q \times R_{thC}$$

$$243 = t_4 + 445 \times .5$$

$$15$$

$$t_4 = 243 - 222.5 = 20.5$$

So $t_4 = 20.5^\circ\text{C}$ indicates it is nearly equal to atmospheric pressure which is possible.

Q 7

A furnace wall consists of 200mm of refractory fire clay bricks, 100mm kaolin bricks and 6mm steel plate. The fire side refractory is at 1150°C and temp at the outside surface of the wall is 30°C . An accurate heat balance over the furnace shows that the heat loss from the wall to be 300w/m^2 . It is that there may be thin layer of air between the layer of bricks and steel. To how many millimeters of kaolin are these air layers equivalent?

Thermal conductivity are as follows

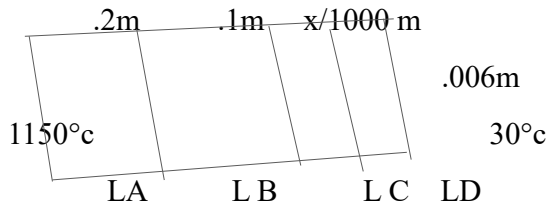
K refractory fire clay brick = $1.7 \text{ W/m}^\circ\text{C}$

K kaolin = $0.17 \text{ W/m}^\circ\text{C}$

K steel = $17 \text{ W/m}^\circ\text{C}$ [Ans 514 mm]

Solution

Given Let x mm is the layer of air equivalent of kaolin.



$$K_{rb}A = 1.7 \text{ W/m}^\circ\text{C} \quad k_{ko}B = 0.17 \text{ W/m}^\circ\text{C} \quad k_sD = 17 \text{ W/m}^\circ\text{C}$$

$$K_{koC} = 0.17 \text{ W/m}^\circ\text{C}$$

$$\Delta t = 1150 - 30 = 1120$$

$$Q/A = 300 \text{ m}^2$$

$$Q = \frac{A \Delta t}{\sum \frac{L}{K}}$$

$$300 = \frac{1120}{\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C} + \frac{L_D}{K_D}}$$

$$= \frac{1120}{\frac{.2}{1.7} + \frac{.1}{.17} + \frac{.00X}{.17} + \frac{.006}{17}}$$

$$= \frac{1120}{.1176 + .588 + .00588X + .00035}$$

$$= \frac{1120}{.00588X + .706}$$

$$0.00588X = \frac{1120}{300} - .706$$

$$= 3.733 - .706$$

$$= 3.027$$

$$X = \frac{3.027}{.00588} = 514.8 \text{ mm. Ans.}$$

Q 8

A furnace wall consisting of 12.5cm wide refractory bricks and 12.5 insulating fire bricks separated by an air gap . The outside wall is covered 1.2 cm thick plaster . Inner surface wall is at 1100°C and room temp is 25°C. i) Calculate the rate at which heat is lost /m² area of wall surface.

$$h_{cf} = \text{heat transfer coefficient from inside w. surface} = 17 \text{ w/m}^2$$

$$R_{air} = \text{resistance of air gap} = 0.16^\circ\text{C/w}$$

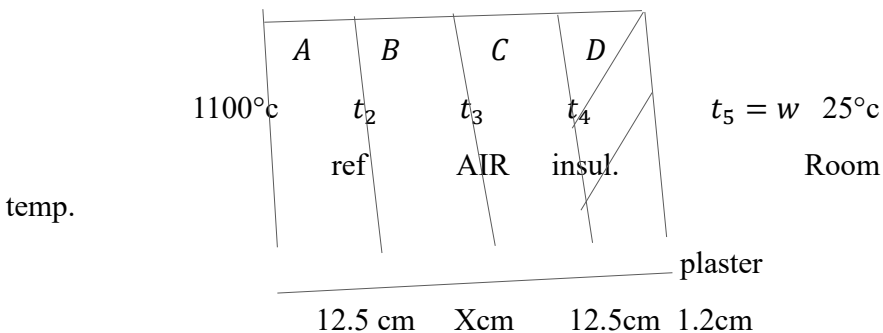
$$k_{rb} = 1.6 \text{ w/m}^\circ\text{C}$$

$$k_{ib} = 0.3 \text{ w/m}^\circ\text{C}$$

$$k_{plaster} = 0.14 \text{ w/m}^\circ\text{C}$$

Also calculate temp of outside wall. [Ans 1338.7 w/m², 103.7°C]

Solution given



$$R_{thcf} = \frac{1}{h_{cf}} = \frac{1}{17} = .0588^\circ\text{C/w}$$

$$R_{air} = .16^\circ\text{C/w}$$

$$R_{rhref} = \frac{.125}{1.6} = .078^{\circ}\text{C/W}$$

$$R_{thins} = \frac{.125}{0.3} = .416^{\circ}\text{C/W}$$

$$R_{thplst} = \frac{.012}{.14} = .0857^{\circ}\text{C/W}$$

$$R_{thtotal} = .7985^{\circ}\text{C/W}$$

$$\begin{aligned} 1) \text{ rate at which heat is lost} &= \frac{\Delta t}{R_{thtotal}} \\ &= \frac{1100-25}{.7985} \\ &= 1346.27 \text{ W/m}^2 \text{ Ans.} \end{aligned}$$

$$11) \text{ Temperature of outside wall } t_5 = t_w$$

From above drawing we may proceed as follows

$$t_1 = t_2 + q \times R_{hcf}$$

$$1100 = t_2 + 1346.27 \times .078$$

$$\text{Or } t_2 = 1100 - 105 = 995^{\circ}\text{C}$$

$$t_2 = t_3 + q \times R_{air}$$

$$\text{Again } 995 = t_3 + 1346.27 \times .16$$

$$t_3 = 995 - 215 = 780^{\circ}\text{C}$$

$$\text{Again } t_3 = t_4 + q \times R_{insb}$$

$$\text{Or } 780 = t_4 + 1346.27 \times .416$$

$$\text{Or } t_4 = 780 - 559 = 221^{\circ}\text{C}$$

$$\text{Now } t_5 = t_w = \text{to find}$$

$$t_4 = t_5 + 1346.27 \times R_{plst}$$

$$221 = t_w + 1346.27 \times .0857$$

$$t_w = t_5 = 221 - 115.375 = 105.6^\circ\text{C Ans.}$$

Q 9

A 2000 w heater element of area $.04 \text{ m}^2$ is protected on the back side with insulation 50mm thick of $k=1.4 \text{ w/m}^\circ\text{C}$ and on the front side by a plate 100mm thick with thermal conductivity $k 45 \text{ w/m}^\circ\text{C}$. The back side is exposed to air at 5°C with convection coefficient $10 \text{ w/m}^2\text{C}$ and front is exposed to air at 15°C with convection coefficient including radiation of $250 \text{ w/m}^2\text{C}$.

Determine i) Temp. of heater elements

ii) heat flow into the room under steady state condition.

[Ans ; 219.4°C , 936.8 w]

Note; left for students.

Q10

Calculate the rate at which heat is lost to the surroundings per unit length of an insulated steam pipe having following dimension and specifications ;

$$d_i = 3\text{cm} \quad r_1 = 1.5\text{cm} \quad d_o = 3.4\text{cm} \quad r_o = 1.7 \text{ cm}$$

$$h_i = 10 \text{ w/m}^\circ \quad h_o = 10 \text{ w/m}^\circ\text{C}$$

$$K_{\text{pipe}} = 15 \text{ w/m}^\circ\text{C}$$

$$K_{\text{insulation}} = 0.5 \text{ w/m}^\circ\text{C}$$

$$t_s = 100^\circ\text{C} \quad t_a = 25^\circ\text{C} \quad (\text{s}=\text{steam} . \text{ a}=\text{air})$$

$$Q = \frac{2\pi L(t_s - t_a)}{\frac{1}{h_i} + \frac{1}{\frac{k_A}{\log \frac{r_o}{r_i}}} + \frac{1}{\frac{k_B}{\log \frac{r_o}{r_i}}} + \frac{1}{h_o}} \quad (L=1\text{m})$$

$$\begin{aligned}
&= \frac{2\pi x 1x(100--25)}{\frac{1}{\frac{1.5}{100}x^{10}} + \frac{1}{\frac{15}{\log \frac{1.7}{1.5}}} + \frac{1}{\frac{.5}{\log \frac{2.7}{1.7}}} + \frac{1}{10 \frac{2.5}{100}}} \\
&= \frac{150\pi}{\frac{100}{15} + \frac{\log \frac{1.7}{1.5}}{15} + \frac{\log \frac{2.7}{1.7}}{.5} + \frac{4}{1}} \\
&= \frac{150\pi}{6.66 + \frac{\log 1.13}{15} + \frac{\log 1.588}{.5} + 4} \\
&= \frac{471.2388}{6.66 + \frac{1.222}{15} + \frac{0.4624}{.5} + 4} = \frac{471.2388}{6.66 + .081 + .9248 + 4} \\
&= \frac{471.2388}{11.66} = 40.415 \text{ w/m Ans.}
\end{aligned}$$

Q 11

A 15 cm outer diameter steam pipe is covered with high temperature insulation ($k=0.85 \text{ w/m}^\circ\text{c}$) and 4 cm low temperature insulation ($k=.72 \text{ w/m}^\circ\text{c}$). The steam is 500°c ambient air is at 40°c . Neglect thermal resistance of steam and air and metal wall. Calculate the heat loss from 1000 long pipe.

[Ans ; 2928kw]

Solution

$$r_1 = .075m \quad r_2 = .125m \quad r_3 = .165m$$

$$t_s = 500 \quad , \quad t_a = 40 \quad , \quad k_a = .85 \quad , \quad k_B = .72 \quad L = 1000 \text{ m long pipe.}$$

$$\begin{aligned}
\text{Heat loss} &= \frac{2\pi L(t_s - t_a)}{\frac{\log \frac{r_2}{r_1}}{k_A} + \frac{\log \frac{r_3}{r_2}}{k_B}} \\
&= \frac{2\pi x 1000(500 - 40)}{\frac{\log \frac{.125}{.075}}{.85} + \frac{\log \frac{.165}{.125}}{.72}}
\end{aligned}$$

$$\begin{aligned}
 &= \frac{2000 \times \pi \times 460}{\frac{.5068 \times .2776}{.85 - .72}} \\
 &= \frac{2890265}{.9815} \\
 &= 2944742.7 \text{ W} \\
 &= 2944.7 \text{ KW. Ans.}
 \end{aligned}$$

Q 12

An insulated steam pipe of 160mm inner diameter and 180mm outer diameter is covered with insulation of 40mm thickness and carries steam at 200°C .

$$\begin{aligned}
 K(\text{pipe}) &= 29 \text{ W/m} & k(\text{insulation}) &= .23 \text{ W/m}, & h_i &= 11.6 \text{ W/m}^2\text{°C} \\
 h_o &= 23.2 \text{ W/m}^2\text{°C}
 \end{aligned}$$

Calculate i) rate of heat loss/m to the surroundings .from the pipe 5 m long.

ii) find interface temperatures.

$$\text{Solution} \quad r_1 = .8 \text{ m} \quad r_2 = .9 \text{ m} \quad r_3 = 1.3 \text{ m}$$

$$t_s = 200^\circ\text{C} \quad t_a = 25^\circ\text{C} \quad L = 5 \text{ m long steam pipe}$$

$$\begin{aligned}
 \text{i) rate of heat loss} &= \frac{2\pi L(t_s - t_a)}{\frac{1}{h_i r_i} + \frac{\log \frac{r_2}{r_1}}{k_{\text{pipe}}} + \frac{\log \frac{r_3}{r_2}}{k_{\text{ins}}} + \frac{1}{h_o r_3}} \\
 &= \frac{2\pi \times 5(200 - 25)}{\frac{1}{11.6 \times .8} + \frac{\ln \frac{.9}{.8}}{.29} + \frac{\ln \frac{1.3}{.9}}{.23} + \frac{1}{23.2 \times 1.3}} \\
 &= \frac{10\pi \times 175}{\frac{1}{.928} + \frac{.0040}{.29} + \frac{1.585}{.23} + \frac{1}{3.016}}
 \end{aligned}$$

$$= \frac{5497.78}{1.077 + .0040 + 1.585 + .33} = 5497.78 / 2.996$$

$$= 1835 \text{ W. Ans.}$$

(Q 12 solution continues)

ii) interface temperatures :

$$\text{a) } t_{hf} - t_1 = \frac{Q}{h_{hf} X r_1 X 2\pi L} = \frac{1835}{11.6 \times .08 \times 2\pi \times 5} = \frac{1835}{29.154} = 62.94$$

$$200 - t_1 = 62.94$$

$$t_1 = 200 - 62.94 = 137.06^\circ\text{C Ans}$$

$$\text{b) } t_1 - t_2 = \frac{Q}{\frac{k_A}{\ln \frac{r_2}{r_1}} 2\pi L} = \frac{1835 \times \ln \frac{.09}{.08}}{2 \times 29 \times \pi}$$

$$= \frac{7.35}{911} = .008$$

$$137.06 - t_2 = .008$$

$$t_2 = 136.99^\circ\text{C Ans.}$$

$$\text{c) } t_2 - t_3 = \frac{Q X \ln \frac{r_3}{r_2}}{k_B X 2\pi L} = \frac{1835 X \ln \frac{.13}{.09}}{.23 \times 2\pi \times 5} = \frac{1835 \times .3646}{7.22} = \frac{668}{7.22}$$

$$136.99 - t_3 = 92.52$$

$$t_3 = 136.99 - 92.52 = 44.47^\circ\text{C Ans.}$$

Q 13

Heat is condensed through a tapered circular rod of 200mm length. The ends A and B having diameters 50mm and 25 mm are maintained at 27°C and 227 °C respectively.

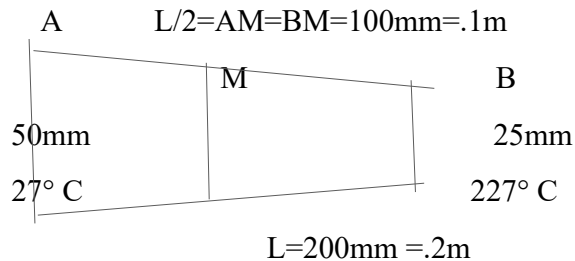
K(rod mat.) = 40 W/m °C

Find i) heat conducted through rod.

ii) Temp at the midpoint of the rod.

Assume there is no temperature gradient at particular cross section and there is no heat transfer through the peripheral surface. [i)39.29wii)93.7 c]

Solution



$$K(\text{rod mat.})=40\text{w/m } ^\circ\text{C}$$

$$\text{Radius B}=.0125\text{ m}$$

$$\text{Radius A}=.025\text{m}$$

$$L=.2\text{m}$$

$$\text{Heat flow through the rod} = Q = \frac{K\pi R_B R_A (t_A - t_B)}{L}$$

$$\text{OR, } Q = \frac{40\pi \times .0125 \times .025 (227 - 27)}{.2} = 39.27 \text{ W Ans}$$

ii) to find temperature at midpoint =t.

radius at midpoint = R_X (Say)

$$= R_A + (R_A - R_B) \frac{x}{L}$$

$$= R_A + (R_A - R_B) X \frac{1}{2} \quad (\text{X being mid point})$$

$$\begin{aligned}
&= \frac{1}{2} (R_A + R_B) \\
\text{So, } Q &= \frac{k\pi R_A R_B (t - t_2)}{L} \\
&= \frac{k\pi \frac{(R_A + R_B) R_A (t - t_2)}{2}}{L/2} \\
39.27 &= \frac{40 \times \pi \times (.0125 + .025) \times .0125 (t - 27)}{.2} \\
(t - 27) &= \frac{39.27 \times .2}{40 \times \pi \times .0125 \times (.0125 + .025)} \\
\therefore t &= \frac{7.854}{.0589} + 27 = 133.34 + 27 = 160.34^\circ\text{C}
\end{aligned}$$

Q 14

A steam pipe having 20mm outer diameter is to be covered with 2 layers of insulation each having a thickness 10mm. The average thermal conductivity of one material is 5 times that of the other. Assuming that inner and outer surface temperature of composite insulation are fixed, show that heat transfer will be reduced by 29.9% when a better insulating material is next to the pipe than when it is away from the pipe ,

Solution

To prove difference between heat transfer of both material will be 29.9%.

Material B insulation K

Material C insulation 5K as given.

$$r_1 = 10\text{mm} = .01\text{m} \quad r_2 = 20\text{mm} = .02\text{m}, \quad r_3 = 30\text{mm} = .03\text{m}$$

Solution 1

$$Q_1 = \frac{2\pi L(t_b - t_c)}{\frac{\ln \frac{r_2}{r_1} + \ln \frac{r_3}{r_2}}{\frac{K_B}{K} + \frac{K_C}{K}}} = \frac{2\pi \times 1 \times (t_B - t_C)}{\frac{\ln \frac{.02}{.01} + \ln \frac{.03}{.02}}{\frac{K}{K} + \frac{5K}{5K}}} = \frac{2\pi(t_B - t_C)}{\frac{5X.693 + .405}{5K}} = \frac{10\pi\{K(t_B - t_C)\}}{3.87}$$

$$= K(t_B - t_C) \times 8.11 \text{ ----- i)}$$

Solution 2 (reversing the insulation B and C interchanged position)

$$Q_2 = \frac{2\pi \times 1 \times (t_c - t_B)}{\frac{\ln \frac{r_2}{r_1} + \ln \frac{r_3}{r_2}}{\frac{K_C}{K} + \frac{K_B}{K}}} = \frac{2\pi(t_c - t_B)}{\frac{\ln \frac{.02}{.01} + \ln \frac{.03}{.02}}{\frac{5K}{5K} + \frac{.405}{K}}} = \frac{2\pi(t_c - t_B)}{\frac{3.465 + .405}{5K + K}}$$

$$= \frac{5K \times 2\pi(t_c - t_B)}{3.465 + 2.025}$$

$$= \frac{10K\pi(t_c - t_b)}{5.49} = K(t_c - t_B) \times 5.72 \text{ ----- ii)}$$

$$\text{Let } k(t_c - t_B) = Y$$

$$Q_1 - Q_2 = 8.11Y - 5.72Y = 2.39Y$$

$$8.11Y \text{ reduced to } 2.39Y$$

1	“	“	$\frac{2.39Y}{8.11Y}$	
100	“	“	$\frac{2.39 \times 100}{8.11}$	= 29.47% Proved

Q 16

Hot air at a temperature of 60 c is flowing through a steel pipe of 100mm diameter, The pipe is covered with two layers of different insulation material of thickness 50mm and 30mm and corresponding thermal conductivities are .23 and .37 w/m c.

The inside and outside heat transfer coefficients are 58 and 12 w/m² c. The atmosphere is at 25 c. Find

i) rate of heat loss from 50 m long pipe ,

neglect the resistance of steel pipe. [Ans 2.334kw]

Solution

$$t_1=60^\circ\text{c} \quad t_o=25^\circ\text{c}$$

$$r_1=50\text{mm}=.05\text{m} \quad r_2=100\text{mm}=.1\text{m} \quad r_3=130\text{mm}=.13\text{m}$$

$$k_A=.23 \quad K_B=.37\text{W/m}^\circ\text{c}, \quad h_i=58\text{w/m}^2\text{c}, \quad h_o=12\text{w/m}^2$$

To find rate of heat loss

$$\begin{aligned} &= \frac{2\pi L(t_{hf}-t_{cf})}{\frac{1}{h_i r_1} + \frac{\ln \frac{r_2}{r_1}}{k_A} + \frac{\ln \frac{r_3}{r_2}}{k_B} + \frac{1}{h_o r_3}} = \frac{2\pi(60-25) \times 50}{\frac{1}{58 \times .05} + \frac{\ln \frac{.1}{.05}}{.23} + \frac{\ln \frac{.13}{.1}}{.37} + \frac{1}{12 \times .13}} \\ &= \frac{2\pi \times 35 \times 50}{\frac{1}{2.9} + \frac{\ln 2}{.23} + \frac{\ln 1.3}{.37} + \frac{1}{1.56}} = \frac{219.9 \times 50}{.3448 + \frac{.693}{.23} + \frac{.2623}{.37} + .641} \\ &= \frac{219.9 \times 50}{.3448 + 3 + .7089 + .641} = \frac{219.9 \times 50}{4.6947} \\ &= 2342 \text{ W} \\ &= 2.342 \text{ KW. Ans.} \end{aligned}$$

Q 17

A steam main 250mm in diameter and 225m long is covered with 50mm high temperature insulation ($k=0.095\text{w/m}^\circ\text{c}$) and 40 mm of low temperature insulation ($k=0.065\text{w/m}^\circ\text{c}$). The inner and outer surface temperature as measured are 400°c and 50°c respectively. Calculate

i) total heat loss

ii) total heat loss $/\text{m}^2$ of outer surface.

iii) heat loss $/\text{m}^2$

iv) temperature between two layers of insulation

neglect heat conduction through pipe material, {Ans 255514kj/h, 873.5kj/h, 1502.5kj/h, 215°c }

$$r_1=.125 \text{ m} \quad r_2=.175 \text{ m} \quad r_3=.215 \text{ m}$$

$$k_{high}=.095 \text{ w/m c} \quad k_{low}=.065 \text{ w/m c}$$

$$L=225 \text{ m} \quad t_1=400 \text{ c} \quad t_2=50 \text{ c}$$

i) total heat loss /hr

$$\begin{aligned} Q &= \frac{2\pi L(t_1 - t_3)}{\frac{\ln \frac{r_2}{r_1}}{k_{hi}} + \frac{\ln \frac{r_3}{r_2}}{k_{low}}} = \frac{2\pi \times 225(400 - 50)}{\frac{\ln \frac{.175}{.125}}{.095} + \frac{\ln \frac{.215}{.175}}{.065}} \\ &= \frac{2\pi \times 225 \times 350}{\frac{\ln 1.4}{.095} + \frac{\ln 1.228}{.065}} = \frac{494800}{\frac{.3364}{.095} + \frac{.2053}{.065}} \\ &= \frac{494800}{3.54 + 3.158} = \frac{493800}{6.698} \\ &= 73872.79 \text{ w/s} \\ &= \frac{73872.79 \times 3600}{1000} \text{ kw/h} \\ &= 265942 \text{ kj/h} \quad \text{Ans,} \end{aligned}$$

iii) total heat loss /m² of pipe surface

$$\begin{aligned} &= \frac{\text{total heat loss /hr}}{2\pi r_1 L} = \frac{265942}{2\pi \times .125 \times 225} \\ &= \frac{265942}{176.7} = 1505 \text{ k j/h} \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \text{ii) total heat loss /m}^2 &= \frac{\text{total heat loss /h}}{2\pi r_3 L} = \frac{265942}{2\pi \times .215 \times 225} \\ &= \frac{265942}{304} = 874.8 \text{ K J/h} \quad \text{Ans} \end{aligned}$$

iv) temperature between 2 layers of insulation ,t₂

$$Q = \frac{2\pi L(t_1 - t_2)}{\frac{\ln \frac{r_2}{r_1}}{k_{high}}} = \frac{2\pi \times 225(400 - t_2)}{\frac{\ln \frac{.175}{.125}}{.095}}$$

$$\begin{aligned}
 73872.79 \text{ W} &= \frac{450\pi(400-t_2)}{3.54} \\
 (400-t_2) &= \frac{73872.79 \times 3.54}{450\pi} = \frac{261509.67}{1413.7} \\
 &= 185
 \end{aligned}$$

So, $t_2 = 400 - 185 = 215^\circ \text{ C}$ Ans.

Q19 A steam pipe of 160mm inside diameter and 5mm thick ($k=58 \text{ W/m}^\circ \text{C}$) is covered with 1st layer of insulating material 30mm thick ($k=.17 \text{ W/m}^\circ \text{C}$) and 2nd layer of insulating material 50mm thick ($k=.093 \text{ W/m}^\circ \text{C}$). The temperature of steam passing through the pipe is 300°C and ambient temperature surrounding the pipe is 30°C . Taking inner and outer heat transfer coefficient 30 and $5.8 \text{ W/m}^2 \text{ }^\circ \text{C}$. Find heat lost per meter length of pipe.

[216W/m]

Solution

$$r_1=80\text{mm}=.08\text{m} \quad r_2=.085\text{m} \quad r_3=.115\text{m}, r_4=.165\text{m}$$

$$t_1=300^\circ \text{C} \quad t_a=30^\circ \text{C}$$

$$k_{steel}=58 \text{ W/m}^\circ \text{C} \quad k_{ins1}=.17 \text{ W/m}^\circ \text{C} \quad k_{ins2}=.093 \text{ W/m}^\circ \text{C}$$

$$h_i=30 \text{ W/m}^2 \text{ }^\circ \text{C} \quad h_o=5.8 \text{ W/m}^2 \text{ }^\circ \text{C}$$

Heat lost/m length of pipe

$$\begin{aligned}
 Q/L &= \frac{2\pi(t_{hf}-t_a)}{\frac{1}{h_1 r_1} + \frac{\ln \frac{r_2}{r_1}}{k_{steel}} + \frac{\ln \frac{r_3}{r_2}}{k_{ins1}} + \frac{\ln \frac{r_4}{r_3}}{k_{ins2}} + \frac{1}{h_o r_4}}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2\pi \times 1 \times (300 - 30)}{\frac{1}{30 \times .08} + \frac{\ln \frac{.085}{.08}}{58} + \frac{\ln \frac{.115}{.085}}{0.17} + \frac{\ln \frac{.165}{.115}}{.093} + \frac{1}{5.8 \times .165}} \\
&= \frac{540\pi}{\frac{1}{2.4} + \frac{\ln 1.06}{58} + \frac{\ln 1.35}{.17} + \frac{\ln 1.434}{.093} + \frac{1}{.957}} \\
&= \frac{540\pi}{.4166 + \frac{\ln 1.06}{58} + \frac{\ln 1.35}{.17} + \frac{\ln 1.434}{.093} + \frac{1}{.957}} \\
&= \frac{1696.46}{.4166 + .001 + 1.776 + 3.870 + 1.0434} \\
&= \frac{1696.46}{7.1086} \\
&= 238.64 \text{ w/m Ans}
\end{aligned}$$

Q 20

A spherical shaped vessel of 1.2m diameter is 100mm thick . Find the rate of heat leakage ,if the temp difference between the inner and outer surface is 200 C. Thermal conductivity of material is 0.3kj/m h C .[Ans 2262 kj/h]

$$1 \text{ k j/m h } ^\circ\text{C} = 0.2778 \text{ W/m } ^\circ\text{C}$$

$$r_2 = .6\text{m} \quad r_1 = .5\text{m}$$

$$t_1 - t_2 = 200 \text{ C (given)}$$

$$\begin{aligned}
\text{Heat leakage} = Q &= \frac{t_1 - t_2}{\frac{r_2 - r_1}{4\pi k r_1 r_2}} = \frac{200}{\frac{.6 - .5}{4\pi \times .3 \times .6 \times .5}} \\
&= \frac{200}{\frac{.1}{4\pi \times .01}} = \frac{200 \times 4\pi \times .09}{.1} = 2261.9 \text{ kj/h}
\end{aligned}$$

Q 21

A voltage of 10 v impressed on a stainless-steel wire (k=22.5w/mc) ,(resistivity $\rho = 70 \times 10^{-6} \text{ ohm cm}$) of 3,2 mm diameter

and 300mm length. The outer surface temp. of the wire is maintained at 93 C. what is the temperature at the center of the wire ?

[138.4 C]

Solution

Given voltage =10v k =22.5w/m°C

Resistivity $\rho = \frac{70 \times 10^{-6}}{100} = 70 \times 10^{-8}$ radius wire $R = 1.6 \text{ mm} = .0016 \text{ m}$

$L = 300 \text{ mm} = .3 \text{ m}$ $t_w = 93^\circ \text{C}$

To find temperature at center =?

$$R_e = \text{electrical resistance} = \frac{\rho L}{A} = \frac{70 \times 10^{-8} \times .3}{\pi (.0016)^2} = .0261$$

$$Q = \frac{V^2}{R_e} = \frac{10^2}{.0261} = 3831.4 \text{ W}$$

$$\therefore q = \frac{Q}{AL} = \frac{3831.4}{\pi (.0016)^2 \times .3} = \frac{3831 \times 10^8}{241.27} = 15.9 \times 10^8$$

$$\text{Temperature at center} = t_w + \frac{q}{4k} R^2$$

$$= 93 + \frac{15.9 \times 10^8}{4 \times 22.5} (.0016)^2$$

$$= 93 + \frac{15.9 \times 10^8}{4 \times 22.5} \times 256 \times 10^{-8}$$

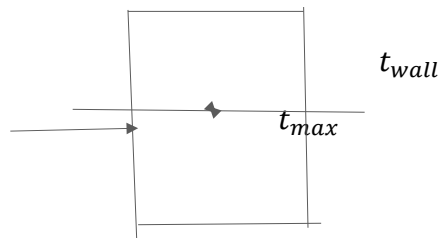
$$= 93 + \frac{4070.4}{90}$$

$$= 93 + 45.226$$

$$= 138.226^\circ \text{C Ans.}$$

Q23

A plane wall ($k=15\text{ w/m}^\circ\text{C}$) 100 mm thick generates heat at the rate of $4 \times 10^4 \text{ w/m}^3$ when an electric current is passed through it. The convective heat transfer coefficient between each face of wall and ambient air is $50 \text{ w/m}^2^\circ\text{C}$. If the ambient temperature is 20°C the surface temperature and maximum temperature in the wall. [60°C 63.33°C] 100mm



Given

$L = \text{thickness of wall} = 100\text{mm} = .1\text{m}$

$q_g = 4 \times 10^4 \text{ w/m}^3$

$h_{con} = 50 \text{ w/m}^2^\circ\text{C}$ $K = \text{Conductive coefficient} = 15 \text{ w/m}^\circ\text{C}$

$t_a = 20^\circ\text{C}$

To find $t_{central}$ and t_{wall} ;--

$$\begin{aligned} t_{max} &= t_a + q_g \left[\frac{L}{2h} + \frac{L^2}{8K} \right] \\ &= 20 + 4 \times 10^4 \left[\frac{.1}{2 \times 50} + \frac{.1^2}{8 \times 15} \right] \\ &= 20 + 40 + 3.3 \\ &= 63.3^\circ\text{C} \quad \text{Ans} \end{aligned}$$

$$t_{wall} = t_a + \frac{L}{2h} \times q_g$$

$$\begin{aligned}
&= 20 + \frac{.1}{2 \times 50} \times 4 \times 10^4 \\
&= 20 + 40 \\
&= 60^\circ \text{ c} \quad \text{Ans}
\end{aligned}$$

Q 25

The temperature on the two surfaces of a 20 mm thick steel plate ($k=50 \text{ W/m } ^\circ\text{C}$) having uniform volumetric heat generation of $40 \times 10^6 \text{ W/m}^3$ are 160°C and 100°C . Neglecting the end effect determine the following

i) position and value of max . temperature

ii) the flow heat from each surface of the plate

[6.25 mm, 175.6°C ,ii)— $250 \times 10^3 \text{ W/m}^2$ iii)— $550 \times 10^3 \text{ W/m}^2$]

Solution

Given $k=50 \text{ W/m } ^\circ\text{C}$, $x=L=20 \text{ mm} = .02 \text{ m}$, $t_{w1}=160^\circ \text{C}$

$t_{w2}=100^\circ \text{C}$, $q_g=40 \times 10^6 \text{ W/m}^3$

Temp .distribution when both walls have different temp is given by ;--

$$\begin{aligned}
t &= \left[\frac{q_g}{2k}(L-x) + \left(\frac{t_{w2} - t_{w1}}{L} \right)x + t_{w1} \right] \\
&= \left[\frac{40 \times 10^6}{2 \times 50}(.02 - x) + \frac{(100 - 160)}{.020}x + 160 \right] \\
&= [8000 - 400000 x^2 - 3000]x + 160 \\
&= 160 + 5000x - 400000x^2
\end{aligned}$$

This is temp distribution position . The distribution will be parabolic in nature when temp is in centigrade and x is in meter.

ii) t_{max} may be found out by differentiating the equation and equating it to 0 .

$$160 + 5000x - 400000x^2 = \frac{dt}{dx}$$

$$5000 - 800000x = 0$$

$$X = .00625$$

$$\text{So } t_{max} = 160 + 5000x - 400000x^2$$

$$\text{Putting value of } x = .00625 \text{ m}$$

$$= 160 + 5000x \cdot .00625 - 400000x \cdot (.00625)^2$$

$$= 160 + 31.25 - 15.625$$

$$= 175.625^\circ \text{ C Ans}$$

iii) flow of heat from each surface q_1 and q_2

$$q_1 = -k A \left(\frac{dy}{dx} \right)_{x=0}$$

$$= -50 \times 1 \times (5000 - 800000x)$$

$$= -50 \times (5000 - 0)$$

$$= -25000 \text{ W}$$

$$= -250 \text{ KW/m}^2 \text{ Ans.}$$

$$q_2 = -KA \left(\frac{dy}{dx} \right)_{x=20\text{mm}} \quad (20\text{mm} = \text{thickness of plate})$$

$$= -50 \times 1 \times (5000 - 800000x)$$

$$= -50 \times (5000 - 800000 \times .02)$$

$$= -50 (5000 - 16000)$$

$$= -50 \times -11000$$

$$= 550000 \text{ W}$$

$$= + 550 \text{ KW } /m^2 \text{ Ans.}$$

Q 26

A plane wall 80mm thickness ($k=.15 \text{ w/m}^\circ \text{ c}$) is insulated on outside while the other side is exposed to environment at 90° c . The rate of heat generation within the wall is $12 \times 10^4 \text{ w/m}^3$. If the convective heat transfer coefficient between wall and environment is $560 \text{ w/m}^2 \text{ }^\circ \text{ c}$, determine the max temperature to which wall will be subjected . [2667° c]

Solution

Given convective heat transfer coefficient = $h_{con}=5.6 \text{ w/m}^2 \text{ }^\circ \text{ c}$
 $k=\text{conductive coeff.}=.15 \text{ w/m}^\circ \text{ c}$

$$L=80\text{mm}=.08 \text{ m}, t_a=90^\circ \text{ c} \quad q_g=12 \times 10^4 \text{ w/m}^3$$

$$t_{max}=?$$

One dimensional steady state heat conduction is given by

$$d^2t/dx^2 + \frac{q_g}{k} = 0 \quad \text{----- i)}$$

Integrating above we get

$$\frac{dt}{dx} = -\frac{q_g}{k} \cdot x + c_1 \quad \text{----- ii)}$$

Further interpretation

$$..t = -\frac{q_g}{k} \frac{x^2}{2} + c_1 x + c_2 \quad \text{----- iii)}$$

$c_1 \ c_2$ are constants of integration.

In order to find $c_1 \ c_2$ following boundary is put

$$\text{I) at } x=0 \quad \frac{dt}{dx} = 0 \quad \text{so } c_1 = 0$$

ii) at $x=L$ heat conduction equals to convective heat flow to the environment

$$\text{so } -KA \left(\frac{dt}{dx} \right)_{x=L} = hA(t_L - t_a)$$

$$\begin{aligned} -\left(\frac{dt}{dx} \right)_{x=L} &= \frac{hA}{KA} [t_L - t_a] \\ &= \frac{h}{k} (t_L - t_a) \quad \text{-----iv)} \end{aligned}$$

Again, from equation ii)

$$\frac{dt}{dx} = -\frac{q_g}{k} x + c_1 = -\frac{q_g}{k} L \quad \text{-----V)}$$

Again, from equation (iv, v)

$$\begin{aligned} -\frac{h}{k} [t_{1L} - t_a] &= -\frac{q_g}{k} L \\ t_{1L} &= t_a + \frac{q_g}{h} L = \frac{-q_g}{2k} L^2 + C_2 \\ C_2 &= t_a + \frac{q_g}{h} L + \frac{q_g}{2k} L^2 \quad \text{-----vi)} \end{aligned}$$

Inserting value of c_1 c_2 in equation iii) we get

$$\begin{aligned} t &= -\frac{q_g}{2k} x + t_a + \frac{q_g}{h} x + \frac{q_g}{2k} L^2 \\ &= t_a + \frac{q_g}{h} L + \frac{q_g}{2k} (L^2 - x^2) \quad \text{-----vii)} \end{aligned}$$

Maximum temperature to which wall will be subjected

When $x=0$,putting the values in vii) above ;--

$$t_{max} = 90 + \frac{12 \times 10^4}{560} \times .08 + \frac{12 \times 10^4}{2 \times .15} \times (.08)^2$$

$$t_{max} = 90 + \frac{120}{7} + \frac{12 \times 64}{.3}$$

$$= 90 + 17.1 + 2560$$

$$= 2667.1 \text{ } ^\circ \text{C} \text{ Ans.}$$

Q27 A 60 mm thick slab of insulating material ($k=0.407 \text{ W/m}^\circ \text{C}$) is placed between and is in contact with parallel electrodes and is thus subjected to dielectric heating (high frequency) at a uniform rate of 40694.4 W/m^3 . On attainment of steady state condition, the coefficient of combined radiation and convection for the exposed surfaces are $12.21 \text{ W/m}^2 \text{ } ^\circ \text{C}$ and $13.96 \text{ W/m}^2 \text{ } ^\circ \text{C}$ respectively. If the ambient temperature is 25°C determine

i) surface temperature

ii) location and magnitude of maximum temperature in the system.

Assume the flow of heat to be unidirectional and each electrode to be at uniform temperature equal to that of the slab which is equal to that of slab which is in contact..

[Ans ; i) 121°C 115°C ii) 29 mm 164°C appx]

Solution

Given t_{w1} t_{w2} = surface temperatures.

L =thickness= $60 \text{ mm} = .06 \text{ m}$, $k=0.407 \text{ W/m}^\circ \text{C}$

$$h_1 = 12.21 \text{ W/m}^2 \text{ } ^\circ \text{C} \quad h_2 = 13.96 \text{ W/m}^2 \text{ } ^\circ \text{C}$$

$$q_g = 40694.4 \text{ W/m}^3 \quad t_a = 25^\circ \text{C} \quad \theta_1 = \text{Electrode 1}$$

θ_2 = Temp. of electrode 2.

i) surface temp. t_{w1} & t_{w2}

The temperature distribution in case dielectric heating is given by

$$\begin{aligned}\theta &= -\frac{q_g}{K}x\frac{x^2}{2} + \frac{h_1\theta_1}{k}x + \theta_1 \\ &= -\frac{40694.4}{.407}x\frac{x^2}{2} + \frac{12.21x\theta_1}{.407}.x + \theta_1 \\ &= -49993x^2 + 30\theta_1 x + \theta_1 \text{----- i)}\end{aligned}$$

$$\begin{aligned}\text{At } x &= .06\text{m and } \theta = \theta_2 \\ \theta_2 &= -49993(.06)^2 + 30 \times (.06) \theta_1 + \theta_1 \\ &= -180 + 1.8 \theta_1 + \theta_1 \\ &= -180 + 2.8 \theta_1 \text{----- ii)}\end{aligned}$$

Under steady state condition

Heat generated due to dielectric heating = heat lost due to

Convection from dielectric surface .

$$\begin{aligned}\text{Or, } q_g A L &= h_1 A \theta_1 + h_2 A \theta_2 \quad (A \text{ is cancelled out }) \\ 40964.4 \times .06 &= 12.21\theta_1 + 13.96 \theta_2 \\ \theta_2 &= 176.06 - .874 \theta_1 \text{----- iii)}\end{aligned}$$

From ii) and iii) we get

$$\begin{aligned}-180 + 2.8 \theta_1 &= 176.06 - .874\theta_1 \\ \theta_1(2.8 + .874) &= 356.06 \\ \theta_1 &= \frac{356.06}{3.674} = 96.91^\circ \text{C} \\ \theta_2 &= 176.06 - .874 \times 96.91 \\ &= 91.36^\circ \text{C}\end{aligned}$$

Thus, electrode temperatures are

$$\theta_1 = t_{w1} - t_a$$

$$t_{w1} = 96.91 + 25 = 121.91^\circ \text{ c Ans.}$$

$$t_{w2} = \theta_2 + t_a = 91.36 + 25 = 116.36^\circ \text{ c Ans}$$

ii) location and magnitude of max . temperature, x and t_{max}

location and maximum temperature can be obtained by differentiating equation i) w.r.to x and equating the same to 0

$$d\theta/dx = d/dx [-49993x^2 + 30\theta_1x + \theta_1]$$

$$\text{or } 0 = -2 \times 49993x + 30\theta_1$$

$$\text{a) } x = \frac{30\theta_1}{49993 \times 2} = \frac{30 \times 96.91}{49993 \times 2} = .0290 \text{ m} = 29 \text{ mm Ans}$$

$$\text{b) } t_{max} = -49993x^2 + 30\theta_1x + \theta_1$$

putting value of x and θ_1 on above equation b)

$$= -49993 (.029)^2 + 30 \times 96.91 \times .029 + 96.91$$

$$= -49993 \times .000841 + 84.31 + 96.91 = 181.22 - 42$$

$$= 139^\circ \text{ c}$$

If we add the ambient temperature 25° c to the maximum temperature, comes to $139 + 25 = 164^\circ \text{ c}$

Q29

The electric cable of thermal conductivity $k=20 \text{ W/m}^\circ \text{ c}$, 3 mm diameter and one meter long has the current flow 200 amp ρ (resistivity) $=70 \mu\text{ohm cm}$. the wire is submerged in a liquid at 100° c and surface heat transfer coefficient (h) is $4000 \text{ W/m}^2^\circ \text{ c}$.

calculate the central tempera of the wire, $[372^\circ \text{ c}]$

solution

given $R = \text{radius of cable} = 3\text{mm}/2 = 1.5\text{mm} = 1.5/1000 \text{ m}$

$$= .0015 \text{ m}$$

$K = 20 \text{ w/m c}$ =thermal conductivity

Resistivity $\rho = 70 \mu \text{ ohm cm} = .7 \mu \text{ohm m}$

$L = 1\text{m}$, $t_a = 100^\circ \text{ c}$, current =200amp= I

Temperature at the center of cable ; -

$$\text{Rate of heat generation} = I^2 R_e = I^2 \frac{\rho L}{A}$$

(R_e =electrical .resistance.)

$$\begin{aligned} q_g &= \frac{Q}{AL} = I^2 \frac{\rho L}{A} \times \frac{1}{AL} = \rho \left(\frac{I}{A}\right)^2 \\ &= 70 \times 10^{-8} \left(\frac{200}{\pi \times .0015^2}\right)^2 \\ &= 7 \times 10^{-7} \left(\frac{200}{.00000706}\right)^2 \\ &= 7 \times 10^{-7} (28328611)^2 \\ &= 7 \times 10^{-7} \times 2.8 \times 2.8 \times 10^{14} \\ &= 54.88 \times 10^7 \text{ w/m}^3 \end{aligned}$$

Temperature at the surface of cable

$$\begin{aligned} t_w &= t_a + \frac{q_g}{2h} R = 100 + \frac{54.88 \times 10^7}{2 \times 4000} \cdot .0015 \\ &= 100 + \frac{54.88 \times 10^3}{8} \times \frac{15}{10000} \\ &= 100 + \frac{54.88 \times 15}{80} \end{aligned}$$

$$=100 + 102.9 = 202.99$$

Temperature at the center of cable

$$\begin{aligned} t_{max} &= t_w + \frac{q_g}{4K} x R^2 = 202.9 + \frac{54.88 \times 10^7}{4 \times 20} (.0015)^2 \\ &= 202.9 + \frac{54.88 \times 22.5}{80} \\ &= 202.9 + 15.4 \\ &= 218.3^\circ \text{ C Ans.} \end{aligned}$$

Q 32

Calculate the amount of energy required to solder together 2 very long pieces of base copper wire 1.625 mm in diameter with solder that melts at 195°C . The wires are positioned vertically in air at 24°C . Assume that the heat coefficient on the wire surface is $17 \text{ W/m}^2 \text{ }^\circ \text{C}$ and thermal conductivity of wire alloy is $335 \text{ W/m}^2 \text{ }^\circ \text{C}$.

[Ans ; 2.65w]

Solution

Given d = diameter of copper wire = .001625m

Melting temperature of solder = $195^\circ \text{C} = t_o$

Air temperature $t_a = 24^\circ \text{C}$

.. h = heat transfer coefficient on wire surface = $17 \text{ W/m}^2 \text{ }^\circ \text{C}$

Thermal conductivity of alloy = $335 \text{ W/m}^2 \text{ }^\circ \text{C}$

Energy required for solder =

$$\text{Area of cross section} = \frac{\pi d^2}{4} = \frac{\pi}{4} \cdot .001625^2 = 2.11 \times 10^{-6} \text{ m}^2$$

$$\text{Perimeter} = \pi d = \pi \times .001625 \text{ m} = 5.18 \times 10^{-3} \text{ m}$$

Heat dissipating in long pin

$$Q_{FIN} = K A m(t_o - t_a)$$

$$= 335 \times 2.11 \times 10^{-6} \times m \times (195 - 24) \text{---i)}$$

$$\therefore m = \sqrt{\frac{hp}{kA}} = \sqrt{\frac{17 \times 5.18 \times 10^{-3}}{335 \times 2.11 \times 10^{-6}}} = \sqrt{124.58}$$

$$= 11.16$$

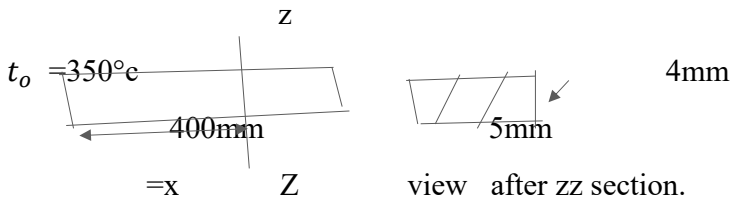
$$\text{So } Q_{fin} = 335 \times 2.11 \times 10^{-6} \times 11.61 (195 - 24)$$

$$= 1.40 \text{ w}$$

So total energy required for 2 long base = $2 \times 1.40 = 2.8 \text{ w}$ Ans.

Q 33

It is required to heat oil to about 350°C for frying purpose. . A ladle is used in frying . The section of handle is $4\text{mm} \times 15\text{ mm}$. The surroundings are at 35°C . The conductivity of the material is 210 w/m c . If the temperature at a distance of 400 mm from oil should not reach 45°C . Determine the convection heat transfer coefficient. [$24.67 \text{ w/m}^2\text{ c}$]



Temperature at zz section = 45°C

Given

$$t_o = 350^\circ\text{C} \quad b = 15\text{mm} = 0.015\text{m}, y = 4\text{mm} = 0.004$$

$$X = L = 400/1000 = .4\text{m. } k = 210\text{W/m}^{\circ}\text{C}$$

To find h = convective heat transfer coefficient (eq 2.132)

Assuming fin to be long one;

$$\frac{t - t_a}{t_o - t_a} = e^{-mx}$$

$$\frac{t_o - t_a}{t - t_a} = e^{mx}$$

$$\frac{350 - 35}{45 - 35} = e^{mx} = e^{mx.4}$$

$$e^{.4m} = \frac{315}{10} = 31.5 \text{ (taking log both side)}$$

$$.4m = \log 31.5$$

$$m = \frac{3.44}{.4} = 8.6$$

$$\text{now } m = \sqrt{\frac{hp}{kA}}$$

$$\text{when } p = \text{perimeter} = 2 \times 15 + 2 \times 4 = 38\text{mm} = .038\text{m}$$

$$A = 15 \times 4 = 60\text{mm}^2 = .00006\text{m}^2$$

$$m^2 = \frac{hp}{kA}$$

$$. h = \frac{m^2 \times kA}{p} = \frac{8.6^2 \times 210 \times .00006}{.038}$$

$$= \frac{9585216}{.038} = 25.22 \text{ W/m}^2 \text{ }^{\circ}\text{C Ans.}$$

Q 34

A temperature rise of 50°C in a circular shaft of 50mm diameter is caused by the amount of heat generated due to friction in the bearing mounted on the crankshaft. The thermal conductivity of shaft material is $55\text{W/m}^{\circ}\text{C}$ and heat transfer coefficient is $7\text{W/m}^2 \text{ }^{\circ}\text{C}$.

- i) develop an expression for temperature distribution .
- ii) determine the amount of heat transferred through the shaft..

assume that the shaft is a rod of infinite length.

$$[\text{ i) } \theta = \theta_0 - e^{-my} \quad \text{ ii) } 17.22 \text{ w }]$$

Solution

Given temperature $= \theta_0 = 50^\circ\text{C}$

Diameter of shaft $= 50\text{mm} = 0.05\text{m}$

Thermal conductivity of shaft $= k = 55 \text{ w/m }^\circ\text{C}$

Heat transfer coefficient of shaft $= h = 7 \text{ w/m}^2 \text{ }^\circ\text{C}$

1) expression of temperature distribution (2.131)

General expression for temperature distribution for an extended surface is given by

$$\theta = c_1 e^{mx} + c_2 e^{-mx} \text{---i)}$$

Using the following boundary condition we have

$$X=0, \quad \theta=\theta_0$$

$$X=\infty \quad \theta=0$$

$$c_1=0 \quad c_2=\theta_0$$

Putting the value in (i)

$$\theta = 0x e^{mx} + \theta_0 e^{-mx}$$

$\theta = \theta_0 e^{-mx}$ required temperature distribution expression.

ii) Amount of heat transferred

$$\begin{aligned}
 Q &= -K A \left(\frac{d\theta}{dx} \right)_{x=0} \\
 &= -k A \left[\frac{d}{dx} (\theta_0 e^{-mx}) \right]_{x=0} \\
 &= K A m \theta_0 \text{ ----- ii)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Value of } m &= \sqrt{\frac{hp}{kA}} = \sqrt{\frac{hx \pi d}{kx \frac{\pi d^2}{4}}} = \sqrt{\frac{4h}{kd}} \\
 &= \sqrt{\frac{4 \times 7}{55 \times .05}} = \sqrt{\frac{28}{2.75}} = 3.19
 \end{aligned}$$

$$\begin{aligned}
 Q &= K A m \theta_0 \\
 &= 55 \times \frac{\pi}{4} (.05)^2 \times 3.19 \times 50 \\
 &= 17.224 \text{ w Ans.}
 \end{aligned}$$

Q 35

One end of a long rod ,30 mm in diameter is inserted into a furnace with other end projecting in the outside air. After the steady state is reached the temperature of the rod is measured at 2 points 150mm apart and found to be 140 c and 100 c .The atmospheric air temperature is 30 c If the heat transfer coefficient is $60 \text{ w/m}^2 \text{ c}$,determine the thermal conductivity of the rod .

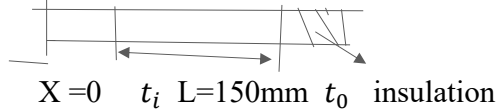
$$[k = 59.79 \text{ w/m c }]$$

Solution

$$\text{Given diameter of rod } = 30 \text{ mm } = .03 \text{ m}$$

$$\text{Air temperature } = t_a = 30 \text{ c } , \text{ ht tr. Co} = h = 60 \text{ w/m}^2 \text{ c}$$

$$L = 150 \text{ mm} = .15 \text{ m}$$



$$t_1 = 140 \text{ }^{\circ}\text{C}$$

$$t_0 = 100 \text{ }^{\circ}\text{C}$$

Eq. 2.134

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = \frac{\cosh\{m(L-x)\}}{\cosh mL}$$

At $x=L$ above eq. reduces to

$$\frac{\theta_L}{\theta_0} = \frac{\cosh\{m(L-L)\}}{\cosh mL} = \frac{1}{\cosh mL} \text{ ----- A}$$

$x=L$ and $x=0$ temperatures are

$$\theta_L = t_0 - t_a = 100 - 30 = 70 \text{ }^{\circ}\text{C}$$

$$\theta_0 = t_i - t_a = 140 - 30 = 110 \text{ }^{\circ}\text{C}$$

Putting above value in eq. A we get

$$\frac{70}{110} = \frac{1}{\cosh mL}$$

$$\text{So, } \cosh mL = \frac{11}{7} = 1.571$$

$$mL = 1.0234$$

$$m = \frac{1.0234}{L} = \sqrt{\frac{h p}{k A}}$$

$$\frac{h p}{k A} = \left(\frac{1.0234}{.15}\right)^2 = 46.5$$

$$K = \frac{h p}{46.5 \times A} = \frac{60 \times \pi d}{46.5 \times \frac{\pi d^2}{4}} = \frac{60 \times 4}{46.5 \times .03}$$

$$= \frac{240}{1.395} = 172 \text{ W/m}^{\circ}\text{C Ans.}$$

Q 36

The heat exchange in certain chemical process is involved from metal surface in distilled water . A number of thin metal film each 60mm long and 3 mm thick are provided to increase the heat transfer rare and their ends are attached to an insulated wall. In order to prevent ionization of water, metal films are coated with 0.15 mm thick layer of plaster . The mean temperature of water and temperature at the base of fins are 25 c and 85 c respectively .The thermal conductivity of the fins material and plastic are 210w/m c and .55w/m c respectively. If the heat transfer coefficient between the plastic coating and water is 200w/m² c. Calculate i) temperature at tip of fin

ii) fin efficiency of fin . [Ans 45.7 c , 54.4%]

solution

given $L = \text{fins} = 60\text{mm} = .06\text{m}$, thickness $= y = .003\text{m}$

mean water temperature $= 25\text{ c} = t_a$

thickness of plastic coating $= \delta_p = .15\text{mm} = .00015\text{m}$

temp. at the base of fin to $= 85^\circ\text{ c}$

thermal conductivity of fin material $k_f = 210\text{w/m c}$

thermal conductivity of plastic $k_p = .55\text{w/m c}$

heat transfer coefficient $h = 200\text{w/m}^2\text{c}$

i) temp. at tip of fin t_1

when making calculation for the value of m given by the expression

$m = \sqrt{\frac{h p}{k A}}$,the value of h is replaced by U as a result of resistance to heat transfer due to plastic coating on the metallic fins , thus

$$\frac{1}{U} = \frac{1}{h} + \frac{\delta_p}{k_p} = \frac{1}{200} + \frac{.00015}{.55} = .005 + .00027 = .00527$$

$$U = \frac{1}{.00527} = 189.75 \text{ W/m}^2 = \text{heat transfer coefficient.}$$

$$\begin{aligned} \text{For fin of rectangular c/s } m &= \sqrt{\frac{2U}{K_f Y}} = \sqrt{\frac{2 \times 189.75}{210 \times .003}} \\ &= \sqrt{\frac{379.5}{.63}} = \sqrt{602.38} = 24.54 = m \end{aligned}$$

For fin with insulated tip

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = \frac{\cosh m(l-x)}{\cosh ml}$$

$$\text{At } x=l, \quad \frac{t_1 - 25}{85 - 25} = \frac{1}{\cosh ml} = \frac{1}{\cosh(24.54 \times .06)}$$

$$\text{Or, } \frac{t_1 - 25}{60} = \frac{1}{\cosh(1.4724)} = \frac{1}{2.2945}$$

$$t_1 = \frac{60}{2.2945} + 25 = 26.14 + 25 = 51.14^\circ \text{ C}$$

= temperature at tip of fin Ans

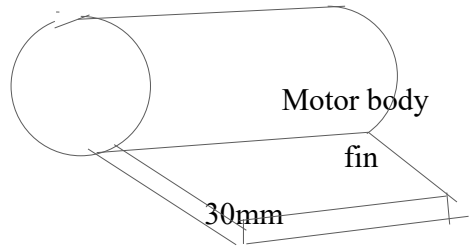
ii) Fin efficiency

$$= \frac{\tanh(ml)}{ml} = \frac{\tanh(24.54 \times .06)}{24.54 \times .06}$$

$$= \frac{\tanh(1.4724)}{1.4724} = \frac{.900}{1.4724} = .61 = 61\% \text{ Ans}$$

Q37 A motor body is 300mm in diameter (outside) and 200mm long. Its surface temperature should not exceed 50 c when dissipating 150 w. Longitudinal fins of 12 mm thickness and 30mm height are proposed . The convective coefficient is 40 w/m c . Determine number of fins required . Atmospheric temperature is 35 °c.

[Ans 18]



200mm

Solution

Given diameter of motor body = 300mm = .3

Fin dimension

Thickness = 12mm = .012m

Height = 30mm = .030m

Length = 200mm = .2m = length of motor body

h = convection coefficient = 40 w/m c

Ambient temp. = 35 c, surface temp = 50 c

Q = Dissipating heat = 150 w , k = 40 w/m c

$$Q_{fin} = \sqrt{phkA} (t_0 - t_a) \left[\frac{\tan(ml) + \frac{h}{km}}{1 + \frac{h}{km} \tan(ml)} \right]$$

$$\begin{aligned} \text{Where } m &= \sqrt{\frac{hp}{kA}} = \sqrt{\frac{h(b+y)x^2}{kx(bxy)}} = \sqrt{\frac{40x(.2+.012)x^2}{40(.2x.012)}} \\ &= \sqrt{\frac{.424}{.0024}} = \sqrt{176.6} = 13.28 \end{aligned}$$

$$\text{So. ml} = 13.28 \times .03 = .3984$$

$$Q_{fin} = \sqrt{(.2 + .012) \times 2 \times 40 \times 40 (.2 \times .012)} \quad (50-35)$$

$$\times \left[\frac{\tanh(.3984) + \frac{40}{40 \times 13.28}}{1 + \frac{40}{40 \times 13.28} \times \tanh(13.28 \times .03)} \right]$$

$$= \sqrt{1.628} \times 15 \left[\frac{\tanh(.3984) + .072}{1 + .072 \times \tanh(.3982)} \right]$$

$$= 19.11 \times \left[\frac{.3785 + .072}{1 + .072 \times .3785} \right]$$

$$= 19.11 \times (.4505 / 1 + .0272) = 19.11 \times \frac{.4505}{1.0272}$$

$$= 19.11 \times .4199 = 8.024 \text{ w}$$

$$\text{Number of fins required} = \frac{Q_{TOTAL}}{Q_{fins}} = \frac{150}{8.02429} = 18.7 \text{ no Ans}$$

Q 38.

A steel rod ($k=30 \text{ w/m c}$) 10 mm diameter and 50 mm long, with insulated end, is to be used as spine. It is exposed to surrounding with a temperature of 65 c and heat transfer coefficient of 50 w/m c. The temperature at the base of the fin is 98 c.

Determine i) The fin efficiency .

ii) The temperature at edge of the spine .

iii) Heat dissipation . [i) 66.6% , ii) 81.9 c iii) 1.72w]

Solution

Given $k=30 \text{ w/m c}$

. $d=10 \text{ mm}=.01 \text{ m}$

$L=50 \text{ mm}=.05 \text{ m}$

$t_a=65 \text{ c}$

$$h = 50 \text{ W/m}^2 \text{ c}$$

$$t_0 = 98 \text{ c}$$

i) fin efficiency (eq no—2.139)

$$= \frac{\tanh(ml)}{ml}$$

$$m = \sqrt{\frac{hp}{kA}} = \sqrt{\frac{h\pi d}{k\frac{\pi d^2}{4}}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 50}{30 \times .01}} = \sqrt{666.66} = 25.8$$

$$\begin{aligned} \text{So, fin efficiency} &= \frac{\tanh(25.8 \times .05)}{25.8 \times .05} = \frac{\tanh(1.29)}{1.29} = \frac{.819}{1.29} = 66.6 \\ &= 66.6\%. \end{aligned}$$

ii) temperature at the edge of the spine, t_1

$$\frac{\theta}{\theta_0} = \frac{t - t_0}{t_0 - t_a} = \frac{\cosh(l - x)}{\cosh ml}$$

$$\text{At } x = l \quad \frac{\theta}{\theta_0} = \frac{1}{\cosh ml}$$

$$\frac{t - 65}{98 - 65} = \frac{1}{\cosh(25.8 \times .05)} = \frac{1}{\cosh 1.29}$$

$$t = 65 + \frac{33}{1.95} = 65 + 16.92 = 81.92 \text{ } ^\circ \text{ c}$$

iii) heat dissipation Q_{fin}

$$= k A m (t_0 - t_a) \tanh(ml)$$

$$= 30 \times \frac{\pi}{4} d^2 \times 25.8 (98 - 65) \tanh(25.8 \times .05)$$

$$= 23.56 \times (.01)^2 \times 25.8 \times 33 \times \tanh(1.29)$$

$$= 23.56 \times .0001 \times 25.8 \times 33 \times .859$$

$$= 1.717 \text{ W Ans.}$$

One end of a copper rod ($k=380 \text{ w/m c}$) 300mm long is connected to wall which is maintained at 300° c . The other end is firmly connected to a wall which is maintained at 100° c . Air is blown across the rod so that the heat transfer coefficient of 20 w/m is maintained. The diameter of the rod is 15mm and temperature of air is 40° c .

ii) The heat conducted to the other end which is at 100°C .

c. $[40.9\text{ W}, 29.05\text{ W}]$

Diagram of a composite wall with a central rod. The wall consists of two vertical sections, wall 1 on the left and wall 2 on the right, separated by a central rod. The total thickness of the wall is 300 mm. The rod has a diameter $d = 0.015 \text{ m}$ and thermal conductivity $k = 380 \text{ W/m}\cdot^\circ\text{C}$. The wall material has thermal conductivity $h = 20 \text{ W/m}\cdot^\circ\text{C}$. The temperature on the left side of wall 1 is $t_1 = 300^\circ\text{C}$, and on the right side of wall 2 is $t_2 = 100^\circ\text{C}$. The rod is 15 mm thick. The distance from the left wall to the rod is 150 mm, and from the rod to the right wall is 150 mm.

49 | Solutions for Unsolved Problems

$$\begin{aligned}\text{And } h p k A &= 20 \times \pi d \times 380 \times \frac{\pi}{4} d^2 \\ &= 20 \times \pi^2 \times (.015)^3 \times 380 = .0632.\end{aligned}$$

$$\begin{aligned}\therefore Q &= \sqrt{.0632} \times (260 + 60) \left[\frac{\cosh\{(1.1235) - 1\}}{\sinh(1.1235)} \right] \\ &= .251 \times 320 \times \left[\frac{1.7 - 1}{1.375} \right] = .251 \times 320 \times .5 = 40.96 \text{ w Ans}\end{aligned}$$

ii) heat conducted to other end where temperature is 100 c

$$Q_{[Cond]}]_{x=l} ; ---$$

$$Q_{cond} = Q_{cond}_{x=0} \quad --Q \quad \text{----- 1.}$$

Temperature distribution is given by

$$\theta = \frac{\theta_1 \sinh\{m(l-x)\} + \theta_2 \sin(mx)}{\sinh(ml)} \quad (\text{eq 2.168})$$

$$\frac{d\theta}{dx} = \frac{-\theta_1 \cosh m(l-x) + \theta_2 m \cosh(mx)}{\sinh(ml)}$$

$$\left[\frac{d\theta}{dx} \right]_{x=0} = \frac{-\theta_1 \cosh ml + \theta_2 m}{\sinh(ml)}$$

$$\text{So } \theta_{cond}]_{x=0} = -k A \left[\frac{d\theta}{dx} \right]_{x=0}$$

$$= k A m \left[\frac{\theta_1 \cosh(ml) - \theta_2}{\sinh(ml)} \right]$$

Putting value in equation 1 , we get

$$Q_{cond}]_{x=l} = K A m \left[\frac{\theta_1 \cosh(ml) - \theta_2}{\sinh ml} \right] - Q$$

=

=

$$= 380 \times \left[\frac{\pi}{4} \times (0.015)^2 \times 3.745 \left[\frac{260 \times \cosh(1.1235) - \theta_2}{\sinh(1.1235)} \right] \right]$$

$$= 0.2515 \left[\frac{260 \times 1.6956 - 60}{1.3693} \right] - 40.96$$

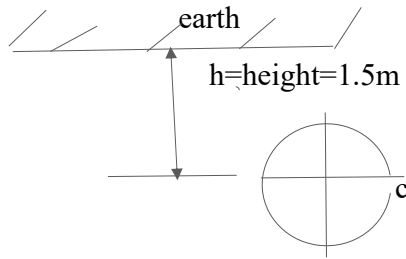
$$= 29.05 \text{ w Ans,}$$

Chapter 3

Conduction – Steady state two and three dimensions

Q 1

A long pipe of .5m outside diameter is buried in earth with axis at a depth of 1.5m. The surface temperature of pipe and earth are 85 c and 20 c respectively. Calculate the heat loss per meter length . conductivity of earth is 0.552w/m c [Ans 85.48 w]



c=center of pipe on axis of pipe

..d =dia of pipe =0.5m

, r = .25m , L=1m k =0.52 w/m c

Surface temperature =85 c

Earth temperature =20 c

To find heat loss /m

(Eq 3.17) $Q = K \cdot s_{fe} (t_p - t_e)$ where s_{fe} is safe factor

$$= \frac{2\pi L}{\ln\left(\frac{2H}{r}\right)} = \frac{2\pi \times 1}{\ln\left(2 \times \frac{1.5}{.25}\right)}$$

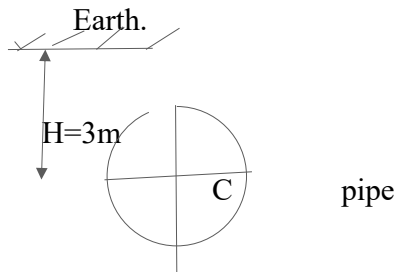
$$= \frac{2\pi}{2.484} = 2.53$$

$$Q = .52 \times 2.53 \times (85 - 20) = 0.52 \times 2.53 \times 65 = 85.5 \text{ W (Ans)}$$

Q 2

A 40m long pipe of 150mm outside diameter carrying steam is buried 3m deep in the ground. The surface temperature of pipe and earth are 200 c and 20 c respectively. Assuming thermal conductivity of soil 0.3 w/m c, calculate the heat loss from the pipe. [Ans 3146.6]

$$\text{Hint ; } S_{fc} = \frac{2\pi l}{\ln\left(\frac{L}{r}\right) \left[1 - \frac{\ln\left(\frac{L}{2H}\right)}{\ln\left(\frac{L}{r}\right)} \right]}$$



Diameter of pipe = 150mm

Radius of pipe = .075m

L=40m=length of pipe.

K=.3w/m c

$t_p = 200 \text{ c}$, $t_e = 20 \text{ c}$

Heat loss = $Q = K S_{fe}(t_p - t_e)$

$$\begin{aligned}
 \text{Where } s_{fe} (\text{as per hint}) &= \frac{2\pi x 40}{\ln\left(\frac{40}{.075}\right) \left[1 - \frac{\ln\left(\frac{40}{2x3}\right)}{\ln\frac{40}{.075}}\right]} \\
 &= \frac{80\pi}{\ln 533.33 \left[1 - \frac{\ln 6.666}{\ln 533.33}\right]} \\
 &= \frac{251.32}{6.279(1-.302)} \\
 &= \frac{251.32}{4.3827} \\
 &= 57.3436
 \end{aligned}$$

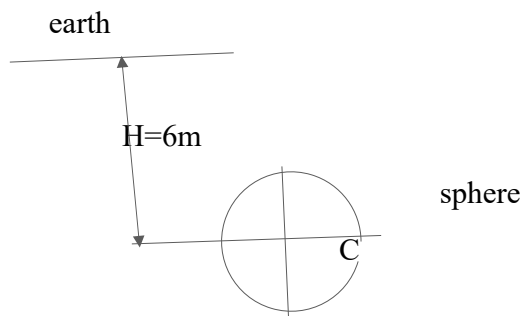
Putting all values;

$$\begin{aligned}
 \text{So, } Q &= .3 \times 57.3436 (200 - 20) \\
 &= .3 \times 57.3436 \times 180 \\
 &= 3096.55 \text{ w}
 \end{aligned}$$

Q 3

A 1.5 m diameter sphere is buried in soil with center in a depth of 6 m.. Heat is generated in the sphere at a rate of 600w . If the conductivity of soil is 0.52w/mc and soil surface temperature is at 5 c, calculate the surface temperature of the sphere under steady condition.
 Ans 119.8 ° c]

Solution



Surface temp. = 5 °C ,

Q = 600 W. r of sphere = 0.75 m

K of soil = 0.52 W/m °C

$$Q = K_s S_{fe} (t_{sph} - t_{sur}) \quad \text{----- 1.}$$

$$\begin{aligned} \text{Where as } S_{fe} &= \frac{4\pi r}{1 - \frac{r}{2H}} \quad (\text{shape factor of sphere}) \\ &= \frac{4\pi \times 0.75}{1 - \frac{0.75}{2 \times 6}} = 10.05 \end{aligned}$$

Putting the values in equation 1 above we get

$$600 \text{ W} = 0.52 \times 10.05 \times (t - 5)$$

$$(t - 5) = \frac{600}{0.52 \times 10.05} = 114.81$$

$$t = 114.81 + 5 = 119.81 \text{ } ^\circ \text{C} \quad \text{Ans.}$$

Q4

A pipe 500 mm diameter carrying steam at a temperature 160 °C (for heating the houses in winter) and another pipe 150 mm diameter carrying water at 11 °C (for supplying drinking water) are buried under the ground with center-to-center distance 1.5 m. The length of each pipe is 60 m and thermal conductivity of soil 0.3 W/m °C. Assuming pipe runs parallel to each other, the earth is infinite and resistance of pipe material is negligible, determine

i) The net heat transfer rate between two pipes.

ii) rise in water temperature due to heat transfer if the velocity of water is 2 m/min.

[Ans i)5991.5 w ii)1.44 c]

Solution ;--

Given

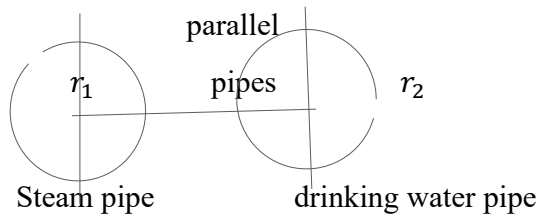
$L = 60 \text{ m}$ for both pipes .

$K_{\text{soil}} = .3 \text{ w/m c}$

$r_1 = 500/2 = 250 \text{ mm} = .25 \text{ m}$

$r_2 = 150/2 = 75 \text{ mm} = .075 \text{ m}$

$D = \text{distance between 2 parallel pipes} = 1.5 \text{ m}$



i)net heat transfer between 2 pipes

$$Q = K S_{fe} (t_{p1} - t_{p2})$$

$$\begin{aligned} \text{Where } S_{fe} &= \frac{2\pi L}{\cosh^{-1}(D^2 - r_1^2 - r_2^2) / 2r_1 r_2} \\ &= \frac{2\pi \times 60}{\cosh^{-1}\left[\frac{1.5^2 - .25^2 - .075^2}{2 \times .25 \times .075}\right]} \\ &= \frac{376.99}{\cosh^{-1}\left(\frac{2.1818}{.0375}\right)} = \frac{376.99}{\cosh^{-1}(58.18)} \\ &= \frac{376.99}{4.756} = 79.266 = S_{fe} \end{aligned}$$

Putting values

$$\begin{aligned} Q &= .3 \times 79.266 \times (160 - 11) \\ &= 3543.19 \text{ W} \end{aligned}$$

II) The rise in water tempt. Δt

Mass of water flowing through the tube

$$m = \rho (\pi r_2^2 v) = 1000 \{ \pi (.075)^2 \times .033 \} = .583$$

$$Q = m \times C_p \times \Delta$$

$$\Delta t = \frac{Q}{m \times C_p} = \frac{3543.19}{.583 \times 4.18 \times 1000} = \frac{3543.19}{2436.94}$$

$$= 1.45 = \text{Rise in water temperature. Ans.}$$

Q 5

The inside dimension of cubical furnace constructed of fire clay bricks ($k = 1.04 \text{ w/m c}$) are .5 m X .5 m x .5 m , the walls being 0.1m thick. The temperature at the inside and outside surfaces are 500 c and 50 c respectively. Determine the heat lost through the walls.

[Ans 8592 W]

Solution

Given

$$A = \text{area} \quad L = .1 \text{ m} = \text{thick} \quad k = 1.04 \text{ w/m c}$$

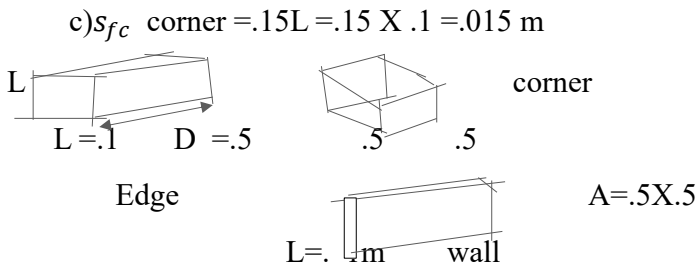
$$t_i = 500^\circ \text{C} \quad t_o = 50^\circ \text{C}$$

The total shape factor computed by adding shape factors

For walls, edges, and corners ;--

$$a) \quad s_{fc \text{ walls}} = \frac{A}{L} = \frac{.5 \times .5}{.01} = 2.5 \text{ m}$$

$$b) \quad s_{fc \text{ edge}} = .54 D = .54 \times 0.5 = .270 \text{ m}$$



1. walls = 6 , 2, edge = 12 , corners = 8

$$s_{fc \text{ total}} = a) + b) + c) = 6 \times 2.5 + 12 \times .270 + 8 \times .015 = 18.36 \text{ m}$$

Q = heat loss through walls

$$= k \times (s_{fc \text{ total}}) \times (t_i - t_o)$$

$$= 1.04 \times 18.36 \times (500 - 50)$$

$$= 8502.48 \text{ w } Q \text{ 6.}$$

Q 6

Three Dimensional Conduction

The inside dimension of a furnace are 3m x 2.5 m x 2m .The walls are 0.2m thick and have thermal conductivity of 1.3 w/m c. If the temperature at the inner and outer surfaces are 300 c and 100 respectively , calculate the heat loss. [Ans 52370 w]

Solution

$$\begin{aligned} \text{Inside surface area} = A_i &= 2[3 \times 2.5 + 2.5 \times 2 + 3.0 \times 2] \\ &= 2[7.5 + 5 + 6] \\ &= 2 \times 18.5 = 37.0 \text{ m}^2 \end{aligned}$$

$$\text{Length of edges} = \sum y = 4(3 + 2.5 + 2) = 4 \times 7.5 = 30 \text{ m}$$

$$\text{Thickness} = \Delta x = .2 \text{ m}$$

$$\begin{aligned}
 \text{Mean area} = A_m &= A_i + (s_{fc})_{edge} \Delta x + (s_{fc})_{corner} \Delta x \\
 &= 37 + 0.54 \sum y \Delta x + 8(.0.15) \Delta x^2 \\
 &= 37 + .54 \times 30 \times .2 + 8 \times .15 \times (.2)^2 \\
 &= 37 + 3.24 + 8 \times .15 \times .04 \\
 &= 37 + 3.24 + 0.48 \\
 &= 40.288
 \end{aligned}$$

$$\text{Rate of heat loss} = Q = K A_m (t_i - t_o) / \Delta x$$

$$= \frac{1.3 \times 40.288 (300 - 100)}{.2} = \frac{10474.88}{.2} = 52370 \text{ W Ans.}$$

Chapter 4

Conduction – Unsteady – State (Transient)

Q1

A copper slab ($\rho = 9000 \text{ kg/m}^3$, $c = 380 \text{ J/kg}^\circ \text{C}$, $k = 370 \text{ W/m}^\circ \text{C}$) measures $400\text{mm} \times 400\text{mm} \times 5\text{mm}$ has a uniform temperature of 250°C . Its temperature suddenly lowered to 30°C . Calculate the time required for the plate to reach the temperature to 90°C . Assume the convective heat transfer coefficient to $90 \text{ W/m}^\circ \text{C}$. [Ans 123.753°C]

Solution

Given $h = 90 \text{ W/m}^\circ \text{C}$, $k = 370 \text{ W/m}^\circ \text{C}$, $c = 380 \text{ J/kg}^\circ \text{C}$ (specific heat)

$A = .4 \times .4 \times 2 = .32 \text{ m}^2$, vol of plate $= .4 \times .4 \times .005 = .0008 \text{ m}^3$

$L_c = \text{Characteristic length} = \frac{V}{A} = \frac{.0008}{.32} = .0025 \text{ m}$

Biot number $= \frac{hL_c}{k} = \frac{90 \times .0025}{370} = .000608 = 6 \times 10^{-4}$

Since B_i is $< .1$ lumped capacitance method may be applied for the solution (Newtonian heating cooling).

Temperature distribution is given by $\frac{t - t_a}{t_i - t_a} = \exp \left[\frac{-hA}{\rho VC} \tau \right]$

$$\frac{90 - 30}{250 - 30} = e \left[\frac{-90 \times .32}{9000 \times .0008 \times 380} \tau \right]$$

$$.2727 = e \left[\frac{-28.8}{2736} \tau \right]$$

$$.2727 = e^{-.010526 \tau}$$

$$e^{.0105} = \frac{1}{.2727} = 3.667$$

Taking log both side

$$.010526 T = \ln 3.667 = 1.299$$

T = Time required to reach 90° c

$$= \frac{1.299}{.010526} = 123.4 \text{ sec. Ans}$$

Q 2.

An aluminium alloy plate .2 m² surface area both side 4mm thick at 200 c is suddenly quenched in liquid oxygen at

−183 c . Determine time required for the plate to reach a temperature of −70 c. Assuming h=500w/m² cC_p =890 J/kg c

$$\rho = 2700 \text{ KG/m}^3 \quad [23.45 \text{ s}]$$

Solution

Given As= surface area of both side = .2m²

$$\text{Volume of plate} = V = .2 \times .004 = .0008 \text{ m}^3$$

$$\text{Characteristic length} = \frac{V}{As} = \frac{.0008}{.2} = .004 \text{ m}$$

$$\text{Biot no} = B_i = \frac{hL_c}{K} = \frac{500 \times .004}{214} = .009$$

Since B_i is <.1 so hence lumps capacitance method can be applied.

Temperature distribution is given by

$$\frac{t-t_a}{t_i-t_a} = e\left(\frac{-hA}{\rho VC} T\right) \text{ where } T \text{ is time to reach } -70^\circ \text{ c}$$

$$\frac{-70-(-183)}{200-(-183)} = e\left(\frac{-500 \times .2}{2700 \times .0008 \times 890} T\right)$$

$$.295 = e^{\left[\frac{-100}{1922.4} T\right]}$$

$$.295 = e^{-.0527 T} = e^{\frac{1}{.052} T}$$

$$e^{.052 T} = \frac{1}{.295} = 3.3898$$

Taking log both side

$$.052 T = \ln 3.3898 = 1.22$$

$$T = \frac{1.22}{.052} = 23.56 \text{ sec. Ans}$$

Q 3

A sphere of 200mm diameter made of cast iron initially at uniform temperature of 400° c is quenched into oil. The oil bath temperature is 40 c. If the temperature of sphere is 100 c after 5 minutes , find the heat transfer coefficient on the surface of sphere.

$$C_p = \text{cast iron} = .32 \text{ kJ/kg c} = .32 \times 1000 \text{ J/kg c.}$$

$$\rho = 7000 \text{ kg/m}^3 \text{ diameter of sphere} = 200 \text{ mm} = .2 \text{ m}$$

$$t_i = 400 \text{ c } t_a = 40, t = 100 \text{ c } , T = \text{Time to heat in 5 min} = 300 \text{ s}$$

$$\text{Vol} = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi (.1)^3 = .0041 \text{ m}^3$$

$$A_s = 4\pi R^2 = 4\pi (.1)^2 = .1256 \text{ m}^2$$

i) to find heat transfer coefficient = h

$$\text{characteristic length} = L_c = \frac{V}{A_s} = \frac{.0041}{.1256} = .033 \text{ m}$$

$$\text{Boit number} = B_i = \frac{h L_c}{k}$$

Since h has to be calculated so assume internal resistance to be negligible .using (eq 4.4 of original book)

$$\frac{t - t_a}{t_i - t_a} = e \left[-\frac{h A s}{\rho V C} T \right] = e \left[\frac{-h \times 1256 \times 300}{7000 \times 0.0041 \times 1000 \times 32} \right]$$

$$\frac{100-40}{400-40} = e \left[-\frac{h \times 37.68}{8960} \right]$$

$$\frac{60}{360} = e^{-.0041h} = .166$$

$$\text{Or } \frac{1}{e^{.0041h}} = .166$$

$$e^{.0041h} = \frac{1}{.166} = 6.024$$

Taking log both side

$$\log e^{.0041h} = \ln 6.024 = 1.7957$$

$$.0041h = 1.7957$$

$$h = \frac{1.7957}{.0041} = 437.97 \text{ W/m}^2 \text{ Ans.}$$

Q 4

An average convective heat transfer coefficient for flow of 100 c air over a flat plate is measured by observing temperature time history of a 30mm thick copper slate ($\rho=9000 \text{ kg/m}^3$), $C=0.38 \text{ KJ}$

$K=370 \text{ W/mc}$ exposed to 100 c air. In one test the initial temperature of the plate was 210 and 5 minutes the temperature decreased by 40 .Find the heat transfer coefficient for the case. Neglect internal thermal resistance. [Ans 77.24 w/m^2]

Solution

$$\text{Given } t_a = 100^\circ \text{c} \quad L = 30 \text{ mm} = .03 \text{ m}$$

$$\rho = 9000 \text{ kg/m}^3, \quad C = .38 \times 1000 \text{ j/kg}^\circ \text{c}$$

$$t_i = 210 \text{ c}, \quad t = 210 - 40 = 170$$

T =time to decrease temp from 210 c to 40 c

$$=5 \text{ min} = 300 \text{ sec.}$$

$$\text{Characteristic length} = L_c = \frac{L}{2} = \frac{.03}{2} = .015 \text{ m}$$

(Applying eq 4.4)

$$\frac{t - t_a}{t_i - t_a} = e^{\left[\frac{-h A_s}{\rho V C} T \right]} \text{-----1.}$$

For that we have

$$\begin{aligned} \frac{h A_s}{\rho V C} &= \frac{h}{\rho C \frac{V}{A_s}} = \frac{h}{9000 \times .38 \times 1000 \times L_c} = \frac{h}{9000 \times .38 \times 1000 \times .015} \\ &= \frac{h}{90 \times 10^5 \times .38 \times .015} = \frac{h \times 10^{-5}}{.513} = 1.94 \times 10^{-5} h \text{ ----- 2} \end{aligned}$$

Putting the value of 2 above in equation 1 we get as follow

$$\frac{170 - 100}{210 - 100} = e^{\left[-h \times 1.94 \times 10^{-5} \times 300 \right]}$$

$$\frac{70}{110} = .6363 = e^{-.00582 h} = \frac{1}{e^{.00582 h}}$$

$$e^{.00582 h} = \frac{1}{.6363} = 1.5715$$

Taking log both side

$$\text{Log } e^{.00582 h} = \ln 1.5715$$

$$.00582 h = .452030$$

$$\text{So } h = \frac{.452030}{.00582} = 77.66 \text{ w/m}^2 \text{ Ans.}$$

Q 5

A cylindrical steel ingot 150mm in diameter and 400mm long passes through a heat treatment furnace which is 6m in length , The ingot must reach a temperature of 850 c before it comes out of the furnace . The furnace gas is at 1280°C and ingot initial temperature is 100 c. , what is maximum speed with which the ingot should move in the furnace to attain the required temperature ? The combined radiation and convective surface heat transfer coefficient is 100 w/m². Take k (steel)=45w/m° c and α (thermal diffusivity)=0.46x 10⁻⁵m²/s, [1.619x10⁻³m/s]

Solution

Given k=45w/m c, h=100w/m²,

diffusivity (α) = .46x10⁻⁵m²/s, furnace length =6m.

D=diameter of ingot =150mm=.15m

L=Length of ingot =400 mm=.4m

t = 850 c of ingot temp. inside furnace

t_i = Furnace gas tempt. = 1280 c

t_a = 100 c

$$\text{Characteristic length} = L_C = \frac{V}{As} = \frac{\frac{\pi D^2 L}{4}}{\pi D L + \frac{\pi D^2 \times 2}{4}} = \frac{DL}{4L + 2D}$$

$$= \frac{.15 \times .4}{4 \times .4 + 2 \times .15} = \frac{.06}{1.6 + .3} = \frac{.06}{1.9} = .031 \text{ m}$$

$$\text{Boit no} = B_i = \frac{h L_C}{K} = \frac{100 \times .031}{45} = \frac{3.1}{45} = .068$$

Since B_i < .1 internal thermal Resistance of the ingot for conduction heat can be neglected.

Time vs temperature relation is given by

$$\frac{t-t_a}{t_i-t_a} = e\left[-\frac{hAs}{\rho VC} T\right]$$

$$\text{Now } \frac{hAs}{\rho VC} T = \frac{K}{K} \frac{hAs}{\rho VC} T = \frac{K}{\rho C} \frac{h}{k} \frac{As}{V} T = \alpha X \frac{h}{k} X \frac{A}{V} T$$

$$= .46 \times 10^{-5} X \frac{100}{45} X \frac{1}{.0368} = \frac{46}{45 \times .0368 \times 10^5}$$

$$= 1.022/3680 = .000277.$$

$$\frac{750}{1180} = e^{(-.000277T)}$$

$$.6355 = \frac{1}{e^{.000277T}}$$

$$e^{.000277T} = \frac{1}{.6355} = 1.57$$

Taking log

$$\text{Log } e^{.000277T} = \ln 1.57$$

$$.000277T = .4510$$

$$T = \frac{.4510}{.000277} = 1628.15 \text{ sec.}$$

So velocity of ingot passing through the furnace

$$= \frac{\text{furnace length}}{\text{time to cross the length of furnace}}$$

$$= \frac{6m}{1628.15 \text{ s}} = 0.003685 \text{ s} = 3.6 \times 10^{-3} \text{ sec. Ans}$$

Q 6

A hot mild sphere ($k=42.5 \text{ W/m}^\circ\text{C}$) having 12 mm diameter is planned to be cooled by an air flow at 27°C . The convective heat transfer coefficient is $114 \text{ W/m}^2 \text{ }^\circ\text{C}$. Determine following

- time required to cool the sphere from 540°C to 95°C
- instantaneous heat transfer rate 2 minutes after start of cooling ,

iii) total energy transferred from sphere during 1st 2 minutes.

Mild steel property

$$\rho = 7850 \text{ kg/m}^3, C = 475 \text{ J/KG } ^\circ\text{C}, \alpha = .043 \text{ m}^2/\text{h}$$

$$[\text{Ans } 2.104 \text{ min}, 3,884 \text{ W}, 1475, 7 \text{ J}]$$

Solution

$$\text{Given } R = .006 \text{ m } k = 42.5 \text{ W/m } ^\circ\text{C}, t_a = 27^\circ\text{C } t_i = 540^\circ\text{C}$$

$$t = 95^\circ\text{C}, h = 114 \text{ W/m}^2 ^\circ\text{C}, \rho = 7850 \text{ KG/m}^3$$

$$C = 475 \text{ J/kg } ^\circ\text{C}, \alpha = 0.043 \text{ m}^2/\text{hr}$$

i) time required to cool sphere from 540°C to 95°C

$$L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = .006/3 = .002 \text{ m}$$

$$\text{Biot no } = \frac{h L_c}{k} = \frac{114 \times .002}{42.5} = .00536$$

$$\text{Fo} = \text{Fourier no.} = \frac{\alpha T}{L_c^2} = \frac{.043 \times T}{.002^2} = 10750 T \text{ (T is in hour)}$$

Since $B_i < .1$ we can use lump theory to solve the problem

$$\frac{t - t_a}{t_i - t_a} = e^{-B_i \text{Fo}} = e^{-\frac{h L_c}{k} \frac{\alpha T}{L_c^2}} = e^{-\frac{114 \times .002}{42.5} \frac{.043 T}{.002^2}}$$

$$\frac{95 - 27}{540 - 27} = .1352 = e^{-.00536 \times 10750 T} = e^{-57.62 T}$$

$$\frac{1}{e^{57.62 T}} = .1352 \text{ OR } e^{57.62 T} = \frac{1}{.1352} = 7.396$$

$$\text{Or } \log e^{57.62 T} = \ln 7.396$$

$$\text{Or } 57.62 T = 2.0009$$

$$T = \frac{2.0009}{57.62} = .0347 \text{ hrs} = .0347 \times 3600 \text{ s} = 124.92 \text{ sec}$$

$$= 2.082 \text{ min. Ans.}$$

ii) Instantaneous heat transfer rate 2 minutes after the start of cooling ;--(follow e qu. 4.7a 0)

$$Q_i = -h A_s (t_i - t_a) e^{B_i F_o}$$

$$B_i x F_o = .00536 \times 10705 \times (2 \text{ min} / 60) \text{ to make it in hrs.}$$

$$= .00536 \times 10705 \times .0333$$

$$= 1.918746$$

$$Q_i = -114 \times 4\pi (.006)^2 (540 - 27) e^{-1.918746}$$

$$= -114 \times 4\pi \times .000036 \times 513 \times 14675$$

$$= -3.8825 \text{ W} \quad \text{Ans .}$$

(---SIGN shows heat is given off by sphere)

Iii) Total energy transferred from sphere in 1st 2 minutes ;-

$$(.0333 \text{ h r s})$$

$$Q^I = \rho v c (t_i - t_a) [e^{B_i F_o} - 1]$$

$$= 7850 \times \frac{4}{3} \pi (.006)^2 \times 475 \times (540 - 27) [e^{-1.918746} - 1]$$

$$= 7850 \times 4.188 \times .000000216 \times 475 \times 513 \times [14675 - 1]$$

$$= -7850 \times 4.188 \times .000000216 \times 475 \times 513 \times 85325$$

$$= 1476,44 \text{ J (Jule) Ans}$$

Q 7

The heat transfer coefficient for the flow of 30 c air over a 12.5mm diameter sphere are measured for observing the temperature -time history of a copper ball of the same dimensions. The temperature of copper ball ($\rho = 8930 \text{ kg/m}^3$,

$C=0.375\text{KJ/kg } ^\circ\text{C}$) was measured by two thermocouples , one located at and other near the surface. Both thermos couples registered within the accuracy of the recording instruments the same temperature at the given instant on one test run ,the initial temperature of the v=ball was of 70°C and in 1.15 minutes the temperature decreased by 7°C . Calculate the convective heat transfer coefficient for this case.[Ans $194.5\text{W/m}^2\text{ }^\circ\text{C}$]

Solution

Given

$$t_a=30^\circ\text{C} \quad t_i=70^\circ\text{C} \quad R(\text{sphere})=.00625\text{m}$$

$$C=.375\text{kJ/kg } ^\circ\text{C} =375 \text{ J/kg } ^\circ\text{C} \quad t =70-7=63^\circ\text{C}$$

$$\rho=8930\text{kg/m}^3 \quad T=\text{time}=1.15 \text{ min}=69 \text{ sec.}$$

To find heat transfer coefficient h

$$\text{Boit no} = \frac{hLc}{k}$$

Since h has to be calculated we will assume internal resistance is negligible and B_i is less than ,1.

Using equ.4.4 we have

$$\frac{\theta}{\theta_1} = \frac{t-t_a}{t_i-t_a} = e \left[-\frac{hAs}{\rho VC} T \right]$$

$$h = \frac{\rho VC}{As T} \ln \left[\frac{t_i-t_a}{t-t_a} \right] = \left[\frac{\frac{4}{3}\pi R^3}{4\pi R^2} \right] \frac{\rho C}{T} \ln \left[\frac{70-30}{63-30} \right]$$

$$\begin{aligned} h &= \frac{R}{3} \times \frac{\rho C}{T} \ln \frac{40}{33} = \frac{.00625}{3} \times \frac{8930 \times 375}{69} \ln 1.21 \\ &= \frac{20929.68}{207} \times 1.922 = 101.1 \times 1.922 \\ &= 19.43 \text{ W/m}^2\text{ }^\circ\text{C} \text{ Ans.} \end{aligned}$$

Q 8

The temperature of an air stream flowing with a velocity of 3m/s is measured by a copper constantan thermocouple which may be approximated as sphere of 3mm in diameter .Initially the junction and air are at a temperature of 25° c . The air temperature suddenly changed to and maintained at 200 °c.

Determine i) time required for the thermocouple to indicate a temp of 150 °c.

ii)thermal time constant and temperature indicated by thermocouple at that instant.

Thermocouple junction properties are

$\rho = 8685 \text{ kg/m}^3$. specific heat = $c = 383 \text{ j/kg}^\circ\text{c}$

$K = 29 \text{ w/m}^\circ\text{c}$ convective co efficient $= h = 150 \text{ w/m}^2\text{c}$

[Ans 13.89s, 11.09s, 155.63° c]

Solution

Given $R = 1.5 \text{ mm} = .0015 \text{ m}$

$t_i = 25^\circ\text{c}$ $t_a = 200^\circ\text{c}$

$t = 150^\circ\text{c}$, $\rho = 8685 \text{ kg/m}^3$

$C = 383 \text{ j/kg}^\circ\text{c}$, $k = 29 \text{ w/m}^\circ$

$h = 150 \text{ w/m}^2$, air stream velocity = 3m/s

i) T is time required to indicate temperature of 150 °c

$$\begin{aligned} \text{Lc} = \text{characteristic length} &= \frac{V}{A_s} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} \\ &= \frac{.0015}{3} = .0005 \text{ m} \end{aligned}$$

$$\text{Thermal diffusivity } \alpha = \frac{k}{\rho C} = \frac{29}{8685 \times 383} = .000008718 \text{ m}^2/\text{s}$$

$$F_o = \text{FOURIER NO} = \frac{\alpha T}{L^2 C} = \frac{.000008718 T}{.0005^2} = 34.872 T$$

$$B_i = \frac{h L C}{k} = \frac{150 \times .0005}{29} = .002586$$

As Boit no is < than .1 lumped capacitation method may be applicable ;--

Temperature distribution is given by;;

$$\frac{\theta}{\theta_1} = \frac{t - t_a}{t_i - t_a} = e^{-B_i F_o}$$

$$\frac{150 - 200}{25 - 200} = e^{-.002586 \times 34.872 T}$$

$$\frac{-50}{-175} = e^{-.09017 T}$$

$$.2857 = e^{-.09017 T}$$

$$\frac{1}{e^{.09017 T}} = .2857 \text{ OR, } e^{.09017 T} = \frac{1}{.2857} = 3.5$$

Taking log

$$.09017 T = \ln 3.5 = 1.2527$$

$$T = \frac{1.2527}{.09017} = 13.66 \text{ S Ans.}$$

Thermocouple requires 13.66s to indicate a temperature of 150 c .
The actual time will be however will be greater because of radiation from the probe and conduction along the thermocouple lead wires,

ii) Time constant T is defined as time required to yield a value of unity for the exponent term in the transient relation.

$$B_i F_o = 1$$

$$.002586 \times 34.872 T = 1$$

$$T^* = \frac{1}{.002586 \times 34.872} = \frac{1}{.09017} = 11.09 \text{ S Ans}$$

iii) at 11.09 s the temperature indicated by the thermocouple is

$$\frac{t-t_a}{t_i-t_a} = e^{-1}$$

$$\frac{t-200}{25-200} = e^{-1}$$

$$\therefore t = e^{-1}x - 175 + 200 = \frac{1}{e} x - 175 + 200$$

$$\text{Or, } = \frac{1}{2.718} x 175 + 200 \quad (\text{value of } e = 2.718.)$$

$$= -64.4 + 200$$

$$= 135.6^\circ \text{C Ans.}$$

As the thermal time constant is 11.09s and time required to effect the temperature variation is 3 secs which is less than thermal time constant hence temperature recovered by the thermocouple may not be reliable.

Q9

A 50mm thick a large steel plate ($k=42.5 \text{ W/m}^\circ\text{C}$, $\alpha = .043 \text{ m}^2/\text{h}$)

Initially at 425°C is suddenly exposed on both sides to an environment with convection heat transfer coefficient $285 \text{ W/m}^2\text{C}$

And temperature 65°C . Determine the center line temperature and temperature inside the plate 12.5mm from the midpoint after 3 minutes.

[no Ans given]

Solution

$$\text{Given } 2L=50\text{mm}=.06\text{m}, k=42.5 \text{ W/m}^2\text{C}$$

$$\alpha = .043 \text{ m}^2/\text{h} \quad t_i = 425^\circ\text{C}$$

$$h=285 \text{ W/m}^2\text{C} \quad t_a=65^\circ\text{C}, \text{ time } = T=3\text{min}=.05\text{hrs}$$

i) temperature at mid point (central line)

$$\text{characteristic length} = L_c = \frac{2L}{2} = L = 50/2 = .025 \text{ m.}$$

$$\text{Fourier no} = F_o = \frac{\alpha T}{L_c^2} = \frac{.043 \times .05}{.025^2} = \frac{.00215}{.000625} = 3.44$$

$$\text{Boit no} = \frac{hL_c}{k} = \frac{285 \times .025}{42.5} = \frac{7.125}{42.5} = .167$$

Since Boit no is greater than .1 internal temperature gradient are not small so internal resistance cannot be neglected .So plate cannot considered as lumped system , Further Boit is less than 100 Heisler chart can be used to find the solution.

Corresponding to the following parameter , values from Hiesler chart (fig 4.8)graph we have

$$F_o = 3.46 \quad \frac{1}{B_I} = \frac{1}{.167} = 5.988 \quad \frac{x}{L} = \text{Mid point}$$

i) from Heisler Chart t_o = center line temperature

$$\frac{t_o - 65}{425 - 65} = .6 \text{ from Heisler chart}$$

$$t_o = .6 \times 360 + 65 = 216 + 65 = 281^\circ\text{C Ans}$$

ii) temperature inside plate 12.5 mm from midpoint after 3 minutes time ;--

$$\frac{x}{L} = \frac{12.5}{25} = .5$$

$$\text{Corresponding to } \frac{x}{L} = .5 \quad \text{and } \frac{1}{B_I} = 5.988 = 6 \text{ (say)}$$

From graph fig (4.11) we have

$$\frac{t - t_a}{t_i - t_a} = .95$$

$$\frac{t - 65}{281 - 65} = .95 \text{ so } t = .95 (281 - 65) + 65 = 266.95^\circ\text{C Ans.}$$

Q 10

A cylindrical bar ($k=17.5 \text{ w/m } ^\circ\text{C}$, $\alpha=.0185 \text{ m}^2/\text{hr}$) of radius 75mm comes out of oven at 811°C through out and is cooled by quenching it in a large bath of 38°C coolant. The surface coefficient of heat transfer between the bar surface and the coolant is $175 \text{ w/m}^2\text{C}$.

Determine i) the time taken by the shaft center to reach 116°C

ii) surface temperature of shaft when its center temperature is 116°C . Also calculate the temperature gradient at the outside surface at the same instant of time,

[Ans i) 2102 s ii) 92.6°C , 546°C]

Solution

Given $R=75 \text{ mm}=.075\text{m}$, $t_1=815^\circ\text{C}$

$t_a=38^\circ\text{C}$ $h=175 \text{ w/m}^2\text{C}$, $t=120^\circ\text{C}$

$K=17.5 \text{ w/m } ^\circ\text{C}$, $\alpha=.0185 \text{ m}^2/\text{h}$

i) time taken by the shaft center to reach 120°C ;---

$$\text{characteristic length} = L_C = \frac{\pi R^2 L}{2\pi RL} = \frac{V}{A} = \frac{R}{2} = .0375\text{m}$$

$$\text{Boit no} = \frac{hL_C}{k} = \frac{175 \times .0375}{17.5} = .3748$$

As $Bi < 100$ hence Heisler chart can be used in this case.

Parametric value of cylinder bar are

$$\frac{1}{Bi} = \frac{1}{.3748} = 2.668$$

$$\frac{t-t_a}{t_i-t_a} = \frac{120-38}{815-38} = \frac{82}{777} = .1$$

$$\frac{r}{R} = 0 \text{ (Center of bar)}$$

$$F_O = \frac{\alpha T}{R^2} = 3.2$$

Corresponding to above value from the chart of infinite cylinder (fig 4.9) of Heisler chart we read Fourier no $Fo = 3.2$

$$\text{So } T = \frac{3.2R^2}{\alpha} = \frac{3.2 \times (.0378)^2}{.0185} = .2421 \text{ hr} = .2421 \times 3600 = 871.5 \text{ s.}$$

ii) a) Temperature at the surface t_s

corresponding to $r/R = 1$

$$\frac{1}{Bi} = 2.668$$

From the Heisler Chart for an infinite cylinder we read

$$\frac{t - 38}{116 - 38} = .85$$

$$t - 38 = .85 \times 78 = 66.3$$

$$t = 66.3 + 38 = 104.3 \text{ } ^\circ\text{C} \text{ (temperature at surface of bar)}$$

ii) b) Temperature gradient at the outer surface $\frac{dt}{dr}$

It is determined by boundary condition $r = R$ at which rate of energy conducted to fluid solid surface from within the solid = rate at which energy is conducted away into the fluid.

$$K A_s \frac{dt}{dr} = h A_s (t_s - t_a)$$

$$\text{Or } k \frac{dt}{dr} = h (t_s - t_a)$$

$$\frac{dt}{dr} = \frac{h}{k} (t_s - t_a)$$

$$\frac{dt}{dr} = \frac{175}{17.5} (104.3 - 38)$$

$$= 10 \times 66.3$$

$$= 663 \text{ } ^\circ\text{C} \text{ Ans.}$$

Q 13

A 50mm thick mild steel plate($\alpha=1.25 \times 10^{-5} m^2/s$) initially at a temperature of 40°C . It is suddenly exposed one side to a fluid which causes the surface temperature to increase to and remain at 90°C .

Determine

i) The maximum time that the slab be treated as a semi infinite body

ii) temperature at the center of the slab one minute after the change in surface temperature. [Ans i) 200s ii) 66°C]

solution

given

$$L = \text{Thickness} = 50 \text{ mm} = .05 \text{ m}$$

$$\therefore \alpha = 1.25 \times 10^{-5} m^2/s$$

$$t_i = 40^\circ \text{C} \quad t_a = 90^\circ \text{C} \quad T = 1 \text{ min.} = 60 \text{ s}$$

i) maximum time that the slab be treated as semi infinite body ; T_{max} ;--

general criteria for infinite solution to apply to a body of finite thickness subjected to one dimensional heat transfer is

$$\frac{L}{2\sqrt{\alpha T}} = .05 \quad \text{OR} \quad \frac{L^2}{4\alpha T} = 0.25 \quad (\text{squaring both side})$$

$$\begin{aligned} \text{So } T_{\text{max}} &= \frac{L^2}{4\alpha \times .25} = \frac{.05^2}{4 \times 1.25 \times 10^{-5} \times .25} = \frac{.0025 \times 10^5}{1.25} \\ &= \frac{250}{1.25} = 200 \text{ S} \end{aligned}$$

ii) temperature at the center of the slab 1min after the change in surface $t^\circ \text{C}$

at the center of slab $x = \frac{.05}{2} = .025 \text{ m}$

$$T = 1 \text{ min} = 60 \text{ s}$$

(Equation 4.14)

$$\frac{t - t_a}{t_i - t_a} = \text{erf}\left(\text{Gaussian error function}\right) \left[\frac{x}{2\sqrt{\alpha T}} \right] = \text{erf}(z)$$

$$\text{Where } z = \frac{x}{2\sqrt{\alpha T}}$$

$$= \text{erf}\left[\frac{.025}{2\sqrt{1.25 \times 10^{-5} \times 60}} \right] = \text{erf}.457 = 0.47 \text{ (erf table 4.1 page 319)}$$

$$\text{So } t = t_a + \text{erf}.457 (t_i - t_a)$$

$$= 90 + .47(40 - 90)$$

$$= 90 - 23.5$$

$$= 66.5^\circ \text{C Ans.}$$

Q 14

A uniform temperature of a thick concrete wall ($1.58 \times 10^{-3} \text{ m}^2/\text{h}$ $K = 0.937 \text{ W/m}$) jet engine test cell is 21°C . Surface temperature of the wall suddenly rises to 315°C , when combination of exhaust gases from the turbojet and spray of cooling water occurs. Determine

- Temperature at a point 75mm from the surface after 7.5 hrs.
- The instantaneous heat flow rate at the specified plane and at the surface itself at the instant mentioned at i).

use the solution for semi infinite solid. [i] 206°C [ii] -1265.6 W/m^2 , -- -- -1425 W/m^2]

solution

$$\text{given } t_i = 21^\circ \text{C} \quad t_a = 315^\circ \text{C}, \quad \alpha = 1.58 \times 10^{-3}$$

$$k = .937 \text{ W/m} \quad T = 7.5 \text{ hrs} \quad x = .075 \text{ m}$$

i) temperature at a point .075 from the surface after 7.5h

$$\frac{t - t_a}{t_i - t_a} = \text{erf} \left[\frac{x}{2\sqrt{\alpha T}} \right]$$

$$\therefore t = \text{erf} \left\{ \frac{x}{2\sqrt{\alpha T}} \right\} (t_i - t_a) + t_a$$

$$\text{When } \text{erf} \left[\frac{x}{2\sqrt{\alpha T}} \right] = \text{erf} \left[\frac{.075}{2\sqrt{1.58 \times 10^{-3} \times 7.5}} \right]$$

$$= \text{erf} \left[\frac{.075}{2\sqrt{.01185}} \right]$$

$$= \text{erf} \left[\frac{.075}{.2176} \right] = \text{erf} [.334]$$

$$= .36 \quad (\text{erf table 4.1 p 319 original book})$$

$$\text{So } t = .36 (t_i - t_a) + t_a$$

$$= .36 (21 - 315) + 315$$

$$= .36 (-294) + 315$$

$$= -105.84 + 315$$

$$= 209.16^\circ \text{C} \quad \text{Ans.}$$

ii) a) instantaneous heat flow rate (Q_i) at specified plane

$$Q_i = -kA (t_i - t_a) \frac{\frac{-x}{2\sqrt{\alpha T}}}{\sqrt{\pi \alpha T}}$$

$$Q_i = -.937 \times 1 \times (21 - 315) \left[\frac{\frac{-.075^2}{e^{4 \times 1.58 \times 10^{-3} \times 7.5}}}{\sqrt{7.5 \times \pi \times 1.58 \times 10^{-3}}} \right]$$

$$= .937 \times 294 \frac{e^{-.11968}}{.1929} = .937 \times 294 \frac{e^{-.11968}}{.1929} = .937 \times 294 \times \frac{.88720}{.1929}$$

$$= .937 \times 294 \times 4.599$$

$$= 275.478 \times 4.599$$

$$=1266.9 \text{ w/m}^2 \text{ Ans}$$

ii)b) heat flow rate at surface itself

$$\begin{aligned} Q_{\text{surface}} &= \frac{kA(t_i - t_a)}{\sqrt{\pi L T}} = \left[\frac{.937 \times 1 \times (21 - 315)}{\sqrt{\pi \times 1.58 \times 10^{-3} \times 7.5}} \right] \\ &= \frac{.937 \times 294}{\sqrt{\frac{37.22}{1000}}} = \frac{-275.47}{\sqrt{.0372}} = \frac{-275.47}{.1928} \\ &= -1428.78 \text{ w/m}^2 \text{ Ans} \end{aligned}$$

Q 15

The initial uniform temperature of a large mass of material ($\alpha = 0.41 \text{ m}^2/\text{hr}$) is 120°C . The surface of the material is suddenly exposed to and held permanently at 5°C . Calculate the time required for reaching 350°C/m . [Ans; 206 s]

Solution

$$\text{Given } t_i = 120^\circ\text{C} \quad t_a = 5^\circ\text{C} \quad \alpha = 0.41 \text{ m}^2/\text{h}$$

$$\left(\frac{dt}{dx} \right)_{x=0} = \text{temp gradient at surface} = 350^\circ\text{C}$$

Time required T ;--

Heat flow at surface ($x=0$) is given by

$$Q_{\text{sur}} = \frac{-kA(t_i - t_a)}{\sqrt{\pi L T}}$$

$$-KA \left(\frac{dt}{dx} \right)_{x=0} = \frac{-kA(t_i - t_a)}{\sqrt{\pi \alpha T}}$$

$$\left(\frac{dt}{dx} \right)_{x=0} = \frac{t_i - t_a}{\sqrt{T \pi \alpha}}$$

Putting the values we get

$$350 = \frac{120 - 5}{\sqrt{\pi \times .41 \times T}}$$

$$\pi \times .41 \times T = \left(\frac{115}{350}\right)^2 = (.3285)^2 = .1079$$

$$T = \frac{.1079}{.41 \times \pi} = \frac{.1079}{1.288} = .083 \text{ Hrs} = .083 \times 3600 \text{ s}$$

$$= 298.8 \text{ s Ans.}$$

Q 16

A motor car of mass 1500kg travelling at 80km/hr is brought to rest within a period of 5 s when breaks are applied .The breaking system consists of 4 breaks with each break band of 350 cm^2 area ; these press against steel drum of equivalent area. The brake lining and drum surface ($k=55\text{w/m c}, \alpha = 1.24 \times 10^{-5} \text{ m}^2/\text{s}$)

Are at the same temperature and heat generated during the stoppage action dissipated flowing across drum. If the drum surface is treated as semi infinite plate , calculate maximum temperature rise . [134.11 s Solution

Given $m=1500\text{kg}$, $V= \text{velocity} = 80\text{km/hr}$

$T = 5 \text{ s}$, $A = \text{area of brake band} = 4(350 \times 10^{-4}) = .140 \text{ m}^2$

$K = 55\text{w/m c}$ $\alpha = 1.24 \times 10^{-5} \text{ m}^2/\text{s}$

Minimum temp rise

After applying brakes when comes to rest its kinetic energy is converted in to heat energy which is dissipated through drums .

Kinetic energy of motor car $= \frac{1}{2} m V^2 = \frac{1}{2} \times 1500 \times \left(\frac{80 \times 1000}{60 \times 60}\right)^2$

$= 370297.5 \text{ j in 5 sec}$

$= 37. \times 10^4 \text{ J in 5 sec.}$

$$\text{Heat flow rate} = \frac{37 \times 10^4}{5} = .74 \times 10^5 \text{ J/S}$$

This equals the instantaneous heat flow rate at the surface

$$(x=0) \text{ which is given by } Q_{\text{sur}} = \frac{-kA(t_i - t_a)}{\sqrt{\pi \lambda \alpha \tau}} = .74 \times 10^5 \text{ J/S or Watt.}$$

This equals the instantaneous heat flow rate at the surface ($x=0$)

Which is given by ;

$$Q_{\text{surface}} = \frac{-kA(t_i - t_a)}{\sqrt{\pi \lambda \alpha \tau}} = .74 \times 10^5$$

$$\text{Or, } \frac{-55 \times 1.40 \times (t_i - t_a)}{\sqrt{\pi \times 1.24 \times 10^{-5} \times 5}} = .74 \times 10^5$$

$$\text{Or, --- } (t_i - t_a) = \frac{.74 \times 10^5 \sqrt{\pi \times 1.24 \times 10^{-5} \times 5}}{55 \times 1.40}$$

$$\text{Or, } (t_a - t_i) = \frac{.74 \times 1395}{7.7} = \frac{1032.3}{7.7}$$

$$(t_i - t_a) = 134.06^\circ \text{C} = \text{Maximum temperature rise. Ans.}$$

Q 17

During periodic heating and cooling of a thick brick wall, the wall temperature varies sinusoidally . The surface temperature ranges from 25°C to 75°C during period of 24 hours.

Determine the time lag of temperature wave corresponding to a point located at 250mm from the wall surface . The properties of wall material are $\rho = 1620 \text{ kg/m}^3$, $k = 0.62 \text{ W/m}^\circ\text{C}$, $C = 450 \text{ J/kg}^\circ\text{C}$.

Solution

[Ans 6.24 h]

Given

$$X=250\text{mm}=0.25\text{m} \quad , \quad \rho =1620\text{kg/m}^3$$

$$K=.62\text{w/m c} \quad , \quad C=450 \text{ J/KG } ^\circ\text{C} \quad ,$$

$$\therefore \eta = \text{frequency of temperature wave} = \frac{1}{24} = 0.04167/\text{hr}$$

Time lag $= \Delta T$;--

$$\Delta T = \frac{X}{2} \sqrt{\frac{1}{\alpha \pi n}}$$

$$\text{When } \alpha = \frac{k}{\rho C} = \frac{.62}{1620 \times 450} = \frac{.62}{729000} = 8.5 \times 10^{-7} \text{ m}^2/\text{s}$$

$$= .00306 \text{ m}^2/\text{s}$$

$$= \frac{8.5}{10000000} \times 3600 \text{ m}^2/\text{hr}$$

$$= .00306 \text{ m}^2/\text{hr}$$

$$\Delta T = \frac{x}{2} \sqrt{\frac{1}{\alpha \pi n}} = \frac{.25}{2} \sqrt{\frac{1}{.00306 \times \pi \times 0.04167}}$$

$$= \frac{.25}{2} \sqrt{\frac{1}{.0004}}$$

$$= \frac{.25}{2} \times \sqrt{2500}$$

$$= \frac{.25}{2} \times 50$$

$$= 6.25 \text{ hr. Ans.}$$

Q 18

A single cylinder ($\alpha = 0.042 \text{ m}^2/\text{h}$ for cylindrical material) 2stroke IC engine operates at 1500 rpm. Calculate the depth where the temperature wave due to variation of cylinder temperatures is damped to 1% of its surface value.

$$[1.775 \text{ mm}]$$

Solution

Given $\alpha = 0.042 \text{ m}^2/\text{hr}$, $n = \text{revolution} = 1500 \text{ rpm} \times 60$

$$\theta_{x.a} = 1\% \quad , \quad = 90000[\text{hr}]$$

The amplitude of temperature excess at any depth x is given by

$$\theta_{x.a} = \theta_{s.a} e^{-x \sqrt{\frac{\pi n}{\alpha}}} \quad [\text{equation-4.23}]$$

$$\frac{\theta_{x.a}}{\theta_{s.a}} = e^{-x \sqrt{\frac{\pi \times 90000}{0.042}}}$$

$$\frac{1}{100} = e^{-x \sqrt{\frac{282743.33}{0.042}}} = e^{-x \sqrt{6731984}}$$

$$\frac{1}{100} = e^{-2594 x} = \frac{1}{e^{2594}}$$

$$\log e^{2594} = \ln 100$$

$$2594x = 4.6$$

$$X = \frac{4.6}{2594} = .001773 \text{ m} = 1.773 \text{ mm} = \text{amplitude, Ans.}$$

Q 19

The temperature distribution across a large concrete slab ($k = 1.2 \text{ W/m}^\circ\text{C}$, $\alpha = 1.77 \times 10^{-3} \text{ m}^2/\text{h}$) 500mm thick heated from one side as measured by thermocouple approximate to the relation

.. $t = 60 - 50x + 12x^2 + 20x^3 - 15x^4$ where t is in $^\circ\text{C}$ and x is in meter.

Considering an area of 5 m^2 , compute

- i) the heat entering and leaving the slab in unit time .
- ii) the heat energy stored in unit time .
- iii) rate of temperature change at both side of the slab and
- iv) the point where the rate of heating or cooling is maximum.

[Ans ; 300w,183w (ii)117w

(iii)42.48x 10³ °c/h,69.03x10⁻³ °c/h(iv)0.33 m]

Solution

Given $A=5m^2$, $x=500mm=.5m$, $k=1.2$ w/m c,

$\alpha = 1.77 \times 10^{-3} m^2 h,$

$,t_i=60-50x+12x^2+20x^3-15x^4$

Temperature distribution in polynomial;--

$$\frac{dt}{dx} = -50 + 24x + 60x^2 - 60x^3$$

i) heat entering and leaving the slab ;

heat entering the slab

$$Q_{in} = -K A \left(\frac{dt}{dx} \right)_{x=0} = -1.2 \times 5 \times (-50 + 24x + 60x^2 - 60x^3)$$

$$= -1.2 \times 5 \times (-50) = 300 \text{ W Ans.}$$

$$Q_{leaving} \text{ the slab} = -k A \left(\frac{dt}{dx} \right)_{x=.5}$$

$$= -1.2 \times 5 \{ -50 + 12 + 15 - 7.5 \} \text{ (putting } x=.5)$$

$$= -6 \times (30.5) = 183 \text{ w Ans.}$$

ii) heat energy stored in unit time

$$= Q_{in} - Q_{out}$$

$$= 300 - 183$$

$$= 117 \text{ W Ans.}$$

iii) rate of temperature change at both side of plate

$$\frac{dt}{dT} = \alpha \times \frac{d^2t}{dx^2} = \alpha (24 + 120x - 180x^2)$$

$$\left(\frac{dt}{dT} \right)_{x=0} = 1.77 \times 10^{-3} (24 + 120 \times 0 - 180 \times 0)$$

$$=1.77 \times 10^{-3} \times 24$$

$$= 42.48 \times 10^{-3} \text{ } ^\circ \text{C} \quad \text{Ans.}$$

$$\left(\frac{dt}{dT}\right)_{x=.5} = 1.77 \times 10^{-3} \{(24 + 120 \times .5 - 180 \times (.5)^2)\}$$

$$= 1.77 \times 10^{-3} (24 + 60 - 45)$$

$$= 1.77 \times 10^{-3} \times 39$$

$$= 69.03 \times 10^{-3} \text{ } ^\circ \text{C} \quad \text{Ans}$$

iv) the point where the rate of heating or cooling is maximum ;---

for that

$$\frac{d}{dx} \left(\frac{dt}{dT} \right) = 0$$

$$\text{Or,} \quad \frac{d}{dx} \left[\frac{d^2 t}{dx^2} \right] = 0$$

$$\text{Or,} \quad \frac{d^3 t}{dx^3} = 0$$

$$\text{Or,} \quad \frac{d}{dx} \left(\frac{d^2 t}{dx^2} \right) = \frac{d}{dx} (48 + 120x - 180x^2) = 0$$

$$= (120 - 360x) = 0$$

$$X = \frac{120}{360} = \frac{1}{3} = 0.33 \text{ m} \quad \text{Ans}$$

Chapter 5

Introduction to Hydrodynamics

There is no unsolved example to solve in this chapter .

Chapter 6

Dimensional Analysis

There is no unsolved example to solve in this chapter.

Chapter 7

Forced Convection LAMINAR FLOW

Q 1

Show that for velocity distribution

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

The ratio of $\delta/\delta^* = 3$ where (δ^*) is displacement thickness.

Solution

$$\begin{aligned}\delta^* &= \int_0^\delta \left(1 - \frac{u}{U}\right) dy \\ &= \int_0^\delta \left[1 - \left\{2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right\}\right] dy \\ &= \int_0^\delta dy - 2 \int_0^\delta \left(\frac{y}{\delta}\right) dy + \int_0^\delta \left(\frac{y}{\delta}\right)^2 dy \\ &= \left[y\right]_0^\delta - \frac{2}{\delta} \left[\frac{y^2}{2}\right]_0^\delta + \frac{1}{\delta^2} \left[\frac{y^3}{3}\right]_0^\delta \\ &= \delta - \frac{2}{\delta} \times \frac{\delta^2}{2} + \frac{1}{\delta^2} \times \frac{\delta^3}{3} \\ &= \delta - \delta + \frac{\delta}{3} \\ &= \frac{\delta}{3}\end{aligned}$$

$$\text{So, } \frac{\delta}{\delta^*} = 3 \quad \text{Proved.}$$

Q 2

The velocity distribution in the boundary layer over a high spillway face was found to have the following form ;

$$\frac{u}{U} = \left(\frac{Y}{\delta}\right)^{0.22}$$

Prove that displacement thickness , momentum thickness and energy thickness in term of δ ,the boundary layer thick , can be expressed as ;

$$\frac{\delta^*}{\delta} = 0.18, \quad \frac{\theta}{\delta} = 0.125, \quad \frac{\delta_c}{\delta} = 0.127$$

Solution

$$\text{Momentum thickness } = \theta = \int_0^\delta \left\{ \frac{u}{U} \left(1 - \frac{u}{U} \right) \right\} \delta y$$

$$\text{Putting values we get } \frac{u}{U} = \left(\frac{Y}{\delta}\right)^{0.22}$$

$$\begin{aligned} \theta &= \int_0^\delta \left\{ \frac{Y^{.22}}{\delta^{.22}} \left(1 - \frac{Y^{.22}}{\delta^{.22}} \right) \right\} dy \\ &= \int_0^\delta \left\{ \frac{y^{.22}}{\delta^{.22}} - \frac{y^{.22} x y^{.22}}{\delta^{.22} x \delta^{.22}} \right\} \delta y \\ &= \int_0^\delta \frac{1}{\delta^{.22}} (y^{.22}) dy - \int_0^\delta \frac{1}{\delta^{.44}} (y^{.44}) dy \\ &= \frac{1}{\delta^{.22}} \left[\frac{y^{.22+1}}{.22+1} \right]_0^\delta - \frac{1}{\delta^{.44}} \left[\frac{y^{.44+1}}{.44+1} \right]_0^\delta \\ &= \frac{1}{\delta^{.22}} \times \frac{\delta^{1.22}}{1.22} - \frac{1}{\delta^{.44}} \times \frac{\delta^{1.44}}{1.44} \\ &= \frac{1}{1.22} \delta - \frac{1}{1.44} \delta, = \delta \left(\frac{1}{1.22} - \frac{1}{1.44} \right) = \delta (.82 - .694) \end{aligned}$$

$$\text{So, } \frac{\theta}{\delta} = .126. \text{ on the same we can prove, } \frac{\delta^*}{\delta} = .18, \frac{\delta_e}{\delta} = .127.$$

Q 3

Find the ratio of displacement thickness to momentum thickness and momentum thickness to energy thickness for the velocity distribution in the boundary layer given by

$$\frac{u}{U} = 2 \left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \quad [\text{Ans} = 2.5, 7/11]$$

Solution

$$\begin{aligned} \text{Momentum thickness } \theta &= \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy \\ &= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy \\ &= \int_0^\delta \left(\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + 2\frac{y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy \\ &= \int_0^\delta \left(\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy \\ &= \left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta \\ &= \left[\frac{2\delta^2}{2\delta} - \frac{5\delta^3}{3\delta^2} + \frac{4\delta^4}{4\delta^3} - \frac{\delta^4}{\delta^4}\right] \\ &= \delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5} = \delta\left(\frac{2}{15}\right) \\ \text{So } \frac{\theta}{\delta} &= \frac{2}{15} \quad \text{----- i.} \end{aligned}$$

Now we compare displacement thickness δ_* and δ

$$\begin{aligned} \delta_* &= \int_0^\delta \left(1 - \frac{u}{U}\right) dy \\ &= \int_0^\delta \left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy \\ &= \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy \\ &= \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta \end{aligned}$$

$$\delta_* = \delta - \frac{2\delta^2}{2\delta} + \frac{\delta^3}{32} = \delta - \delta + \frac{\delta}{3}$$

$$= \frac{\delta}{3}$$

$$\frac{\delta_*}{\delta} = \frac{1}{3} \quad \text{----- ii)}$$

$$\frac{\text{displacement thickness}}{\text{momentum thickness}} = \frac{\delta_* - \delta_*}{\theta} \times \frac{\delta}{\theta} = \frac{1}{3} \times \frac{15}{2} = 2.5 \text{ Ans.}$$

Similarly we can show ratio $\frac{\theta}{\delta_e} = \frac{7}{11}$

For that we have to begin as follows ;

$$\delta_e = \int_0^\delta \frac{u}{U} \left[1 - \frac{u^2}{U^2} \right] dy \quad (\text{Equ 7.3})$$

Replacing $u=y$ and $U=\delta$ in above equation we have to go ahead and finally after integration replacing $y=\delta$ to arrive at the result .

Q4

For the velocity profile in laminar layer given as

$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$. Find the thickness of the boundary layer at the end of the plate and the drag force on the side of the plate 1 m long and .8 m wide when in water flowing with velocity of 0.15m/s. Calculate value of coefficient of drag. μ of water .001Ns/m² [Ans; 14.15mm,.0338 N,.00376]

Solution

Velocity distribution

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Length of plate =L =1m

Width of plate =B= .8m

Free stream velocity = $U = 0.15 \text{ m/s}$

;; μ for water = $.001 \text{ Ns/m}^2$

i) Thickness of boundary layer δ

Re_L = Reynold no at 1 m distance from leading edge

$$\frac{\rho U L}{\mu} = \frac{1000 \times 0.15 \times 1}{.001} = 1,50,000$$

Since $Re_L < 5 \times 10^5$ so this is a case of laminar boundary .

So thickness of boundary layer at 1m

$$\begin{aligned} \delta &= \frac{5.48 x}{\sqrt{150000}} = \frac{5.48 x 1}{\sqrt{1,50000}} = \frac{5.48}{387.29} = .014149 \text{m} \\ &= 14.149 \text{ mm Ans} \end{aligned}$$

ii) coefficient of drag

$$\bar{c}_f = \frac{1.46}{c_f} = \frac{1.46}{\sqrt{150000}} = \frac{1.46}{387.29} = .003769 \text{ Ans.}$$

Q 5

Air flows on a smooth flat plate with a velocity of 10 m/s . The velocity profile is in the form of

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

The length of plate is 1.2 m and width is $.9 \text{ m}$. If the laminar boundary layer exists up to a value of $Re = 2 \times 10^5$ and kinematic viscosity of air is 0.15 stokes . Find

i) maximum distance from leading edge up to which laminar boundary layer exists.

ii) maximum thickness of boundary layer.

[Ans 0.3 m ii) 3.67 mm]

Solution

Velocity of air $=u=10\text{m/s}$

$L=1.2\text{ m}$ width $=b=.9\text{m}$

$R_e = 2 \times 10^5$ kinematic velocity $=\nu = 0.15$ stoke.

$$= .15 \times 10^{-4} \text{m}^2/\text{s}$$

i) max distance leading edge to boundary layer $=x$

$$R_e = \frac{uX}{\nu} = \frac{10X}{.15 \times 10^{-4}} = 2 \times 10^5$$

$$\text{So } X = \frac{2 \times 10^5 \times 10^{-4} \times .15}{10} = .3 \text{ m Ans}$$

ii) max. thickness of boundary layer

$$\delta = \frac{5.48 X}{\sqrt{R_e}} = \frac{5.48 X .3}{\sqrt{2 \times 10^5}} = \frac{1.644}{447.21} = .00367 \text{ m}$$

$$= 3.67 \text{ mm Ans.}$$

Q 6

A plate of 500mm and width 250mm has been placed longitudinally in a stream of crude oil which flows with a velocity of 6m/s. If oil has a specific gravity 0.9 and kinematic viscosity of 1 stoke, calculate

i) boundary layer thickness at the middle of plate .

ii) shear stress at the middle of plate.

iii) friction drag on one side of plate.

$$[\text{Ans; i) } 10.5 \text{ mm, ii) } 87.8 \text{ N/m}^2, 12.36 \text{ N}]$$

Solution

Plate

$$L=500\text{mm}=.5\text{m}$$

$$B=250\text{mm}=.25\text{m}$$

Velocity $U=6\text{m/s}$, specific gravity of oil ='.9

V =Kinematic viscosity of oil =1 stoke= $1 \times 10^{-4} \text{m}^2/\text{s}$

i) boundary layer thickness at middle of plate

$$\begin{aligned} \text{Reynold no} = R_e &= \frac{\rho UX}{\mu} = \frac{UX}{V} = \frac{6 \times \frac{L}{2}}{1 \times 10^{-4}} = \frac{6 \times \frac{.5}{2}}{10^{-4}} = \frac{3 \times 10^4}{2} \\ &= 1.5 \times 10^4 = 15000. \end{aligned}$$

Since $R_e < 5 \times 10^5$, so boundary layer is laminar. And so, Blasius solution will be applicable here as

$$\delta = \frac{5x}{\sqrt{R_{ex}}} = \frac{5 \times \frac{.5}{2}}{\sqrt{15000}} = \frac{2.5}{2 \times 122.47} = .010206 \text{ m} = 10.206 \text{ mm Ans.}$$

ii) shear stress at middle of plate

according to Blasius local coefficient of drag

$$= (C_{fx}) = \frac{.664}{\sqrt{R_{ex}}} = \frac{.664}{\sqrt{15000}} = \frac{.664}{122.64} = .0054.$$

By definition

$$C_{fx} = \frac{\text{shear stress}}{\frac{1}{2} \rho U^2} = \frac{\tau_0}{\frac{1}{2} \rho U^2}$$

$$\begin{aligned} \tau_0 &= C_{fx} \times \frac{1}{2} \rho U^2 = .0054 \times .5 \times (.9 \times 1000) \times 6^2 \\ &= .0027 \times .9 \times 1000 \times 36 \\ &= 2.7 \times .9 \times 36 \\ &= 87.48 \text{ N/m}^2 \text{ Ans.} \end{aligned}$$

iii) friction drag on one side F_D

$$R_{eL} = \frac{UL}{V} = \frac{6 \times .5}{1 \times 10^{-4}} = 30000$$

As the boundary layer is laminar even at trailing edge, therefore

Average frictional drag coefficient

$$\overline{C_f} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{30000}} = \frac{1.328}{173.2} = .00766$$

So $F_D = \overline{C_f} \times \frac{1}{2} \rho U^2 \times \text{Area of plate on one side}$

$$= .00766 \times .5 \times (.9 \times 1000) \times 6^2 \times (.5 \times 2.5)$$

$$= 15.51 \text{ N Ans,}$$

Q 7

Atmospheric air at 20°C is flowing to a flat plate at a velocity of 3m/s. Assessing cubic velocity profile and using exact Blasius solution estimate the boundary layer thickness and the coefficient of drag at x=1 m from the leading edge of the plate. Also find the deviation of the approximate solution from the exact solution.

[Ans 11.376 mm, 1.511×10^{-3} , 7.2%, 2.78%]

Solution

Note; - since velocity profile is not given but Blasius solution is indicated we may go ahead on that.

Given velocity of air $U=3\text{m/s}$.

distance from the leading edge $=x=1\text{m}$

$$Re_x = \frac{UX}{\nu} = \frac{3 \times 1}{15.4 \times 10^{-6}} = 194805$$

Blasius solution

$$\begin{aligned} \text{i) Boundary layer thickness } \delta &= \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 1}{\sqrt{194805}} = \frac{5}{441.36} \\ &= .01132 = 11.32\text{mm.} \end{aligned}$$

$$\text{ii) local coefficient drag} = C_{fx} = \frac{.646}{\sqrt{Re_x}} = \frac{.646}{\sqrt{194805}} = \frac{.646}{441.36} \\ = .001463 = 1.463 \times 10^{-3}$$

iii) deviation of approx. solution from the exact solution

a) approx. solution

$$\text{i) } \delta = \frac{4.64x}{\sqrt{Re_x}} = \frac{4.64 \times 1}{\sqrt{194805}} = \frac{4.64}{441.36} = .0105\text{m} = 10.5 \text{ mm}$$

$$\text{ii) } C_{fx} = \frac{.64}{\sqrt{194805}} = \frac{.64}{441.36} = .001450\text{m}$$

b) deviation from exact solution

$$\text{deviation for } \delta = \frac{11.32 - 10.5}{11.32} \times 100 = 7.24\%$$

$$\text{deviation for } C_{fx} = \frac{.001463 - .001450}{.001463} \times 100 = 88\%$$

Q 8

Air is flowing over a plate 4m x 2m with a velocity of 5m/s at 15° c.
If $\rho = 1.208 \text{ kg/m}^3$ and $\nu = \text{kinematic viscosity} = 1.47 \times 10^{-5} \text{ m}^2/\text{s}$

Calculate

i) length of plate over which the boundary layer is laminar and thickness of boundary layer.

ii) shear stress at the location where boundary layer ceases to be laminar and

iii) total force on both sides on the portion of plate where boundary is laminar.

$$[\text{Ans } 1.47, 10.39\text{mm} \text{ ii) } .01418 \text{ N/m}^2 \text{ iii) } 0.1662 \text{ N}]$$

Plate

$L=4\text{M}$, $B=\text{width}=2\text{m}$, velocity of air $=U=5\text{m/s}$

Density of air $=\rho=1.208\text{ kg/m}^3$,

Kinematic of velocity $=V=1.47 \times 10^{-5}\text{m}^2/\text{s}$

i) length of plate over which boundary layer is laminar $=x$

$$\text{Reynold no} = R_{ex} = \frac{UL}{V} = \frac{4 \times 5}{1.47 \times 10^{-5}} = 1.361 \times 10^6$$

$R_{ex} > 5 \times 10^5$ hence front portion is laminar but rear portion turbulent

$$\text{So } R_{ex} = 5 \times 10^5 = \frac{Ux}{V} = \frac{5 \times x}{1.47 \times 10^{-5}}$$

$$x = \frac{5 \times 10^5 \times 1.47 \times 10^{-5}}{5} = 1.47 \text{ m Ans,}$$

$$\text{thickness of boundary layer } \delta = \frac{.5x}{\sqrt{R_{ex}}} = \frac{5 \times 1.47}{\sqrt{5 \times 10^5}} = \frac{7.35}{707} = .01039\text{m}$$

$$= 10.39 \text{ mm Ans.}$$

ii) Shear stress at the location where boundary layer ceases to be laminar $=\tau_0$

$$\text{local coefficient of drag} = C_{fx} = \frac{.646}{\sqrt{R_{ex}}} = \frac{.646}{\sqrt{5 \times 10^5}} = .000939$$

$$\begin{aligned} \tau_0 &= C_{fx} \times \frac{1}{2} \times \rho U^2 = .000939 \times .5 \times 1.208 \times 5^2 \\ &= .014178 \text{ N/m}^2 \text{ Ans} \end{aligned}$$

iii) total drag force on both sides of plate F_D

$$= 2 \bar{C}_1 \times \frac{1}{2} \times \rho \times A \times U^2$$

Where $\bar{C}_1$ = average coefficient of drag

$$= \frac{1.328}{\sqrt{R_{ex}}} = \frac{1.328}{\sqrt{5 \times 10^5}} = 1.878 \times 10^{-3}$$

$$A = \text{Area} = x.2 = 2x = 2 \times 1.47 = 2.94 \text{ m}^2$$

$$\therefore F_D = 2 \times (1.878 \times 10^{-3} \times 5 \times 1.208 \times 2.94 \times 5^2)$$

$$= \frac{166.74}{10^3} = 0.16674 \text{ N Ans.}$$

Q 9

A submarine can be assumed to have cylindrical shape with rounded nose. Assuming its length 55m and diameter 6.0m, determine the total power required to overcome boundary friction if it cruises at 8.0 m/s velocity in water at 20 ° c. Take $\rho = 1030 \text{ kg/m}^3$, $V = 1 \times 10^6 \text{ m}^2$ [Ans 476.5 kw]

Solution

Submarine

$$L = 55 \text{ m, diameter} = 6 \text{ m, } U = 8 \text{ m/s.}$$

$$\text{Density of sea water } = \rho = 1030 \text{ kg/m}^3$$

$$\text{Kinematic viscosity} = 1 \times 10^{-6} \text{ m}^2/\text{s}$$

i) Total power required for boundary friction P

$$\text{Reynold no} = \frac{UL}{\nu} = \frac{8 \times 55}{1 \times 10^{-6}} = 4.4 \times 10^8$$

The length over which boundary layer is given by

$$\frac{UX}{\nu} = 5 \times 10^5$$

$$X = \frac{5 \times 10^5 \times 1 \times 10^{-6}}{8} = \frac{5}{80} = 0.0625 \text{ m}$$

This is very small contribution to total drag force from laminar boundary layer is negligible hence $\overline{C_f}$ average coefficient of drag is given by

$$= \frac{0.455}{(\log_{10} Re_L)^{2.58}} = \frac{0.455}{[\log_{10}(4.4 \times 10^8)^{2.58}]} \\ = \frac{.455}{(8.643)^{2.58}} = \frac{.455}{260.97} = .0017435 = \overline{C_f}$$

$$\text{Area} = \pi D L = \pi \times 6 \times 55 = 1036.7 \text{ m}^2$$

$$\text{Drag force} = F_D = \overline{C_f} \times \frac{1}{2} \times \rho \times A \times U^2 \\ = .0017435 \times .5 \times 1030 \times 1036.7 \times 8^2 \\ = 59574.57 \text{ N}$$

So total power required to overcome the boundary friction = $P = \frac{F_D \times U}{1000} = \frac{59574.75 \times 8}{1000} \text{ KW}$
 $= 59.574 \times 8 = 476.6 \text{ KW. Ans,}$

Q10

Air at 30°C and at atmospheric pressure flows over a flat plate at a velocity of 1.5 m/s. If the length of the plate is 2 m and is maintained at 90°C, Calculate the heat transfer rate per unit width using i) exact and ii) approximate method ‘

[Ans 406.8 W, 305.04 W]

$$\text{Given } t_\infty = 30^\circ \text{C } t_s = 90^\circ \text{C } U = 1.5 \text{ m/s}$$

$$L = 2 \text{ m}$$

$$\text{Reynold no} = \frac{UL}{\nu} = \frac{1.5 \times 2}{18.9 \times 10^{-6}} = 1.58 \times 10^5$$

So, the flow is laminar

i) using exact solution

taking help of average Nusselt no; --

$$\overline{Nu} = 0.664 Re^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

$$\frac{\overline{h}L}{K} = 0.664 \times (1.58 \times 10^5)^{\frac{1}{2}} = 0.664 \times 397.49 \times 0.887 = 234.1$$

$$\overline{h} = \frac{234.1 \text{ K}}{L} = \frac{234.1 \times 0.0289}{2} = 3.38 \text{ W/m}^2\text{°C}$$

∴ heat transfer rate for the plate

$$Q = \overline{h} \times A_s (t_s - t_{\infty}) = 3.38 \times (2 \times 1) \times (90 - 30) \\ = 3.38 \times 2 \times 60 = 405.6 \text{ W. Ans.}$$

ii) approximate method

$$\overline{Nu} = \overline{h}L/K = 0.646 (Re_L)^{\frac{1}{2}} (Pr)^{\frac{1}{3}}$$

$$= 0.646 (1.58 \times 10^5)^{\frac{1}{2}} \times (.696)^{\frac{1}{3}}$$

$$= 0.646 \times 397.49 \times .887 = 178.67$$

$$\therefore \overline{h} = \frac{178.67 \text{ K}}{L} = \frac{178.67 \times 0.0289}{2} = 2.58 \text{ W/m}^2\text{°C}$$

Heat transfer rate from the plate $Q = \overline{h} \times A_s (t_s - t_{\infty})$

$$= 2.58 \times (2 \times 1) \times (90 - 30) = 309.6 \text{ W. Ans.}$$

Q 11

Air at a temperature of 30°C flows past a flat plate at a velocity of 2m/s. The flat surface has sharp leading edge and in total length equals 800 mm. Calculate the following

- i) the average skin friction or drag coefficient
- ii) the average shear stress.

iii) the ratio of average stress to the shear stress at trailing edge.

[Ans .004199 ii).009783 N/m² iii)2]

Solution

Properties of air at 30° c

$$\rho = 1.165 \text{ kg/m}^3, \mu = 6.717 \times 10^{-2} \text{ kg/hm}$$

$$V = 16 \times 10^{-6} \text{ m}^2/\text{s}$$

Solution

$$\text{Given } t_{\infty} = 30 \text{ c, } U = 2 \text{ m/s, } L = 800 \text{ mm} = .8 \text{ m}$$

i) average skin friction or drag coefficient

$$\overline{C_f}$$

$$Re_L = \frac{UL}{\nu} = \frac{2 \times .8}{16 \times 10^{-6}} = 10^5$$

Boundary layer over the entire plate is laminar in nature.

$$\overline{C_f} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{10^5}} = .0042$$

$$\begin{aligned} \text{ii) average shear stress } \tau_w &= \frac{1}{2} U^2 \rho \times \overline{C_f} \\ &= \frac{1}{2} 2^2 \times 1.165 \times .0042 \\ &= .009786 \text{ N/m}^2 \text{ Ans.} \end{aligned}$$

iii) The ratio of average shear stress to the shear stress at the trailing edge; --

skin friction coefficient at the trailing edge

$$C_{fx} = \frac{.664}{\sqrt{Re_x}} = \frac{.664}{\sqrt{10^5}} = \frac{.664}{316.2} = .0020998$$

$$\begin{aligned} \tau_{wx} &= \frac{1}{2} \rho U^2 C_{fx} = .5 \times 1.165 \times 2^2 \times .0020998 \\ &= .00489 \end{aligned}$$

$$\therefore \frac{\tau}{\tau_w} = \frac{.009786}{.00489} = 2.00 \text{ Ans.}$$

Q 12

Air at 20°C with a velocity of 3m/s over a plate (1000mm) length x 500mm width x 20 mm thickness. The top surface of the plate is maintained at 100° c. If thermal conductivity of the plate material 23w/m° c; ---

Calculate the following

i) heat lost by the plate

ii) bottom temperature of plate for the steady state condition

--- at 60°C thermos physical properties of air are

$$\rho = 1.06 \text{ KG/M}^3, c_p = 1.005 \text{ kJ/kg}^\circ \text{ c}$$

$$K = 0.02894 \text{ w/m}^\circ \text{ c} \quad v = 18.97 \times 10^{-6} \text{ m}^2/\text{s}$$

$$p_r = 0.696 \text{ [Ans; } 270.72 \text{ w, } 100.47^\circ \text{ c]}$$

Solution

$$t_\infty = 20^\circ \text{ c}, t_s = 100^\circ \text{ c} \quad U = 3 \text{ m/s}, k = 23 \text{ w/m}^\circ \text{ c}$$

$$L = 1000 \text{ mm}, B = 500 \text{ mm}, \text{ thickness} = 20 \text{ mm.}$$

$$\delta = .020 \text{ m}$$

Heat loss by the plate

$$R_{eL} = \frac{UL}{v} = \frac{3 \times 1}{18.97 \times 10^{-6}} = 1.58 \times 10^5$$

So, flow is laminar as $R_{eL} < 5 \times 10^5$ through out the length.

(Equa. 7.68)

$$\overline{Nu} = 0.664 (Re_L)^{\frac{1}{2}} X (Pr)^{\frac{1}{3}} = \frac{\bar{h} L}{K}$$

$\bar{h}$ = average heat transfer coefficient

$$\begin{aligned} \frac{\overline{Nu} k}{L} &= \frac{0.664 (1.58 \times 10^5)^{\frac{1}{2}} (696)^{\frac{1}{3}} k}{1} \\ &= .664 \times 397.49 \times .887 \times .02894 \\ &= 6.775 \text{ w/m}^2\text{ }^\circ \end{aligned}$$

$\therefore$ heat lost by the plate $Q = \bar{h} A_s (t_s - t_\infty)$

$$= 6.775 (1 \times .5) (100 - 20) = 6 \times .5 \times 80 = 271 \text{ w Ans.}$$

ii) to find bottom temperature t_b

heat lost due to plate Q calculated above must be conducted through the plate, hence exchange from top to bottom surface is

$$\begin{aligned} Q &= \frac{K_{plate} A (t_s - t_\infty)}{\delta} = \frac{23 \times (1 \times .5) (t_b - 100)}{.020} \\ 271 &= \frac{23 \times .5 \times (t_b - 100)}{.020} \end{aligned}$$

$$t_b - 100 = (271 \times .020) / 11.5 = .47$$

$$t_b = 100 + .47 = 100.47^\circ \text{ C Ans.}$$

Q 13

Air at 20 c and at atmospheric pressure flows at a velocity of 2 m/s over a plate maintained at 100° c. The length and width of the plate are 800mm and 400mm respectively. Using exact solution calculate the heat transfer rate from

i) first half of the plate ii) full plate iii) next half of the plate.

Properties of air at 60 c = $\rho = 1.06 \text{ kg/m}^3$, $\mu = 7.211 \text{ kg/mh}$

$$V = 18.97 \times 10^{-6} \quad Pr = .696$$

solution

Given $t_{\infty}=20^{\circ}\text{C}$, $U=2\text{m/s}$, $t_s=100^{\circ}\text{C}$, $L=800\text{mm}=0.8\text{m}$

Width $=b=400\text{mm}=0.4\text{m}$, $k=0.02894$

i) heat transfer rate from 1st half of the plate

$$x = \frac{0.8}{2} = 0.4 \text{ m}$$

$$R_{ex} = \frac{Ux}{\nu} = \frac{2 \times 0.4}{18.97 \times 10^{-6}} = \frac{8 \times 10^5}{18.97} = 0.42 \times 10^5 = 42000$$

Since $R_{ex} < 5 \times 10^5$ hence flow is laminar.

(eqn. 7.65) the local Nusselt number is given by

$$\begin{aligned} \bar{Nu} &= 0.332(R_{ex})^{\frac{1}{2}} (Pr)^{\frac{1}{3}} \\ &= 0.332 \times (42000)^{\frac{1}{2}} \times (0.696)^{\frac{1}{3}} \\ &= 0.332 \times 204.9 \times 0.887 \\ &= 60.339 \end{aligned}$$

$$\text{But } \bar{Nu}x = \frac{h_x x}{k}$$

$$h_x = \frac{Nu_x k}{x} = \frac{60.339 \times 0.02894}{0.4} = \frac{1.746}{0.4} = 4.365 \text{ W/m}^2\text{ }^{\circ}\text{C}$$

Average heat transfer coefficient

$$\bar{h} = 2h_x = 2 \times 4.365 = 8.730 \text{ W/m}^2\text{ }^{\circ}\text{C}$$

Heat transfer rate from the 1st half of plate

$$\begin{aligned} Q_1 &= \bar{h} A_s (t_o - t_{\infty}) = 8.730 \times (0.4 \times 0.4) (100 - 20) \\ &= 8.73 \times 0.16 \times 80 \\ &= 111.744 \text{ W} \quad \text{Ans.} \end{aligned}$$

ii) heat transfer rate from full plate

$$x = L = 0.8 \text{ m}$$

$$R_{eL} = \frac{2 \times 8}{18.97 \times 10^{-6}} = \frac{1.6}{18.97} \times 10^6 = 0.84 \times 10^6 = 84000$$

$$\begin{aligned} \bar{Nu} &= .664(R_{eL})^{\frac{1}{2}} (P_r)^{\frac{1}{3}} = .664 \times (84000)^{\frac{1}{2}} (.696)^{\frac{1}{3}} \\ &= .664 \times 289.82 \times .887 = 170.69 = \frac{\bar{h} L}{K} \end{aligned}$$

$$\text{So } \bar{h} = \frac{\bar{Nu}}{L} \times K = \frac{170.69 \times .02894}{.8} = 6.174 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

Heat transfer for entire plate

$$\begin{aligned} Q_2 &= \bar{h} \times A \times (t_s - t_\infty) \\ &= 6.174 \times (.8 \times .4) (100 - 20) \\ &= 1.97 \times 80 = 158.05 \text{ W Ans.} \end{aligned}$$

iii) heat transfer for next of the plate

$$\begin{aligned} &= (Q_2 - Q_1) = (158.05 - 111.744) \\ &= 46.3 \text{ W Ans.} \end{aligned}$$

Q 14

Castor oil at 30 °C flows at a velocity of .08 m/s past a flat

plate in a certain process. If the plate is 5 m long and it is maintained at a uniform temperature of 90 °C. Calculate the following using exact solution

- i) hydrodynamic and thermal boundary layer thickness on the side of the plate.
- ii) the total drag force /unit width on one side of the plate.
- iii) local heat transfer coefficient at the trailing edge
- iv) heat transfer rate.

physical properties of oil at mean film temperature

$$\frac{30+90}{2}=60 \text{ c}, \rho=956.8 \text{ kg/m}^3, \alpha=7.2 \times 10^{-8} \text{ m}^2/\text{s}$$

$$K=.213 \text{ W/mc } V=\text{Kinematic viscosity}=.65 \times 10^{-4} \text{ m}^2/\text{s}$$

[Ans i) 318.7mm, 32.98mm ii) .2591N/m iii) 10.916W/m²
iv) 6429 W]

Solution

Given $t_{\infty}=30\text{c}$, $t_s=90\text{c}$. plate $L=5\text{m}$

$$U=.08 \text{ m}, \rho=956.8, \alpha=7.2 \times 10^{-8}$$

$$K=0.213 \text{ W/mc}, V=\text{Kinematic viscosity}=.65 \times 10^{-4} \text{ m}^2/\text{s}$$

i) hydrodynamic and thermal boundary layer δ, δ_{th}

$$\text{Reynold no} = \frac{UL}{V} = \frac{.08 \times 5}{0.65 \times 10^{-4}} = 6150$$

$$\text{So } Re_L < 5 \times 10^5$$

Hence the flow is laminar

The hydrodynamic boundary layer thickness

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5xL}{\sqrt{6150}} = \frac{5 \times 5}{78.42} = .3187 \text{ m} = 318.7 \text{ mm Ans.}$$

Thermal boundary layer thickness according to Ohlhausen is

$$\delta_{th} = \frac{\delta}{Pr^{\frac{1}{3}}} \text{ where } Pr = \frac{v}{\alpha} = \frac{.65 \times 10^{-4}}{7.2 \times 10^{-8}} = 902.77$$

$$= \frac{.3187}{902.77^{\frac{1}{3}}} = \frac{.3187}{9.447} = 33.73 \text{ mm Ans}$$

ii) total drag force /unit width on one side of the plate F_D ;---

The average skin friction coefficient is given by

$$\overline{C_f} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{6150}} = \frac{1.328}{78.42} = .01693 U^2$$

The drag force $= F_D = \overline{C_f} x \frac{1}{2} \rho x U^2$ area of plate for one side

$$= .01693 \times .5 \times 956.8 \times (.08)^2 \times (5 \times 1)$$

$$= 0.2591 \text{ N per m width. Ans.}$$

iii) the local heat transfer coefficient at the trailing edge

$$h_x (\text{at } x=L)$$

$$\overline{Nu}_x = \frac{h_x x}{k} = .332 (\sqrt{Re_x} \times (Pr)^{\frac{1}{3}}) = .332 \times 78.42 \times (902.77)^{\frac{1}{3}}$$

$$= 251.03$$

$$h_x = \frac{245.95 \times k}{x} = \frac{245.95 \times .213}{5} = 10.694 \text{ w/m}^2 \text{ } ^\circ\text{C Ans}$$

iv) heat transfer rate $Q = \bar{h} A (t_s - t_\infty)$

$$\text{where } \bar{h} = 2h_x = 2 \times 10.694 = 21.388 \text{ w/m}^2 \text{ } ^\circ\text{C}$$

$$Q = 21.388 \times (5 \times 1) \times (90 - 30)$$

$$= 21.388 \times 5 \times 60$$

$$= 6416.4 \text{ W Ans.}$$

Q 15

Air at 30°C and at atmospheric pressure flow plate at a velocity of 4m/s past a flat plate with a sharp leading edge.

The entire plate surface is maintained at a temperature of 50°C .

Assuming the transition occurs at a critical Reynold number of 5×10^5 find the distance from the leading edge at which flow in the boundary layer changes from laminar to turbulent.

At the location calculate the following; -

- i) thickness of hydrodynamic layer.
- ii) thickness of thermal boundary layer
- iii) local and average convective heat transfer coefficient
- iv) heat transfer rate from both side for unit width of plate.
- v) mass entrainment in the boundary layer.
- vi) the skin friction coefficient.

$$\text{mean film temperature} = (50+30)/2 = 80/2 = 40^\circ \text{ c}$$

$$\rho = 1.128 \text{ kg/m}^3 \quad V = 16.96 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = .02755 \text{ w/m}^\circ \text{ c}$$

$$[\text{Ans; I)13.8mm, ii)15.258mm iii)2.625w/m}^2\text{c iv)445.2 w} \\ \text{v)140.76 kg/h vi).000913}]$$

solution

$$t_\infty = 30^\circ \text{ c}, t_s = 50^\circ \text{ c} \quad U = 4 \text{ m/s}$$

At the transition point Re_c = Critical Reynold no = $\frac{UX_c}{\nu}$

X_c = Critical distance from leading edge at which the flow in the boundary changes from laminar to turbulent.

$$x_c = \frac{Re_c \nu}{U} = \frac{5 \times 10^5 \times 16.96 \times 10^{-6}}{4} = 2.12 \text{ m}$$

$$\text{i) thickness of hydrodynamic layer } \delta = \frac{4.64 x_c}{\sqrt{Re_c}} = \frac{4.64 \times 2.12}{\sqrt{5 \times 10^5}} = .01391 \text{ m} \\ = 13.91 \text{ mm.}$$

$$\text{ii) thickness of thermal boundary layer } \delta_{th} = \frac{0.975 \delta}{(Pr)^{\frac{1}{3}}} \\ = \frac{.975 \times .01391}{(.669)^{\frac{1}{3}}} = \frac{.01356}{.875} \\ = .01549 \text{ m} = 15.49 \text{ mm.}$$

iii) local and average convective heat transfer coefficient (eq7.67)

$$\text{Nu at } x = X_c \text{ so } \text{Nu}_c = 0.322(R)_{ec}^{\frac{1}{2}} x (Pr)^{\frac{1}{3}} = 0.322(5 \times 10^5)^{\frac{1}{2}} x (.699)^{\frac{1}{3}} \\ = 0.322 \times 707.1 \times .875 = 208.34$$

$$\text{But } \text{Nu}_x = \frac{h_s x x_c}{k}, \quad h_s = \frac{\text{Nu}_x k}{x_c}$$

(h_c = local heat transfer coefficient at $x = x_c$)

$$(a) \quad h_c = \frac{\text{Nu}_x k}{x_c} = \frac{208.34 \times 0.2753}{2.12} = 2.705 \text{ W/m}^2\text{°C}$$

(b) Average heat transfer coefficient

$$\bar{h} = \frac{1}{x_c} \int_0^{x_c} h_x dx = 2h_c = 2 \times 2.705 = 5.410 \text{ w/m}^2$$

i v) heat transfer rate from both sides for unit width of the plate
 $Q = \bar{h} (2A_s) \Delta t = 5.410 (2 \times 2.12 \times 1) (50 - 30)$

$$= 5.410 \times 4.24 \times 20$$

$$= 458.768 \text{ w}$$

v) Mass entrainment in the boundary layer

$$m = 5/8 \rho U (\delta_2 - \delta_1)$$

$$= 5/8 \times 1.128 \times 4 \times (.01234 - 0)$$

$$= .625 \times 1.128 \times 4 \times .01234$$

$$= 125.27 \text{ kg/h Ans.}$$

vi) the skin friction coefficient C_{fx}

$$C_{fx} = \frac{0.646}{\sqrt{R_{ex}}} = \frac{0.646}{\sqrt{5 \times 10^5}} = \frac{.646}{707.1} = .000913.$$

Q16 A steam of water at 20°C ($\rho = 1.205 \text{ kg/m}^3$, $\mu = 0.06533 \text{ kg/hm}$) flow at a velocity of 1.5 m/s over a plate 0.5m long and placed at 0 angle of incidence. Using exact solution, calculate the following

I, stream wise velocity component at midpoint of the boundary layer.

2. maximum value of normal component at the trailing edge of the plate. [Ans i) 1.104 m/s ii) 11.2 mm]

$$\text{Given } t_{\infty} = 20^\circ\text{C} \quad \rho = 1.205 \text{ kg/m}^3, L = 0.5 \text{ m}$$

$$U = 1.5 \text{ m/s}$$

i) streamwise velocity component at the midpoint of the boundary layer U ;

Boundary layer thickness by exact calculation is given by –

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5}{\sqrt{\frac{UX}{\nu}}} = 5 \sqrt{\frac{\nu X}{U}} = 5 \left[\text{Midpoint } \frac{\delta}{2} \right]$$

$$\text{Occurs at } n = y \sqrt{\frac{U}{\nu X}}$$

Streamwise velocity component is obtained from Blasius solution in the tabular form (ref table no 7.1 from original book)

$$\text{At } n = y \sqrt{\frac{U}{\nu X}} = 2.5$$

$$\text{We get } \frac{u}{U} = 0.736$$

$$u = 0.736U = 0.736 \times 1.5 = 1.104 \text{ m/s} \quad \text{Ans}$$

ii) maximum boundary layer thickness $= \delta_L$

$$\text{maximum boundary layer thickness occurs at } x = 0.5 \text{ m thus } Re_L = \frac{\rho UL}{\mu} = \frac{1.205 \times 1.5 \times 0.5}{0.06533/3600} = (1.205 \times 1.5 \times 0.5 \times 3600) / 0.06533$$

$$= \frac{3253.5}{0.06533} = 49801 \quad \text{Hence flow is laminar.}$$

Boundary layer thickness at the trailing edge

$$\delta_L = \frac{5L}{\sqrt{Re_L}} = \frac{5 \times 5}{\sqrt{49801}} = \frac{2.5}{223.16} = .011202\text{m} = 11.2 \text{ mm}$$

iii) maximum value of normal component of velocity at the trailing edge =v

maximum value of normal component of velocity occurs outer edge of the boundary layer where $u=U$. hence $\frac{u}{U} = 1$, we have

$$\frac{v}{U} \sqrt{Re_L} = 0.86 \text{ (Ref table 7.1)}$$

$$v = \frac{0.86U}{\sqrt{49801}} = \frac{.86 \times 1.5}{223.16} = \frac{1.29}{223.16} = .00578\text{m/s} \quad \text{Ans}$$

Q17

At 25 °c flow past a flat plate at 2.5 m/s. The plate measures 600mmx 300 mm and is maintained at uniform temperature of 95°C. Calculate heat loss from the plate of the air flow parallel to 600mm side. How would this loss be affected if the flow of air is made parallel to the 300 mm side?

[Ans 100.5w ,142.128 w]

Solution

Given $U=2.5\text{m/s}$, $L=600\text{mm}=.6\text{m}$, width= $300\text{mm}=.3\text{m}$

Assumed $\nu=18.97 \times 10^{-6}$, $k=0.025\text{w/m c}$, $p_r=0.7$

i) heat loss per hr from the plate

case 1

when flow is parallel to 600mm side

$$\overline{Nu} = \frac{\bar{h}L}{K} = .664 (Re_L)^{\frac{1}{2}} (Pr)^{\frac{1}{3}} \text{ (equal. 7.68)}$$

$$R_{CL} = \frac{UL}{V} = \frac{2.5 \times .6}{18.97 \times 10^{-6}} = 1.31 \times 10^5$$

$$\bar{h} = \frac{k}{L} \times 0.664 (1.31 \times 10^5)^{\frac{1}{2}} (.7)^{\frac{1}{3}}$$

$$= \frac{.025}{.6} \times 0.664 \times (1.31 \times 10^5)^{\frac{1}{2}} \times .889$$

$$= .0276 \times \sqrt{31000} \times .889$$

$$= 8.88 \text{ W/m}^2\text{°C}$$

$$Q = \bar{h} A_s (t_s - t_\infty) = 8.88 \times (.6 \times .3) (95 - 25) = 111.8 \text{ W Ans}$$

Case ii

When the flow is parallel to 300mm side;

$$R_{eL} = \frac{UL}{V} = \frac{2.5 \times 3}{18.97 \times 10^{-6}} = .0363 \times 10^6 = 3.63 \times 10^4$$

$$\bar{h} = \frac{k}{L} \times 0.664 (R_{eL})^{\frac{1}{2}} (P_r)^{\frac{1}{3}}$$

$$= \frac{.025}{.3} \times 0.664 \times \sqrt{3.63 \times 10^4} \times (.7)^{\frac{1}{3}} = .083 \times 0.664 \times 190.52 \times .888$$

$$= 9.324 \text{ W/m}^2\text{°C}$$

$$\text{So } Q = \bar{h} A (t_s - t_\infty) = 9.324 \times (.3 \times .6) (95 - 25)$$

$$= 9.324 \times .18 \times 70$$

$$= 117.48 \text{ W Ans}$$

Q18

Air at 1 bar and at a temperature of 30 °C ($\mu = .06717$) flows at a speed of 1.25m/s over a flat plate .

Determine i) boundary layer thickness at distances of 200mm and 400mm from the leading edge of the plate. Also calculate II) mass

entrainment between the 2 sections. Assume the parabolic velocity distribution as $\frac{u}{U} = \frac{3}{2}\left(\frac{Y}{\delta}\right) - \frac{1}{2}\left(\frac{Y}{\delta}\right)^3$

[Ans 7.478mm, 10.5mm, 9.77kg/h]

Solution

Given $t_{\infty} = 30^{\circ}\text{C}$ $\mu = .06717$, $U = 1.25$ m/s

$P = 1\text{ bar} = 1 \times 10^5$

Boundary layer thickness ;----

Density of air $\rho = \frac{p}{RT} = \frac{1 \times 10^5}{287 \times (30 + 273)} = 1.15 \text{ kg/m}^3$

i) a) $x = 200 \text{ mm} = .2 \text{ m}$

Reynold number $R_{ex} = \frac{\rho U x}{\mu} = \frac{1.15 \times 1.25 \times .2 \times 3600}{.06717} = 15408.66$

Boundary layer thickness $= \delta_1 = \frac{4.64 x}{\sqrt{R_{ex}}} = \frac{4.64 \times .2}{\sqrt{15408.66}}$

$= 7.47 \text{ mm Ans.}$

ii) b)

at $x = 400 \text{ mm} = .4 \text{ m}$

$R_{ex} = \frac{\rho U x}{\mu} = \frac{1.15 \times 1.25 \times .4 \times 3600}{.06717} = 30817.32$

$\delta_2 = \frac{4.64 X}{\sqrt{30817.32}} = \frac{1.856}{175.54} = 0.01057 \text{ m} = 10.57 \text{ mm (Ans)}$

Mass entrainment

The mass flow rate at any position in the boundary layer is given by

$$m_x = \int_0^{\delta} \rho U dy = \int_0^{\delta} \rho \left[U \left\{ \frac{3}{2} \left(\frac{Y}{\delta} \right) - \frac{1}{2} \left(\frac{Y}{\delta} \right)^3 \right\} \right] dy$$

$$= P u \left[\frac{3}{4} \left(\frac{y}{\delta} \right)^2 - \frac{1}{8} \left(\frac{y}{\delta} \right)^4 \right] \delta$$

$$= \frac{5}{8} \rho U \delta \text{ ----- } **$$

Therefore, the mass entrainment between two section that is at 200mm and 400mm on the basis of (**) above;

$$= \frac{5}{8} \rho U (\delta_2 - \delta_1)$$

$$= \frac{5}{8} \times 1.15 \times 1.25 (.01057 - .00747)$$

$$= .625 \times 1.15 \times 1.25 \times .0031$$

$$= .002785 \text{ kg/s}$$

$$= .002785 \times 3600 \text{ kg/hr}$$

$$= 10.02 \text{ kg/hr. (Ans)}$$

Q 20 Turbulent flow

The crank case of an I C Engine measuring 900 mm x 250 mm may be idealized as a flat plate. Engine runs 100km/h and the crank case is cooled by the air flowing past it at the same speed, calculate; -- heat loss from the crank surface maintained at 80 °c to the ambient air at 20° c. The boundary layer becomes turbulent from the leading edge itself due to road induced vibration.

Solution

$$\text{Given } U = 100 \text{ km/hr} = 27.77 \text{ m/s}$$

$$L = 900 \text{ mm} = .9 \text{ m} \quad B = 250 \text{ mm} = .25 \text{ m}$$

$$t_s = 80^\circ \text{C} \quad t_\infty = 20^\circ \text{C}$$

$$\text{Properties of air at } t_f = (80 + 20)/2 = 50^\circ \text{C}$$

$$K = 0.02824 \text{ W/mC}, \quad \nu = 17.95 \times 10^{-6} \text{ m}^2/\text{s}$$

$$p_r = 0.698$$

i) heat loss from crank case Q; ----

$$Re_L = \frac{UL}{\nu} = \frac{27.77 \times .9}{17.95 \times 10^{-6}} = 1.39 \times 10^6$$

It is more than 5×10^5 so flow is turbulent.

$$\begin{aligned} \overline{Nu} &= \frac{\bar{h}L}{K} = .036(Re_L)^.8 (Pr)^.333 \\ &= .036 (1.39 \times 10^6)^.8 (.698)^.333 \\ &= .036 \times 82112 \times .887 = 2622.0 \end{aligned}$$

$$\bar{h} = \frac{k}{L} \times \overline{Nu} = \frac{.02824 \times 2622}{.9} = 82.27 \text{ w/m}^2\text{°c}$$

$$\begin{aligned} Q &= \bar{h} A_x (t_s - t_\infty) = 82.27 \times (.9 \times .25) (80 - 20) \\ &= 82.27 \times .225 \times 60 \\ &= 1110.6 \text{ w Ans} \end{aligned}$$

Q 21

In a gas turbine system hot gas at 1000°C flow at 75 m/s over the system of combustion chamber which is at uniform temperature of 300°C . Determine heat loss from gases to the combustion chamber which can be idealized as a flat plate measuring 1m x .5m. The flow is parallel to 1 m side transition Reynold no. is equal to 5×10^5 . Take properties of gas

$$\rho = .496 \text{ kg/m}^3 \quad \nu = 935 \times 10^{-6} \quad k = .0744 \text{ w/m}^\circ\text{C} \quad p_r = 0.625$$

[Ans 23.8kw]

Solution

$$\text{Flat plate area} = 1 \text{ m} \times .5 \text{ m} = .5 \text{ m}^2$$

$$U = 75 \text{ m/s} \quad t_s = 1000^\circ\text{C} \quad t_\infty = 300^\circ\text{C}$$

Heat loss from the gases to the combustion; --

Average Nusselt no is applicable for both Laminar and Turbulent boundary layer

$$\overline{Nu} = [0.664(R_{eL})^{.5} + 0.36 \{R_{eL}^{.8} - R_{eL}^{.8}\}] (Pr)^{.333}$$

$$\text{Reynold no} = \frac{UL}{\nu} = \frac{75 \times 1}{93.5 \times 10^{-6}} = 8.021 \times 10^5$$

This shows that both laminar and turbulent are present

Since R_e value is showing both laminar and turbulent presence we apply the following equation to get the result;

$$\overline{Nu} = [0.036(8.021 \times 10^5)^{.8} + 0.625] (625)^{.333}$$

$$= [0.036(802100)^{.8} + 0.625] (.625)^{.33}$$

$$= [1904.08 + 0.625] \times (.625)^{.33}$$

$$= 1068 \times 0.857$$

$$= 915.276$$

$$\text{So } \frac{\overline{h}L}{K} = 915.276$$

$$\overline{h} = 915.276 \times \frac{k}{L} = \frac{915.276 \times 0.0744}{1} = 68.09 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

Heat loss from gases

$$Q = \overline{h} \times A \times \Delta t$$

$$= 68.09 \times (1 \times .5) (1000 - 300)$$

$$= 68.09 \times .5 \times 700$$

$$= 23837.5 \text{ W}$$

$$= 23.831 \text{ kW,}$$

Q23

A refrigerated truck is moving on a high way at 95 km/hr in a deserted area when the ambient temperature is 50°C . The body of the truck may be considered as a rectangular box measuring 10m length x 4m width x 3 m high. Assuming that boundary layer on the four walls is turbulent heat transfer takes place only from the four surfaces and surfacing the truck is maintained at 10°C .

Neglecting the heat transferring from the front and back and assuming flow to be parallel to 10m long side, calculate the following

- i) heat loss from 4 surfaces
- ii) tonnage of refrigeration
- iii) power required to overcome the resistance acting on the four surfaces.

Properties of air at 30°C $\rho = 1.165 \text{ kg/m}^3$, $C_p = 1.005 \text{ KJ/kg}^\circ\text{C}$

$$K = 0.02673 \text{ W/m}^2\text{}^\circ\text{C}, \nu = 16 \times 10^{-6} \text{ m}^2/\text{s}, \quad p_r = 701$$

$$[\text{Ans; I) } 320.08 \text{ kW, ii) } 82.3 \text{ TR III) } 3.88 \text{ KW}]$$

Solution

$$\text{Given } U = 95 \text{ km/hr} = \frac{95 \times 1000}{3600} = 26.38 \text{ m/s}$$

$$t_\infty = 50^\circ\text{C} \quad t_s = 10^\circ\text{C} \quad L = 10 \text{ m} \quad B = 4 \text{ m} \quad H = 3 \text{ m}$$

i) heat lost from 4 surfaces; -----

$$\text{Reynold no at end of side} = R_{eL} = \frac{UL}{\nu} = \frac{26.38 \times 10}{16 \times 10^{-6}} = 16.43 \times 10^6$$

So, the boundary layer is turbulent.

$$\therefore \bar{Nu} = \frac{\bar{h}L}{K} = 0.36 (R_e^8)(p_r^{333})$$

$$=.036 \times (16 \times 10^6)^{\cdot 8} (.701)^{\cdot 333}$$

$$=.036 \times 592256.79 \times .888$$

$$=18933.26$$

$$\bar{h} = \frac{\bar{N} K}{L} = \frac{18933.26 \times .02673}{10} = 50.6 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

$$Q = \text{Heat loss 4 surfaces.} = \bar{h} \times A \times \Delta t$$

$$= 50.6 \times 2(4+3) \times 10 \times (50-10)$$

$$= 283.36 \text{ KW Ans.}$$

ii) the tonnage of refrigerator

cooling capacity required

$$= \frac{283.36 \times 3600}{14000} = 72.8 \text{ TR (Tonnage refrigeration)}$$

$$(1 \text{ tonnage} = 14000 \text{ k joule/ hr})$$

iii) power required to overcome the resistance acting on 4 surfaces; -- P

$$\overline{C_f} = \frac{0.072}{R_e^{0.2}} = \frac{.072}{(16.43 \times 10^6)^{.2}} = \frac{.072}{(16430000)^{.2}} = \frac{.072}{27.72}$$

$$=.00259 = 2.59 \times 10^{-3}$$

$$F_D = \overline{C_f} X \frac{1}{2} \rho A U^2$$

$$= 2.59 \times 10^{-3} \times .5 \times 1.165 \times 2 \{(4+3) \times 10\} \times 26.38^2$$

$$= 156.54 \text{ N}$$

$$\text{Power } p = F_D \times U = 156.54 \times 26.38 = 3865.72 \text{ W} = 3.865 \text{ kW. Ans}$$

Q 24

A fluid is flowing in a pipe which is 250mm in diameter ,3m long and whose surface is maintained at constant temperature. The

temperature of the wall surface at the inlet section of the pipe exceeds fluid by 30 °c. Determine i) what is the rise in the fluid temperature at the end section of the pipe?

Reynolds analogy holds good which is given by $st = \frac{f}{g}$

Friction factor = .02. (ST = Stanton no)

[Ans = 3.39° c]

Given

D = 250mm = .25m, L = 3m, friction factor = .02.

i) Rise in fluid temperature at the end section of pipe

$(t_0 - t_i)$; ----

The energy balance yields

$$Q = \bar{h} A (t_s - t_\infty) = m \times C_p (t_0 - t_i)$$

$$\text{Or, } h(\pi D L) (t_s - \frac{t_0 + t_i}{2}) = [\rho \cdot \frac{\pi}{4} D^2 \times V] C_p (t_0 - t_i)$$

Divide both side by $\pi \rho v C_p D$ (set $= \frac{h}{\rho v C_p}$)

$$\frac{hL}{\rho v C_p} \left[\frac{t_s - t_0 + (t_s - t_i)}{2} \right] = \frac{D}{4} (t_0 - t_i) - (t_s - t_0)$$

$$\text{Or, s.t. } \frac{L}{2} [(t_s - t_0) + (t_s - t_i)] = \frac{D}{4} (t_0 - t_i) - (t_s - t_0)$$

$$\text{Or } \frac{f}{8} \times \frac{L}{2} [(t_s - t_0) + (t_i - t_i)] = \frac{D}{4} (t_s - t_i) - (t_s - t_0)$$

Inserting the value

$$\frac{.02}{8} \times \frac{3}{2} [(t_s - t_0) + 30] = \frac{.25}{4} [30 - (t_s - t_0)]$$

$$\frac{.03}{8} (t_s - t_0) + \frac{.25}{4} (t_s - t_0) = \left(\frac{.25 \times 30}{4} - \frac{.03 \times 30}{8} \right)$$

$$(t_s - t_0) \left(\frac{.03}{8} + \frac{.25}{4} \right) = 30 \left(\frac{.25}{4} - \frac{.03}{8} \right) = \frac{7.05}{4}$$

$$(t_s - t_0) = \frac{1.76}{\frac{.03}{8} + \frac{.25}{4}} = \frac{1.76}{.06625} = 26.56^\circ \text{ c}$$

The rise in temperature of fluid at the end

$$(t_0 - t_i) = (t_s - t_i) \text{ --- } (t_s - t_0)$$

$$= (30 - 26.56)$$

$$= 3.44^\circ \text{ C Ans.}$$

Q25

In a straight tube of 50 mm diameter water is flowing at a velocity 15 m/s. The tube surface temperature is maintained

at 60° c and the flowing water is heated from inlet temperature 15° c to an outlet temperature 45° c , Calculate the following;

i) heat transfer coefficient from tube surface to the water.

ii) the length of the tube.

thermos physical properties

$$\rho = 991.7 \text{ kg/m}^3 \quad C_p = 4.174 \text{ K J/kg c} \quad k = 61.718 \times 10^{-6}$$

$$V = .805 \times 10^{-6} \text{ m}^2, \quad m = 26.24 \text{ m}, p_r = 5.42$$

$$D = 50 \text{ mm} = .05 \text{ m} \quad U = 15 \text{ m/s}, t_i = 15^\circ \text{ c}, t_s = 60^\circ \text{ c}, t_0 = 45^\circ \text{ c}$$

$$\text{Fluid mean bulk temp.} = \frac{15 + 45}{2} = 30^\circ \text{ c} = t_b$$

i) Heat transfer coefficient from tube surface to water

$$\text{Reynold no} = \frac{U D}{\nu} = \frac{15 \times .05}{.805 \times 10^{-6}} = .9316 \times 10^6$$

Hence flow is turbulent.

$$\bar{N} u = \frac{\bar{h} D}{K} = .023 (R_e^8 \times (p_r^{.333}) [\text{eq u 7.151}]$$

$$\begin{aligned}\bar{h} &= \frac{61.718 \times 10^{-2}}{.05} \times .023 \times (.9316 \times 10^6) \times (5.42)^{.333} \\ &= 12.34 \times .023 \times 59618 \times 1.755 \\ &= 29695.97 \text{ W/m}^2 \text{ } ^\circ \text{C Ans.}\end{aligned}$$

ii) length of tube

$$Q = \bar{h} A (t_s - t_b) = \bar{h} (\pi D L) (t_s - t_b)$$

We have to find value of Q first.

$$\begin{aligned}Q &= m \times C_p \times (t_0 - t_i) = (\rho \times \pi / 4 \times D^2 \times L) \times C_p (t_0 - t_i) \\ &= 995.7 \times \frac{\pi}{4} \times (.05)^2 \times 15 \times 4.174 \times 10^3 (45 - 15) \\ &= 995.7 \times .7854 \times .0025 \times 15 \times 30 \times 4174 \\ &= 3672183.47 \text{ W}\end{aligned}$$

$$\begin{aligned}Q &= \bar{h} A (t_s - t_b) \\ &= 29695.97 (\pi D L) (60 - 30) \\ &= 29695.97 (\pi \times .05 \times L) \times 30 = 139938.96 L\end{aligned}$$

$$\text{So, } L = \frac{Q}{139938.96} = \frac{3672183.47}{139938.96} = 26.24 \text{ m Ans.}$$

Q 26

Air at a temperature of 20°C flows across a flat plate at a velocity of 6.5 m/s maintained at a temp $,600^\circ\text{C}$. Calculate amount of heat transferred per meter width from both sides of the plate over a distance of 300mm from leading edge. The following relation holds good in the case of large temperature between plate and the fluid;

$$Nu_x = 0.332 (P_r^{\frac{1}{3}}) (R_e^{\frac{1}{2}}) \left(\frac{T_s}{T_\infty} \right)^{0.117}$$

Where T_s & T_∞ are absolute temp. of plate surface and free stream respectively and all fluid properties are evaluated at mean film temperature. [7850.9 w]

Solution

$$T_m = 20 + 273 = 293^\circ\text{K}, \quad T_s = 600 + 273 = 873^\circ\text{K}$$

$$X = 300\text{mm} = \frac{300}{1000} = .3\text{m}, \quad u = 6.5\text{m/s}$$

i) Amount of heat transferred per unit width;

$$\text{the mean film temp.} = t_f = \frac{600 + 20}{2} = 310^\circ\text{C}$$

thermophysical properties $\rho = .614\text{kg/m}^3$, $C_p = 1.046\text{KJ/kg}^\circ\text{C}$

$$k = .04593\text{ W/m}^\circ\text{C}, \quad \mu = 29.7 \times 10^{-6}\text{kg/m s}, \quad p_r = 0.675$$

$$\text{Reynold number} = \frac{\rho vx}{\mu} = \frac{.614 \times 6.5 \times .3}{29.7 \times 10^{-6}} = 40313$$

$$\text{Nusselt number} = N_{u_x} = 0.332(p_r^{\frac{1}{3}})(R_e^{\frac{1}{2}})\left(\frac{T_s}{T_\infty}\right)^{0.117}$$

$$= .332(.675)^{\frac{1}{3}}(40313)^{\frac{1}{2}}\left(\frac{873}{293}\right)^{0.117}$$

$$= .332 \times .877 \times 200.78 \times 1.13625$$

$$= 66.43$$

$$\text{Now } h_x = \frac{k}{x} \times 66.43 = \frac{.04593 \times 66.43}{.3} = 10.19\text{W/m}^2\text{C}$$

Average heat transfer coefficient is

$$\bar{h} = \frac{1}{L} \int_0^L h_x dx = 2h_x = 2 \times 10.19$$

$$= 20.38\text{ W/m}^2\text{C}$$

Heat transfer from both sides of plate per meter width

$$Q = 2 [\bar{h} \times A_s \times (t_s - t_\infty)] = 2[20.38 \times (.3 \times 1) \times (600 - 20)]$$

$$= 2[20.38 \times 3 \times 580]$$

$$= 7092.24 \text{ W} \quad \text{Ans.}$$

Q 27

A surface condenser consists of 200 thin-walled circular tube (each tube is 25mm diameter and 5 m long) arranged in parallel through which water flows. If the mass flow rate of water through the tube bank is 150 kg/s and its inlet and outlet temperature are known to be 20°C and 30°C, calculate the average heat transfer coefficient associated with flow of water.

$$[\text{Ans } 5941.9 \text{ W/m}^2\text{°C}]$$

Solution

Given $D=25\text{mm}=0.025\text{m}$, $L=5\text{m}$, $t_i=20^\circ\text{C}$, $t_o=30^\circ\text{C}$

No of tubes = 200. $\dot{m}$ = mass flow = 150 kg/s

Average heat transfer coefficient $\bar{h}$;--

$$\text{Mean bulk temperature } = t_b = \frac{t_i + t_o}{2} = \frac{20 + 30}{2} = 25^\circ\text{C}$$

Thermophysical properties of water at 25°C

$$\rho = 996.65 \text{ kg/m}^3 \quad \mu = 0.862 \times 10^{-3} \text{ kg/ms}$$

$$K = 0.6079 \text{ W/m}^\circ\text{C} \quad C_p = 4.178 \text{ kJ/kg}^\circ\text{C}$$

$$\text{Prandtl no } = \text{Pr} = \frac{\mu C_p}{K} = \frac{0.862 \times 10^{-3} \times 4.178 \times 10^3}{0.6079} = 5.924$$

Mass flow rate of water through each tube; --

$$\dot{m} = \frac{150}{200} = 0.75 \text{ kg/s}$$

$$\begin{aligned}\text{Reynold no} &= \frac{\rho U D}{\mu} = \frac{\rho A U D}{A \mu} = \frac{m D}{A \mu} = \frac{D m}{\frac{\pi}{4} X D^2 \mu} = \frac{4 m}{\pi D \mu} \\ &= \frac{4 \times 75}{\pi \times 0.0225 \times 0.862 \times 10^{-3}} = \frac{3000}{0.0677} \\ &= 44313.14\end{aligned}$$

$Re > 2500$ hence flow is turbulent since water is being heated in condenser hence, we may use McAdams's correlation as follows; $\bar{Nu} = 0.023(R_e^8)(p_r^4)$

$$= 0.023 \times (44313.14)^8 (5.924)^4 = 248.47$$

$$\bar{Nu} = \frac{\bar{h} D}{K} = 248.47$$

$$\bar{h} = \frac{248.47 \times k}{D} = \frac{248.47 \times 6079}{.925} = 6041.79 \text{ W/m}^2 \text{ } ^\circ \text{C Ans.}$$

Q 28

0.05kg/s of hot air flows through an uninsulated sheet metal duct of 150 mm diameter. The air enters the duct at a temperature 105 °C and after a distance of 5 m gets cooled to a temperature of 80 °C. If the heat transfer coefficient between the outer surface of the duct and cold ambient air at 5° C is 6 w/m²°C.

Calculate the heat loss from the duct over 5 m length ii) heat flux

And duct surface temperature at a length of 5m.

$$[\text{Ans i)} 1261.25 \text{ W ii)} 299.69 \text{ W/m}^2 \text{ } ^\circ \text{C}, 54.92^\circ \text{C}]$$

Solution

$$\text{Given } m = 0.05 \text{ kg/s, } D = 150 \text{ mm} = 0.15 \text{ m, } h_o = 6 \text{ W/m}^2 \text{ } ^\circ \text{C}$$

$$\text{Distance} = 5 \text{ m, mean bulk temp. } (t_b = \frac{t_i + t_x}{2} = \frac{105 + 80}{2} = 92.5^\circ \text{C})$$

Thermos physical properties

$$\rho = 0.972 \text{ kg/m}^3 \quad c_p = 1.009 \text{ kJ/kg} \cdot ^\circ\text{C}, \quad k = 0.03127 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$V = 22.1 \times 10^{-6} \quad p_r = 0.69$$

$$\text{Reynold number} = \frac{\rho V D}{\mu} = \frac{4m}{\pi D \mu} = \frac{4 \times 0.05 \times 10^6}{\pi \times 0.15 \times 22.14} = 1.9 \times 10^4$$

$R_e > 2500$ so flow is turbulent.

$$\begin{aligned} \text{i) heat loss from the duct } Q &= m \dot{C}_p \Delta t = \frac{.05 \times 1.009 (105 - -80)}{1} \\ &= 1261.25 \text{ W} \quad \text{Ans.} \end{aligned}$$

ii) heat flux and duct surface temperature at a length of 5 m

$$\frac{Q}{A} = \frac{(80 - -t)}{\sum R_{th}} = \frac{\Delta t}{\frac{1}{h_i} + \frac{1}{h_o}} \quad \text{where } h_o = 6 \text{ W/m}^2 \cdot ^\circ\text{C}$$

And h_i = inside convective coefficient is calculated from the following correlation

$$\text{Nu} = 0.023 (R_e)^{.8} (p_r)^{.333} = .023 \times (1.9 \times 10^4)^{.8} \times (.69)^{.333}$$

$$= .023 \times 2648 \times 2648 \times .883 = 53.778 \frac{h_i D}{K}$$

$$\text{So } h_i = \frac{53.778 \times k}{D} = \frac{53.778 \times 0.03127}{.15} = 11.2 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$\begin{aligned} \text{Therefore resistance} &= \sum R_{th} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{11.2} + \frac{1}{6} = .089 + .166 \\ &= 0.255 \end{aligned}$$

$$\text{Heat flux} = \frac{Q}{A} = \frac{\Delta t}{\sum R_{th}} = \frac{(80 - -5)}{.255} = 294.1 \text{ W/m}^2$$

$$\begin{aligned} \text{From resistance network} &= \frac{Q}{A} = \frac{80 - t}{\frac{1}{h_i}} = \frac{(80 - t) h_i}{1} \\ 294.1 &= 11.2 (80 - t) \end{aligned}$$

$$t = 80 - 294.1/11.2 = 80 - 26.25$$

= 53.75° c duct surface temperature. Ans.

Q 29

Liquid mercury flows at the rate of 1.25kg/s through a copper tube of 20mm diameter. The mercury enters the tube at 15 ° c and after getting heated it leaves the tube at 25 °c. Calculate the tube length for constant heat flux at the wall which is maintained at an average temperature of 40 °c

For liquid metal flowing through a tube the following empirical correlation is presumed to agree for well experienced result: --

$$\bar{Nu} = 7 + 0.025(p_e)^{.8} \text{ where } p_e \text{ is Peclet number } (p_e = p_r R_e)$$

. [Ans .437 m]

Solution

$$\text{Given } D=20\text{mm}=.02\text{m}, m=1.25\text{kg/s } t_i=15 \text{ c } t_o=25 \text{ c}$$

$$L=\text{Tube length}=? \quad t_{avg}=40 \text{ c}$$

$$\text{Mean bulk temp} = \frac{15+25}{2} = 20 \text{ c}$$

Thermos physical properties at 20 c

$$\rho = 13582 \text{ kg/m}^3 \quad k = 8.69 \text{ w/m c}, \quad C_p = 140 \text{ j /kg c}$$

$$V = 1.5 \times 10^{-7} \text{ m}^2/\text{s}, p_r = 0.0248$$

Renold number $= \frac{\rho U D}{\mu} = \text{-----} = \frac{4m}{\pi D}$ [Earlier sums full calculation has been done)

$$= \frac{4 \times 1.25}{\pi (.02)(13582 \times 1.5 \times 10^{-7})}$$

$$=39060.36$$

$$\overline{Nu} = 7 + .025(p_r R_e)^{.8} = 7 + .025 (.0248 \times 39060.36)^{.8}$$

$$= 7 + 6.122 = 13.122$$

$$\bar{h} = 13.122 \times \frac{k}{D} = 13.122 \times \frac{8.69 \text{ W/m}^2 \text{ c}}{.02} = 5701.59 \text{ W/m}^2 \text{ c}$$

Heat gained by mercury due to convection is

$$Q = \bar{h} A \Delta t = \bar{h} \times (\pi \times .02 \times L) (40 - 20)$$

$$= 5701.59 \times (\pi \times .02 \times L \times 20)$$

$$= 7164 L \text{ -----i)}$$

$$\text{Heat gained by mercury} = Q = m C_p (t_o - t_1)$$

$$= 1.25 \times 140 (25 - 15)$$

$$= 1750 \text{ ----- ii)}$$

Heat gained by mercury due to convection (i)

= heat gained by mercury (ii)

$$\text{Or, } 7164 L = 1750$$

$$\text{So, } L = \frac{1750}{7164} = .244 \text{ m} = \text{length of tube Ans.}$$

Q 30

A square channel of side 15 mm and length 200m in a heat transfer problem carries water at a velocity of 6m/s. The mean temperature of water along the length of channel is found to be 30 °c while inner channel surface temperature is 70° c.

Calculate the heat transfer coefficient from channel water to water.
Use the following correlation

$$\overline{Nu} = 0.021(R_e^8)(p_r^{0.43})\left(\frac{p_r}{p_{rs}}\right)^{.25}$$

Thermos physical properties

A=area of channel, p= perimeter of channel

We can evaluate mean bulk temperature but not p_{rs} and P_r s

Physical properties of water at 30 ° c

$$\rho = 995.7 \text{ kg/m}^3 \quad k = 0.6175 \text{ W/m c}, \quad \nu = .805 \times 10^{-6} \text{ m}^2/\text{s}$$

$$p_r = 5.42, p_{r_s} \text{ at } 70 \text{ c} = 2.55 \quad [\text{Ans } 25456.6 \text{ W/m}^2 \text{ c}]$$

Solution

Given side of channel = 15mm = .015m, V=6m/s

$$t_b = 30 \text{ c} \quad t_s = 70 \text{ c}$$

Heat transfer coefficient = $\tilde{h}$

$$\begin{aligned} \text{Equivalent diameter of channel} = D_{eqv.} &= \frac{4A}{P} = \frac{4(.015 \times .015)}{4 \times .015} \\ &= .015 \text{ m} \end{aligned}$$

$$\text{Reynold number} = \frac{VD_{eq}}{\nu} = \frac{6 \times .015}{.805 \times 10^{-6}} = 111801.24.$$

Nusselt number = $\overline{Nu}$ (as in this Q30)

$$\begin{aligned} &= .021(111801.24)^8 (5.42)^{.43} \left(\frac{5.42}{2.55}\right)^{.25} \\ &= .021 \times 10933.45 \times 2.068 \times 1.20743 \\ &= 573.3 \end{aligned}$$

$$\text{Also } \overline{Nu} = \frac{\tilde{h}}{k} \times D_{eqv.} = 573.3$$

$$\text{Therefore } \bar{h} = \frac{573.3 \times k}{Deqv} = \frac{573.3 \times 6175}{.015} = 23600.85 \text{ w/m}^2 \text{ } ^\circ\text{C Ans.}$$

Q31 A copper bus bar 20mm diameter is cooled by air (in cross flow) at 30 °C and flowing past the bus bar with a velocity of 2m/s. If the surface temperature of the bar is not to exceed 80 °C and resistivity of copper is $0.0175 \times 10^{-6} \text{ ohm-m}^3/\text{m}$.

Calculate the following

- i) heat transfer coefficient from surface to the air.
- ii) permissible current density from the bus bar.

Following correlation may be applicable for a single cylinder placed in cross flow;

$$\text{For } 10 < Re < 10^3 \quad \bar{Nu} = 0.44 (Re)^{0.5}$$

$$\text{For } 10^3 < Re < 2 \times 10^5 \quad \bar{Nu} = 0.22 (Re)^{0.6}$$

Thermos physical properties at $t_{\infty} = 30^\circ\text{C}$

$$K = 0.02673 \text{ w/m } ^\circ\text{C} \quad \nu = 16 \times 10^{-6} \text{ m}^2/\text{s}$$

$$[\text{Ans i) } 32.14 \text{ w/m}^2 \text{ } ^\circ\text{C ii) } 1346 \text{ amp}]$$

Solution

$$\text{Given } D = 20\text{mm} = .02\text{m}, \quad t_{\infty} = 30^\circ\text{C} \quad t_s = 80^\circ\text{C}$$

$$U = 2 \text{ m/s} \quad \rho_{\text{copper}} = \text{resistivity} = 0.0175 \times 10^{-6} \text{ ohm-m}^3/\text{m}$$

i) heat transfer coefficient from surface to the air $\bar{h}$

$$\begin{aligned} \text{Reynold number } = Re &= \frac{VD}{\nu} = \frac{2 \times .02}{16 \times 10^{-6}} = .0025 \times 10^6 \\ &= 2500 \end{aligned}$$

Since $Re = 2500$ so we go for 2nd relation

$$\bar{Nu} = \frac{\bar{h} D}{K} = 0.22(R_e)^{0.6} = 0.22 \times (2500)^{0.6}$$

$$= 24.05$$

$$\bar{h} = \frac{k}{D} \times 24.05 = \frac{0.02673}{0.02} \times 24.05 = 32.14 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

ii) permissible current intensity for the bus bar 'I'

Heat dissipation to air $= \bar{h} A (t_s - t_i)$

$$= 32.14 (\pi \times D \times l) (80 - 30)$$

$$= 32.14 (\pi \times 0.02 \times 1) \times 50 = 100.97 \text{ W/min} \text{ -----(i)}$$

Heat generated in bus bar $= I^2 X_r = I^2 x \frac{\rho L}{A_c} = I^2 x \frac{0.0175 \times 10^{-6} \times 1}{\frac{\pi}{4} \times (0.02)^2}$

$$= I^2 \times 10^{-6} \times \left(\frac{0.07}{0.00125} \right) = 56 \times 10^{-6} \times I^2 \text{ -----(ii)}$$

From (i) and (ii) we get

$$56 \times 10^{-6} \times I^2 = 100.97$$

$$\text{So } I^2 = \frac{100.97}{56 \times 10^{-6}} = \frac{100.97 \times 10^6}{56}$$

$$= 1803065$$

$$I = \sqrt{1803065}$$

$$= 1342.78 \text{ amp Ans.}$$

Q 32

Air at 25°C flows across an elliptical tube 60mm x 120mm perpendicular minor axis with a velocity of 3m/s. Tube surface is

maintained at 55 °c. Determine the value of convection coefficient [Ans; 25.1w/m²k]

Solution

Given $t_{\infty}=25^{\circ}\text{c}$ $t_s=55^{\circ}\text{c}$. $U=3\text{m/s}$. $D_1=60\text{mm}=.06\text{m}$

$$D_2=120\text{mm}=.12\text{m}$$

$$\text{Film temperature} = t_f = \frac{t_{\infty} + t_s}{2} = \frac{25 + 55}{2} = 80/2 = 40^{\circ}\text{c}$$

Properties at 40° c

$$V = 17.6 \times 10^{-6} \text{m}^2/\text{s} \quad k = 0.0265 \text{w/mk}$$

$$p_r = 0.71$$

$$\begin{aligned} \text{Renold number} &= \frac{UD_1}{V} = \frac{3 \times .06}{7.6 \times 10^{-6}} = \frac{3 \times .06 \times 10^6}{17.6} = .0102 \times 10^6 \\ &= 10200. \end{aligned}$$

$$\text{Now Nusselt no } \overline{Nu} = C(R_e)^n(p_r)^{\frac{1}{3}}$$

When value of C and n for table 7.3 are C=.224, n=.612

Since cross section is Eclipse (60 mmx200 mm) and C=.224,

$$n=.612, R_e = 2.5 \times 10^3 = 2500 \text{ to } 1.5 \times 10^4 = 15000$$

Substituting the value, we get

$$\begin{aligned} \overline{Nu} &= C (R_e^n) (p_r^{\frac{1}{3}}) = .224 \times (10200)^{.612} \times (.71)^{.333} \\ &= .224 \times 283.96 \times .8922 \\ &= 56.75 \end{aligned}$$

$$\text{Bue } \overline{Nu} = \frac{h D_1}{K}$$

$$\text{So, } h = \text{convection coefficient} = \frac{\overline{Nu}}{D_1} \times k = \frac{56.75 \times 0.0265}{.06}$$

$$= 25.06 \text{ W/m}^2\text{k} \quad \text{Ans.}$$

Q 34

Liquid sodium is to be heated from 120 °c to 149° c at a rate of 2.3 kg /s in a 25mm diameter electrically heated tube (constant heating flux.). Calculate the minimum length of the tube if its wall temperature is not to exceed 20° c, [0.364m Ans]

Solution

Given $D=25\text{mm}=0.025\text{m}$, $m=2.3\text{kg/s}$, $\Delta t = 149-120=29^\circ\text{C}$

$t_w=t_s=20^\circ\text{C}$, average bulk temp. $=\frac{149+120}{2}=134.5^\circ\text{C}=t_b$

Properties of sodium at 134.5

$\rho=916\text{ kg/m}^3$, $v=0.594 \times 10^{-6}\text{m}^2/\text{s}$, $p_r=0.0087$.

$C_p=1.3565\text{ KJ/Kg K}$ $k=84.9\text{W/mk}$.

Now heat transfer (Q) at constant heat flow condition is calculated as;

$$Q = m C_p \Delta t = 2.3 \times 1.3565 \times (149-120) = 90.47\text{kw.}$$

But $m = \rho A U$

$$U = \text{Velocity of flow} = \frac{m}{A \cdot \rho} = \frac{2.3}{\frac{\pi}{4} \times 0.025^2 \times 916} = \frac{9.2}{1.79856} = 5.11\text{m/s}$$

$$\text{Reynold No, } R_e = \frac{UD}{v} = \frac{5.11 \times 0.025}{0.594 \times 10^{-6}} = \frac{127750}{.594} = 215067.34$$

$$\begin{aligned} \text{And Peclet No} = P_e &= R_e P_r = 215067.34 \times 0.0087 \\ &= 1871.08 \end{aligned}$$

Using following relation, we have

$$\begin{aligned} \bar{Nu} &= 4.82 + 0.0185 (P_e)^{.827} \quad [\text{equ; 7.308}] \\ &= 4.82 + 0.0185 (1871.08)^{.827} \\ &= 4.82 + 9.40 = 14.22 \end{aligned}$$

$$\text{Also } \bar{Nu} = \frac{\bar{h}D}{k} = 14.22$$

$$\bar{h} = \frac{14.22 \times 84.90}{.025} = 48291.12 \text{ W/m}^2\text{K}$$

Now by energy balance we get length of the tube; --

$$Q = \bar{h} A (t_w - t_b) = \bar{h} \pi D L (t_w - t_b)$$

$$90.47 \times 1000 = 48291.12 \times \pi \times .025 \times L (200 - 134.5)$$

$$\therefore L = \frac{90470}{48291.12 \pi \times .025 \times 65.5} = \frac{90470}{248426.8} = .364 \text{ m} \quad \text{Ans.}$$

Chapter 8

Free Convection

Q 1

A hot plate 1.1 wide ,300mm high and 120°C is exposed to the ambient still air at 20 ° c. Calculate

- i) maximum velocity boundary layer thickness and local heat transference coefficient at 150 mm from the leading edge of the plate.
- ii) total mass flow through the boundary
- iii) heat loss from the plate
- iv) rise in temperature of the air passing through the boundary.

[i) 0.39m/s ,11,4 mm,5.199 ii).004372kg/s]

Solution

Given hot plate width =1.1m, height =.3m, $t_s = 120^\circ\text{C}$.

$t_\infty = 20^\circ\text{C}$, $x = .15\text{m}$ from leading edge.

Mean film temp. $\frac{120+20}{2} = 70^\circ\text{C}$

Thermophysical properties at 70 C

$K = 0.02964 \text{ W/m }^\circ\text{C}$, $\rho = 1.029 \text{ kg/m}^3$, $\nu = 20.02 \times 10^{-6} \text{ m}^2/\text{s}$

$p_r = 0.694$, $\beta = \frac{1}{T} = \frac{1}{273+70} = \frac{1}{343} = 0.002915$

Grashoff number $= G_{rx} = \frac{x^3 g \beta (t_s - t_\infty)}{\nu^2} = \frac{9.81 \times 0.002915 \times (120 - 20) \cdot .15^3}{(20.02 \times 10^{-6})^2}$
 $= 1.787 \times 10^7 = y$

$G_{rL} = Y \left(\frac{high}{x} \right)^3 = 1.787 \left(\frac{.3}{.15} \right)^3 = 14.29 \times 10^7$

i) maximum velocity U_{max} from .15m from leading edge

$$\begin{aligned}
 &= .766 \sqrt{(.952 + \frac{v}{\alpha})^{-\frac{1}{2}} [G_{rx}]^{-\frac{1}{2}} x^{\frac{1}{2}}} \quad [p_r = \frac{v}{\alpha} = .694] \\
 &= .766(20.02 \times 10^{-6}) \quad \times \\
 & (.952 + .694)^{-\frac{1}{2}} \left[\frac{9.81 \times .002915(120 - 20)}{(20.02 \times 10^{-6})^2} \right]^{-\frac{1}{2}} (.15)^{\frac{1}{2}} \\
 &= .766 x^{\frac{.0000202 \times .7794}{.0000202}} [.0286 \times 100]^{-\frac{1}{2}} x (.15)^{\frac{1}{2}} \\
 &= .766 \times .7794 \times (2.86)^{-\frac{1}{2}} \times .387 \\
 &= .766 \times .7794 \times 1.691 \times .387 \\
 &= .390 \text{ m/s} \quad \text{Ans.}
 \end{aligned}$$

ii) local heat transfer coefficient through the boundary;

$$\begin{aligned}
 \bar{Nu} = \frac{h_x x}{K} &= 0.508 (P_r)^{\frac{1}{2}} (.952 + p_r)^{-\frac{1}{4}} \\
 (G_x)^{\frac{1}{4}} &= \frac{2x}{\delta} \quad (\text{equ. 8.31}) \quad (\delta = \text{boundary thickness})
 \end{aligned}$$

We have to find out boundary thickness first;

$$\begin{aligned}
 \frac{\delta}{x} &= 3.93 (.952 + P_r)^{\frac{1}{4}} (G_{rx})^{-\frac{1}{4}} (p_r)^{-\frac{1}{2}} \quad [\text{Eq u , 8.27}] \\
 &= 3.93 (.952 + .694)^{\frac{1}{4}} (24128001)^{-\frac{1}{4}} (.694)^{-\frac{1}{2}} \\
 &= 3.93 \times (1.646)^{\frac{1}{4}} (.01426) \times 1.2 \\
 &= 3.93 \times 1.132 \times .01426 \times 1.2 \\
 &= .0761 \quad (\text{now we have to multiply } \delta \text{ with } x \text{ i. e. } .15 \text{ m})
 \end{aligned}$$

$$\text{So } \delta = .0761 \times .15 = .01142 \text{ m} = 11.42 \text{ mm} \quad \text{Ans}$$

iii) local heat transfer coefficient

$$h_x = \frac{2k}{\delta} = \frac{2 \times .02964}{.01142} = 5.19 \text{ W/m}^2 \text{ } ^\circ\text{C} \quad \text{Ans.}$$

iv) total mass flow through the boundary (m_f)

on the basis of following equation, we calculate the mass flow; --

$$m_f = 1.7 \rho v \left[\frac{Gr_L}{(Pr^2)(Pr + 0.952)} \right]^{\frac{1}{4}}$$

$$\begin{aligned} \text{We have } Gr_L &= Gr_x \times \left(\frac{\text{Height}}{x} \right)^3 = 24128001 \left(\frac{.3}{.15} \right)^3 \\ &= 2412800 \times 8 \end{aligned}$$

$$\begin{aligned} m_f &= 1.7 \times 1.029 (20.02 \times 10^{-6} \left[\frac{2412800 \times 8}{.694^2 (.694 + .950)} \right]^{.25}) \\ &= 35.02 \times 10^{-6} \left[\frac{24128001}{.4816 \times 1,644} \right]^{.25} \\ &= .00003502 \times \left[\frac{24128001}{.7917504} \right]^{.25} \\ &= .00003502 \times [30474251.73]^{.25} \\ &= .00003502 \times 74.299 \\ &= .003601 \text{ kg/s } \text{ Ans} \end{aligned}$$

Q 2

A 300 mm long glass plate is hung vertically in the air at 27°C while its temperature is maintained at 77°C . Calculate boundary layer thickness at the trailing edge of the plate. If a similar plate placed in a wind tunnel and air is blown over it at a velocity of 4 m/s find the boundary layer thickness at its trailing edge. Also determine heat transfer coefficient for natural and forced convection for the above-mentioned data.

$$[\text{Ans}; 15.2 \text{ mm}, 5.67 \text{ mm}, 4.9 \text{ W/m}^2 \text{ } ^\circ \text{C}, 14.11 \text{ W/m}^2 \text{ } ^\circ \text{C}]$$

Solution

Given glass; - $L = .3 \text{ m}$, $t_\infty = 27^\circ \text{C}$. $U = 4 \text{ m/s}$, $v = 18.41 \times 10^{-6}$

$$t_s = 77^\circ \text{C}, k = 28.15 \times 10^{-3} \text{ W/m}^\circ \text{C}, Pr = 0.7$$

$$t_f = \frac{t_s + t_\infty}{2} = \frac{77 + 27}{2} = 52^\circ\text{C} \quad \beta = \frac{1}{T} = \frac{1}{52 + 273} = \frac{1}{325} = 3.07 \times 10^{-3} \text{K}^{-1}$$

I) a) Boundary layer thickness to find;

$$\text{Grashoff number} = \frac{g\beta L^3 (t_s - t_\infty)}{v^2} = \frac{9.81 \times 3.07 \times 10^{-3} (77 - 27) (.3)^3}{(18.41 \times 10^{-6})^2}$$

$$= 1.2 \times 10^8 = G_{rL}$$

$$\text{Reynold number} = G_{rL} P_r = 1.2 \times 10^8 \times .7 = .8 \times 10^8$$

According to equ. 8.36 indicates a laminar boundary layer,

So, using equation 8.27 we get

$$\frac{\delta}{x} = 3.93 (0.952 + P_r)^{.25} (G_r)^{-.25} (P_r)^{-.5}$$

$$\frac{\delta}{x} = 3.93 (.952 + .7)^{.25} \times 1.2 \times 10^8)^{-.25} \times (.7)^{-.5}$$

$$= 3.93 \times 1.133 \times .00955 \times 1.195$$

$$= 0.050815$$

$$\therefore \delta = X \times 0.050815$$

$$= .3 \times 0.050815$$

$$= .015244 \text{ m}$$

$$= 15,244 \text{ mm Ans.}$$

i) b)

for forced convection;

$$\text{Reynold number} = \frac{UL}{V} = \frac{4 \times .3}{18.41 \times 10^{-6}} = \frac{1.2}{18.41 \times 10^{-6}} = .065 \times 10^6$$

$$\text{So, } Re_L = 6.5 \times 10^4$$

So, boundary layer is laminar

$$\delta = \frac{5L}{\sqrt{6.5 \times 10^4}} = \frac{5 \times .3}{2.849 \times 10^2} = \frac{1.5}{254.9} = .00588 \text{ m} = 5.88 \text{ mm.}$$

ii) heat transfer coefficient (free convection)

$$\begin{aligned}\bar{Nu}_L &= \frac{\bar{h}L}{K} = .677(P_r)^{.5}(0.952 + P_r)^{-.25}(G_r)^{.25} \\ &= .677(.7)^{.5}(0.952 + .7)^{-.25}(1.2 \times 10^8)^{.25} \\ &= .677 \times .836 \times .862 \times 104,66 \\ &= 52.244\end{aligned}$$

$$\bar{h} = \frac{\bar{Nu}_L K}{L} = \frac{52.244 \times 28 \times 10^{-3}}{.3} = \frac{1470.66 \times 10^{-3}}{.3} = 4.9 \text{ W/m}^2 \text{ } ^\circ\text{C} \quad \text{Ans}$$

In case of Forced convection

$$\begin{aligned}\bar{Nu}_L &= \frac{\bar{h}L}{K} = 0.664 (Re_L)^{.5} (P_r)^{.333} \\ \bar{h} &= \frac{K}{L} 0.664 (6.5 \times 10^4)^{.5} (.7)^{.333} \\ &= \frac{28.15 \times 10^{-3}}{.3} \times 0.664 \times (65000)^{.5} (.7)^{.333} \\ &= .09383 \times .664 \times 254.95 \times .888 \\ &= 14.1 \text{ W/m}^2 \text{ } ^\circ\text{C} \quad \text{Ans.}\end{aligned}$$

Q 3

A nuclear reactor with its core construction of parallel vertical plates 2.25 m high and 1.5 m wide has been designed on free convection heating of liquid bismuth. The maximum temperature of the plate surface is limited 975 °c while the lowest allowable temperature of bismuth is 325 °c. Calculate the possible heat dissipation from both sides of each plate.

For the convection coefficient the appropriate correlation is

$Nu=0.13(G_r P_r)^{.333}$ where parameter is evaluated at the mean film temperature. [153 MW]

Solution

Given height = 2.25m. wide = 1.5 m. Max temp = $t_{max}=975^\circ\text{C}$

Lowest temperature = $t_{lowest}=325^\circ\text{C}$

Mean film temperature = $\frac{975+325}{2}=650^\circ\text{C}$

Thermophysical properties

$C_p=150.7\text{ J/kg}^\circ\text{C}$, $\rho=10^4\text{ kg/m}^3$, $\mu=3.12\text{ kg/m h}$

$K=13.02\text{ W/m}^\circ\text{C}$, $\beta=\frac{1}{T}=\frac{1}{650+273}=1.08\times 10^{-3}\text{ K}^{-1}$

$P_r=\frac{\mu C_p}{K}=\frac{3.12}{3600}\times\frac{150.7}{13.02}=.01$

$$G_r=\frac{L^3\rho^2g\beta\Delta t}{\mu^2}=\frac{2.25^3\times(10^4)^2\times 9.81\times 1.08\times 10^{-3}(975-325)}{\frac{3.12^2}{3600}}$$

$$=\frac{11.39\times 10^8\times 9.81\times .00108\times 650}{(.000866)^2}$$

$$=\frac{7843860180}{.00000075}=10458450240\times 10^6$$

$$=1.04\times 10^{16}$$

$(G_r P_r)=1.04\times 10^{16}\times(.01)=1.04\times 10^{15}$

$Nu=\frac{hL}{K}=0.13(G_r P_r)^{.333}=0.13(1.04\times 10^{15})^{.333}$

$h=\frac{k}{L}[.013][46610]=\frac{13.02}{2.25}\times .013\times 46610$

$$=46610\times .7522=35060\text{ W/m}^2\text{C}$$

Heat dissipation from both sides of each plate = Q

$=2 h A \Delta t=2\times 35060(2.25\times 1.5)(975-325)$

$$=153.82 \times 10^6 = 153.8 \text{ M W. Ans.}$$

Q 4.

Find the convective heat loss from a radiator 0.5m wide and 1m high maintained at a temperature of 84°C in a room at 20°C . Consider the radiator at a vertical plate. [110.08 w]

Solution

Given $L=1\text{m}$. width $=b=.5\text{m}$, $t_s = 84^\circ \text{C}$ $t_\infty = 20^\circ \text{C}$

$$\text{Film temperature} = (84+20)/2 = 52^\circ \text{C}$$

Thermophysical properties

$$K = 28.15 \times 10^{-3} \text{ W/m}^\circ \text{C}, \nu = 18.41 \times 10^{-6} \text{ m}^2/\text{s},$$

$$\beta = \frac{1}{52+273} = \frac{1}{325} = 3.077 \times 10^{-3} \text{ K}^{-1}$$

$$\begin{aligned} \text{Rayleigh no} = R_{aL} &= (G_{rL} \cdot P_r) = \frac{g\beta(t_s - t_\infty)L^3}{\nu^2} \times P_r \\ &= \frac{9.81 \times 3.077 \times 10^{-3} (84-20) \times 1^3}{(18.41 \times 10^{-6})^2} \times .7 \\ &= 3.9 \times 10^9 \end{aligned}$$

Since $G_r P_r > 10^9$ we will follow equation 8.40 as per M r Churchill and M r Chu as below; --

$$\begin{aligned} \bar{N}uL &= \left[.825 + \frac{.387(G_r P_r)^{.166}}{\left[1 + \left(\frac{.492}{P_r} \right)^{.5625} \right]^{.296}} \right]^2 \\ &= \left[.825 + \frac{.387(3.9 \times 10^9)^{.166}}{\left[1 + \left(\frac{.492}{.7} \right)^{.5625} \right]^{.296}} \right]^2 \\ &= \left[.825 + \frac{15.127}{1.194} \right]^2 = [.825 + 12.67]^2 \\ &= [13.495]^2 = 182.11 \\ \bar{N}uL &= 182.11 \end{aligned}$$

$$\text{So } \bar{h} = \frac{K}{L} x \bar{N} U I = \frac{28.15 \times 10^{-3}}{1} \times 182.11$$

$$= .02815 \times 182.11$$

$$= 5.126 \text{ W/m}^2\text{°C}$$

$$\text{Convection heat loss } = Q = \bar{h} A \Delta t$$

$$= 5.126 \times (1 \times .5) \times (84 - 20)$$

$$= 5.126 \times .5 \times 64$$

$$= 164.03 \text{ W} \quad \text{Ans}$$

Q 5

A steam pipe 60mm in diameter is covered with 20mm thick layer of insulation which has a surface emissivity of 0.92. The surface temperature of insulation is 75° c and the pipe is placed in atmospheric air at 25° c considering heat loss both by radiation and natural convection, Calculate

i) heat loss from 5 m length of the pipe.

ii) Overall heat transfer coefficient and heat transfer coefficient due to radiation alone, [Ans; 1049.83 w ,ii) 7.07 w/m²°c]

solution

$$\text{given } D = 60\text{mm} + 2 \times 20\text{mm (insulation)} = .1 \text{ m}$$

$$t_s = 75^\circ\text{C}, \quad t_\infty = 25^\circ\text{C} \quad \text{emissivity} = \epsilon = .92$$

$$\text{The film temperature } = t_f = \frac{75 + 25}{2} = 50^\circ\text{C}$$

Thermophysical properties of air at 50 c

$$C_p = 1007 \text{ J/KG }^\circ\text{C}, \quad \rho = 1.092 \text{ KG/m}^3, \quad \mu = 19.57 \times 10^{-6} \text{ kg/m}$$

$$K=27.81 \times 10^{-3} \text{ W/m } \cdot \text{ }^{\circ}\text{C}, \beta = \frac{1}{T} = \frac{1}{50+273} = \frac{1}{323} = 3.096 \times 10^{-3} \text{ K}^{-1}$$

$$P_r = \frac{\mu C_p}{K} = \frac{19.57 \times 10^{-6} \times 1007}{27.81 \times 10^{-3}} = .708$$

$$\begin{aligned} G_{rL} &= \frac{g \beta D^3 \rho^2 \Delta t}{\mu^2} = \frac{9.81 \times 3.096 \times 10^{-3} (.1)^3 (1.092)^2 (75-25)}{(19.57 \times 10^{-6})^2} \\ &= \frac{.0018107643312}{.00000000038298} = 4728091.1 \\ &= 4.7 \times 10^6 \end{aligned}$$

Using equation 8.45 we get $\bar{Nu} = \frac{\bar{h}D}{K} = .53(G_r P_r)^{.25}$

$$\begin{aligned} \bar{h} &= \frac{27.81 \times 10^{-3}}{.1} \times .53 (4.7 \times 10^6 \times .708)^{.25} \\ &= 0.278 \times .53 \times (3327600)^{.25} \\ &= 0.278 \times .53 \times 42.71 \\ &= 6.29 \text{ W/m}^2 \end{aligned}$$

i) heat lost from 5 m length of pipe; ---

a) heat lost by convection; $Q_{con} = \bar{h} A \Delta t$

$$\begin{aligned} Q_{con} &= 6.29 \times (\pi \times D \times 5) (75-25) \\ &= 6.29 \times 1.57 \times 5 \times 50 = 494 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{b) heat lost by radiation} &= Q_{rad} = \epsilon \times \sigma \times A \times (T_1^4 - T_2^4) \\ &= .92 \times (5.67 \times 10^{-8}) (\pi \times .1 \times 5) \{ (75+273)^4 - (25+273)^4 \} \\ &= .92 \times (.0000000567) (1.57) \{ (348)^4 - (298)^4 \} \\ &= .92 \times .0000000567 \times 1.57 \times 6780028400 \\ &= 555.267 \text{ W} \end{aligned}$$

Therefore, total heat loss $Q = Q_{con} + Q_{rad}$

$$= 494 + 555.267 = 1049.267 \text{ W Ans.}$$

ii) overall heat transfer coefficient (h_f) and

b) due to radiation alone (h_{rad})

$$a) Q_t = h_t A \Delta t$$

$$h_t = \frac{Q_t}{A \Delta t} = \frac{1049}{1.57 \times 50} = 13.35 \text{ w/m}^2$$

b) heat radiation = heat total --- heat convection

$$= 13.35 \text{ --- } 6.29$$

$$= 7.06 \text{ w /m}^2 \text{ } ^\circ \text{C Ans}$$

Q 6

Calculate the heat transfer from a 60w incandescent bulb at 125°C to ambient air 25°C , Assume the bulb as a sphere of 50 mm diameter. Also find %age power loss by free convection. The correlation is given by $Nu = 0.60(G_r P_r)^{\frac{1}{4}}$, [7.71 w, 19.278 %]

Solution

Given

$$D = \text{Diameter of sphere (bulb)} = 50 \text{ mm} = .05 \text{ m}$$

$$t_s = 125, ^\circ\text{C} \quad t_\infty = 25 ^\circ\text{C} \quad \text{film temp} = t_f = \frac{125 + 25}{2} = 75 ^\circ\text{C}$$

Thermophysical properties of air at $75 ^\circ\text{C}$

$$k = 2.964 \times 10^{-2}, \quad \nu = 20.02 \times 10^{-6}, \quad p_r = .694$$

$$\beta = \frac{1}{t + 273} = \frac{1}{75 + 273} = \frac{1}{348} = 2.87 \times 10^{-3}$$

$$G_r = \frac{g\beta\Delta t D^3}{\nu^2} = \frac{9.81 \times 2.87 \times 10^{-3} (125-25)(.05)^3}{(20.02 \times 10^{-6})^2}$$

$$= 8.78 \times 10^5$$

Characteristic length = L = D in this case.

Using the given correlation as below;

$$\bar{N}u = \frac{\bar{h}L}{k} = 0.60 (G_r P_r)^{.25}$$

$$\bar{h} = \frac{\bar{N}ux k}{L} = \frac{0.60 (8.78 \times 10^5 \times 6.94)^{.25}}{.05} \times \frac{2.964 \times 10^{-2}}{1}$$

$$= 335268 \times .02964 = 9.93$$

$$\text{Heat transfer} = \bar{h} A \Delta t = 9.93 \times [\pi \times (.05)^2] (125-25)$$

$$= 7.799 \text{ W}$$

% age of power lost by free convection

$$= \frac{\text{heat transfer}}{\text{bulb power}}$$

$$= \frac{7.799}{60} \times 100$$

$$= 12.998 \% \text{ Ans.}$$

Q 7

A vertical slot of 15mm thickness is formed by 2 2m x 2m square plates. If the temperature of the plates are 120 c and 20 c

Respectively. Calculate the following;

i) the effective thermal conductivity.

ii) the rate of heat flow through the slot.

$$[\text{ i) } 0.031 \text{ W/m c ii) } 845.3 \text{ W}]$$

Solution

$$\text{Given } L = 15 \text{ mm} = .015 \text{ m}, t_1 = 120^\circ \text{C}, t_2 = 20^\circ \text{C}$$

i) effective conductivity k_e

$$\text{mean temperature} = \frac{t_1 + t_2}{2} = \frac{120 + 20}{2} = 70^\circ \text{C}$$

properties of air at 70°C are

$$k = 0.0295 \text{ W/m c}, \nu = 2.0 \times 10^{-5} \text{ m}^2/\text{s},$$

$$\beta = \frac{1}{70 + 273} = \frac{1}{343} = 2.915 \times 10^{-3}$$

The Grashoff number based on gap L is

$$\begin{aligned} G_{rL} &= \frac{L^2 g \beta \Delta t}{\nu^2} = \frac{(.015)^2 \times 9.81 \times 2.915 \times 10^{-3} (120 - 20)}{(2 \times 10^{-5})^2} \\ &= \frac{0.000009651200625}{.0000000004} \\ &= 24128.0015625 \\ &= 2.41 \times 10^4 \end{aligned}$$

The effective thermal conductivity is given by

$$\frac{k_e}{k} = 0.064 (G_{rL})^{\frac{1}{3}} \left(\frac{H}{L}\right)^{-\frac{1}{9}} \quad [\text{EQN-8.50}]$$

$$\begin{aligned} K_e &= .0295 [.064 \times (2.41 \times 10^4)^{.333} \left(\frac{2}{.015}\right)^{-1.11}] \\ &= .0295 [.064 \times 28.799 \times .58] \\ &= .031536 \text{ W/m}^\circ \text{C} \quad \text{Ans.} \end{aligned}$$

ii) The rate of heat flow through the slot Q

$$\begin{aligned}
 &= k_e X A X \frac{(t_1 - t_2)}{L} \\
 &= .031536 \times (2 \times 2) \frac{(120 - 20)}{.015} \\
 &= \frac{12.6144}{.015} \\
 &= 840.96 \\
 &= 841 \text{ Ans}
 \end{aligned}$$

Q 8 Two horizontal plates separated by a distance of 25.4 mm contain air at atmospheric pressure. The temperature of lower and upper panels is 60 ° c and 15.6 ° c respectively. Calculate the free convection heat transfer per meter square of the panel surface. [Ans; 167.2 w/m²]

Solution

$$\text{Given } L = 25.4 \text{ mm} = .0254 \text{ m}, t_1 = 60^\circ \text{ c } t_2 = 15.6^\circ \text{ c}$$

$$\text{Mean temperature of 2 panel} = \frac{60 + 15.6}{2} = 37.8^\circ \text{ c}$$

Properties of air at 37.8 ° c

$$K = 29.2 \times 10^{-3} \text{ w/m}^\circ \text{ c}, \rho = 1.121 \text{ kg/m}^3$$

$$\beta = \frac{1}{37.8 + 273} = 3.22 \times 10^{-3} \text{ K}^{-1}$$

$$V = .171 \times 10^{-4}$$

i) Free convection heat transfer /m² of the panel surface Q

$$\begin{aligned}
 G_{rL} &= \frac{L^3 g \beta \Delta t}{\nu^2} = \frac{(.0254)^3 \times 9.81 \times (3.22 \times 10^3) (60 - 15.6)}{(.171 \times 10^{-4})^2} \\
 &= 7.3 \times 10^4
 \end{aligned}$$

$$\overline{Nu} = 0.195(G_{rL})^{\frac{1}{4}} = 0.195 (7.3 \times 10^4)^{.25}$$

$$= 3.186$$

$$\text{Also } \overline{Nu} = \frac{q \cdot L}{K(t_1 - t_2)} \quad \left[\text{where } q = \frac{Q}{A} \right]$$

$$q = \frac{\overline{Nu} \cdot K(t_1 - t_2)}{L} = \frac{3.186 \times 29.2 \times 10^{-3} (60 - 15.156)}{.0254}$$

$$= \frac{3.186 \times .0292 \times 44.4}{.0254}$$

$$= 162.62 \text{ W/m}^2$$

Chapter 9

Boiling and Condensation

Q 1

Water at atmospheric pressure is to be boiled in a polished copper pan. The diameter of the pan is 300mm and kept at 111°C. Calculate the following

i) power of the burner to maintain boiling.

ii) rate of evaporation in kg/hr.

take the properties of water at 100

$$\rho_1 = 958 \text{ kg/m}^3, \rho_2 = 0.597 \text{ kg/m}^3, \mu_f = 278 \times 10^{-6} = \mu_l$$

$$c_{pf} = 4216 \text{ J/kg} = c_{pl}, h_{fg} = 2257 \text{ kJ/kg}, p_r = 1.723,$$

$$\sigma = 58.9 \times 10^{-3} \text{ N/m [i) 13.664 kW, ii) 21.8 kg/h]}$$

solution

given

$$\Delta t_e = \text{excess temp.} = t_s - t_{swl} = 111 - 100 = 11 \text{ }^\circ\text{C}$$

$$D = 300 \text{ mm} = 0.3 \text{ m}$$

For polished copper pan $C_{sl} = 0.013$

i) Power of burner to maintain boiling

As per boiling curve $\Delta t_e = 11 \text{ }^\circ\text{C}$, nuclear pool boiling will occur and for that following correlation will hold good [eq 9.8, table 9]

$$\begin{aligned}
 q_s &= \mu_l \times h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{0.5} \left[\frac{C_{pl} \Delta t_e}{C_{CL} x h_{fg} P_{rl}^n} \right]^3 \\
 &= 279 \times 10^{-6} \times 2257 \times 10^3 \left[\frac{9.81(958.4 - .5955)}{58.9 \times 10^{-3}} \right]^{.5} \left[\frac{4220 \times 11}{0.013 \times 2257 \times 10^3 \times 1.75} \right]^3 \\
 &= 629.7 \times 399.4 \times (.9)^3 \\
 &= 183345.08 = 183.354 \times 10^3 \text{ W/ } m^2
 \end{aligned}$$

So, power of burner

$$\begin{aligned}
 Q &= q_s \times A = 183.345 \times 10^3 \times \frac{\pi}{4} \times (D)^2 \\
 &= 183.345 \times 10^3 \times \frac{\pi}{4} \times (.3)^2 \\
 &= 12959.89 \text{ W} \\
 &= 12.959 \text{ KW Ans.}
 \end{aligned}$$

ii) rate of evaporation m_w

under steady state condition all the heat added to the pan will result in evaporation of water, thus

$$\begin{aligned}
 Q &= m_w \times h_{fg} \\
 m_w &= \frac{Q}{h_{fg}} = \frac{12959}{2257000} = .0057 \text{ kg/s} \\
 &= 20.67 \text{ kg/h r Ans.}
 \end{aligned}$$

Q2

A wire of 10mm diameter and 150mm length is submerged horizontally in water at 7 bars. The wire carries a current 131.5 A with an applied voltage 2.15 volt. If the surface of the wire is maintained at 180 c , calculate i) heat flux ii) boiling heat transfer coefficient , [Ans i) 0.6 MW ii) 39920 w/m² c]

solution

given = $d = 1\text{ mm} = .001\text{ m}$, $L = 150\text{ mm} = .15\text{ m}$

$I = 131.5\text{ A}$, $V = 2.15\text{ Volt}$, $t_s = 180\text{ c}$ = surface temp.

$t_{\text{saturated}} = t_{\text{sat}}$, t_e = excess temp, $= t_s - t_{\text{sat}}$

i) heat flux = q

electric energy input is given by $Q = VI = 2.15 \times 131.5$
 $= 282.725\text{ W}$

Surface area of wire = $A = \pi d l = \pi .001 \times .15 = .00047\text{ m}^2$

$$q = \frac{Q}{A} = \frac{282.725}{.00047} = 61117 = .6 \times 10^6\text{ MW./m}^2 \text{ Ans}$$

ii) boiling heat transfer coefficient h

corresponding to 7 bar saturated temp. $t_{\text{sat}} = 164.97\text{ c}$

$$\therefore q = h(t_s - t_{\text{sat}}) = h(180 - 164.97) = h \times 15.03$$

$$\therefore h = \frac{.6 \times 10^6}{15.03} = 39920\text{ W Ans.}$$

Q 3

An electric wire of 1.5mm diameter and 200mm long is laid horizontally and submerged in water at atmospheric pressure. The wire has an applied voltage of 16 v and carries a current of 40 amperes

Calculate i) heat flux and ii) excess temperature.

[i) 0.679 MW /m^2 ii) 18.52° c]

Solution

$$\text{Given } d = 1.5 \text{ mm} = \frac{1.5}{1000} = .0015 \text{ m}$$

$$L = 200 \text{ mm} = .2 \text{ m}$$

$$V = 16 \text{ volt} \quad I = 40 \text{ amp.}$$

$$A = \text{surface area of wire} = \pi d l = \pi \times .0015 \times .2 = .00094 \text{ m}^2$$

$$\text{Electrical energy input into wire} = Q = VI = 40 \times 16 = 640 \text{ W}$$

$$\begin{aligned} \text{i) } q = \frac{Q}{A} &= \text{Heat flux} = \frac{640}{.00094} = 680851 \text{ w} \\ &= .680 \text{ MW Ans.} \end{aligned}$$

$$\text{ii) } t_e = \Delta t_e = \text{excess temperature.}$$

Using correlation

$$1.58 \left[\frac{Q}{A} \right]^{.75} = 5.62 (\Delta t)^3 \text{ w/m}^2 \text{ } ^\circ \text{C}$$

$$1.58 [680 \times 10^6]^{.75} = 5.62 (\Delta t)^3$$

$$\begin{aligned} (\Delta t_e)^3 &= \frac{1.58 (680 \times 10^6)^{.75}}{5.62} \\ &= .281 \times (680000)^{.75} \\ &= .281 \times 23679.99 \\ &= 6654.67 \end{aligned}$$

$$\Delta t = (6654.67)^{\frac{1}{3}} = 18.75 \text{ } ^\circ \text{C} \text{ Ans.}$$

Q 4

A nickel wire of 1.5 mm diameter and 500 mm long, carrying current is submerged in water bath which is open to atmosphere. Calculate the voltage at the burnout point if at this point the wire carries a current of 200 amp. [Ans; 17.9 v]

Solution

Given $d=1.5\text{mm}=0.0015\text{m}$, $L=500\text{mm}=0.5\text{m}$, $I=200\text{ AMP}$

Thermophysical properties at 100°C

$$\rho_l = \rho_f = 958.4 \text{ kg/m}^3 \quad \rho_v = 0.5955 \text{ kg/m}^3,$$

$$h_{fg} = 2257 \text{ kJ/kg} \quad \sigma = 58.9 \times 10^{-3} \text{ N/m}$$

i) voltage at the burn out point V_b

A burn out means the point of critical heat flux, the correlation is

As per (equation; 9.11)

$$\begin{aligned} q_{sc} &= 0.18(\rho_v)^{\frac{1}{2}} \times h_{fg} \times [g \sigma (\rho_l - \rho_v)^{\frac{1}{4}}] \\ &= 0.18(0.5955)^{\frac{1}{2}} \times (2257 \times 10^3) [9.81 \times 58.9 \times 10^{-3} (958.4 - 0.5955)]^{\frac{1}{4}} \\ &= 0.18 \times 0.7716 \times 2257000 \times [9.81 \times 0.0589 \times 957.8]^{\frac{1}{4}} \\ &= 313470.216 \times 4.85 \\ &= 1504657.0368 \end{aligned}$$

Electrical energy input for the nickel wire $= Q$

$$\begin{aligned} &= V_b \times I \\ q_{sc} &= \frac{Q}{A} = \frac{V_b I}{A} \end{aligned}$$

$$\begin{aligned} \text{SO } V_b &= \frac{A \times q_{sc}}{I} = \frac{\pi d L q_{sc}}{I} \\ &= \frac{\pi \times 0.0015 \times 0.5 \times 1504657.03}{200} \\ &= 17.726 \text{ Volt Ans.} \end{aligned}$$

Q 5

A metal clad heating element is of 8mm diameter and emissivity 0.95. The element is horizontally immersed in water bath. The surface temperature of the metal is 260 c under study stabilizing. Calculate the power dissipation per unit length for the heater if water is exposed to atmospheric pressure and is at uniform temperature. [1.75 kw Ans]

Solution

Given $D=8\text{mm}=0.008\text{m}$, $\epsilon=.95$, $t_s=260\text{ c}$

Thermophysical properties of water at 100 c from table are;

$$\rho_l = \rho_f = 958.4 \text{ kg/m}^3, h_{fg} = 2257 \text{ kJ/kg}$$

Thermophysical properties of vapor at 260 c from table are

$$\rho_v = 4.807 \text{ kg/m}^3, C_{pv} = 2.56 \text{ kJ/kg K}$$

$$k_v = 0.0331 \text{ W/mK}, \mu_v = \mu_g = 14.85 \times 10^{-6} \text{ Ns/m}^2$$

Power dissipation /unit length for the heater:

$$\text{Excess } \Delta t = t_s - t_{sat} = 260 - 100 = 160 \text{ c}$$

As per boiling curve at $\Delta t = 160 \text{ c}$ there exists a film pool boiling condition. In this case heat transfer is due to both convection and radiation.

The heat transfer coefficient h (approx.) is calculated as per equation (9.13); --

$$h = h_{conv} + \frac{3}{4} h_{rad} \quad \text{-----}$$

$$h_{conv} = 0.62 \left[\frac{k_v^3 \rho_v (\rho_l - \rho_v) g (h_{fg} + 0.4 C_{pv} \Delta t_e)}{\mu_v D \Delta t_e} \right]$$

$$= 0.62 \left[\frac{(0.0331)^3 4.807 (958.4 - 4.807) \times 9.81 (2257.7 \times 10^3 + 0.4 \times 2.56 \times 10^3 \times 160)}{14.85 \times 10^{-6} \times 0.008 \times 160} \right]^{.25}$$

$$= 0.62 [207606433554.29]^{.25}$$

$$=.62 \times 675.009$$

$$=418.5 \text{ w/m}^2 \text{ } ^\circ \text{C}$$

Radiation heat transfer coefficient (equation 9.15)

$$\begin{aligned} h_{\text{radiation}} &= \frac{5.67 \times 10^{-8} \times \epsilon \times (T_s^4 - T_{\text{sat}}^4)}{(T_s - T_{\text{sat}})} \\ &= \frac{\{5.67 \times 10^{-8} \times .95 \{ (260 + 273)^4 - (100 + 273)^4 \} \}}{(260 + 273) - (100 + 273)} \\ &= \frac{5.67 \times 10^{-8} \times .95 (80706559921 - 19356878641)}{160} \\ &= \frac{3304.6}{160} \\ &= 20.65 \text{ w/m}^2 \text{ } ^\circ \text{C} \end{aligned}$$

$$\begin{aligned} \therefore h &= 418.5 + \frac{3}{4} 20.65 \\ &= 433.98 \text{ w/m}^2 \text{ } ^\circ \text{C} \end{aligned}$$

Hence power dissipation per unit length for the heater

$$\begin{aligned} &= h (\pi D \times 1) (260 - 100) \\ &= 433.98 \times \pi \times .008 \times 1 \times 160 \\ &= 1745.137 \text{ watt/m} \\ &= 1.745 \text{ kw/m Ans.} \end{aligned}$$

Q6

A vertical plate 44=50 mm high and maintained at 30 °C is exposed to saturated steam atmospheric pressure. Calculate

i) rate of heat transfer ii) the condensation rate per hour per meter for plate width film condensation.

property of water film at the mean temperature are

$$\rho = 980.3 \text{ kg/m}^3 \quad k = 66.4 \times 10^{-3} \text{ W/m}^\circ\text{C} \quad \mu = 434 \times 10^{-6} \text{ kg/m}^\circ\text{s}$$

$$h_{fg} = 2256.9 \text{ kJ/kg} \quad [439.9 \times 10^3 \text{ kJ/h}, 218.8 \text{ kg/hr}]$$

Solution

$$\text{Given } L = 400 \text{ mm} = .4 \text{ m} \quad \text{width} = B = 1 \text{ m} \quad \rho = 980.3 \text{ kg/m}^3$$

$$t = 30^\circ\text{C}$$

i) rate of heat transfer per meter width Q

$$\begin{aligned} \bar{h} &= .943 \left[\frac{\rho_i(\rho_i - \rho_s)k^3 g h_{fg}}{\mu L (t_{sat} - t_s)} \right]^{\frac{1}{4}} \quad (\text{equation 9.29}) \quad (\text{neglecting } \rho_v) \\ &= .943 \left[\frac{(980.3)^2 \times (66.4 \times 10^{-2})^3 \times 9.81 \times (2256.9 \times 10^3)}{434 \times 10^{-3} \times .4 \times (100 - 30)} \right]^{\frac{1}{4}} \\ &= .943 \left[\frac{6228683061744.42}{.012152} \right]^{.25} = .943 \times 4758.13 = 4486.91 \text{ W/m}^2\text{ }^\circ\text{C} \end{aligned}$$

$$\begin{aligned} \therefore Q &= \bar{h} A (t_{sat} - t_s) = 4486.91 \times (.4 \times 1) (100 - 30) \\ &= 125633.2 \text{ watt} \\ &= \frac{125633.2 \times 3600}{1000} \text{ kJ/hr} \\ &= 452278.8 \text{ kJ/hr} \\ &= 452.278 \times 10^3 \text{ kJ/hr} \quad \text{Ans.} \end{aligned}$$

ii) the condensate rate per meter width m

$$\begin{aligned} m &= \frac{Q}{h_{fg}} \\ &= \frac{452.278 \times 10^3}{2256.9} \\ &= .20039 \times 10^3 \\ &= 200.39 \text{ kg/hr} \quad \text{Ans} \end{aligned}$$

Q 7

A vertical plate in the form of fin is 500mm in height and is exposed to steam at an atmospheric pressure. If the surface of the plate is maintained at 60 C, Calculate

i) film thickness at trailing edge of film.

ii) overall heat transfer coefficient

iii) heat transfer rate

iv) condensate mass flow rate

assume laminar flow condition and unit in δ th. of the plate

[Ans i).1732 mm, ii) 6227.52 w/m iii) 124550w iv) 0.055 kg/s]

Solution

Given $L = 500 \text{ mm} = .5 \text{ m}$

$t_s = 60^\circ \text{C}$ $t_{\text{saturation}} = 100^\circ \text{C}$, mean film temp $= \frac{100 + 60}{2} = 80^\circ \text{C}$

Properties saturated vapour at 80°C

$\rho_1 = 971.8 \text{ kg/m}^3$

$k = 67.413 \times 10^{-2} \text{ W/m}$

$\mu = 355.3 \times 10^{-6} \text{ Ns/m}^2$ or kg/ms

Properties of vapour at atmospheric pressure.

$h_{fg} = h_{fs} = 2257 \text{ kJ/kg}$

i) film thickness at the trailing edge of the plate $= \delta = x = L = .5 \text{ m}$

$$\delta_x = \left[\frac{4k\mu(t_{\text{sat}} - t_s)x}{\rho_1(\rho_1 - \rho_2)gh_{fs}} \right]^{\frac{1}{4}} \text{ (equation 9.24)}$$

$$= \delta_L = \left[\frac{4(67.413 \times 10^{-2}) \times (355.3 \times 10^{-6})(100 - 60) \times .5}{971.8 \times (971.8 - .596) \times 9.81 \times (2257 \times 10^3)} \right]^{\frac{1}{4}}$$

$$= \left[\frac{.01916}{2.08970 \times 10^{13}} \right]^{.25}$$

$$=.000173$$

$$=1.73 \times 10^{-4} \text{ m}$$

$$=.1730 \text{ mm Ans}$$

$$\begin{aligned} \text{ii) } \bar{h} &= \frac{4}{3} h_L = \frac{4}{3} X \frac{K}{D_L} = \frac{4}{3} X \frac{67.413 \times 10^2}{1.73 \times 10^{-3}} = \frac{1.33}{1.73} X 6741.3 \\ &= 5177.3 \text{ W/m}^2 \end{aligned}$$

Using McAdam's correction factor which is 20% higher than

Nusselt's number $= 5177.3 \times 1.2 = 6212.78 \text{ w/m}^2 \text{C Ans.}$

$$\text{iii) heat transfer rate } Q = \bar{h} A (t_{sat} - t_s)$$

$$= h (LXB) (t_{sat} - t_s)$$

$$= 6212.78 (.5 \times 1) (100 - 60)$$

$$= 124254 \text{ w (Ans)}$$

$$\text{iv) the condensate mass flow rate } m = \frac{Q}{h_{fg}}$$

$$= \frac{124254}{2257 \times 10^3}$$

$$= \frac{124254}{2257000} = .0550 \text{ kg/s (Ans)}$$

Q 8.

A vertical plate 2.8m high is maintained at 54°C in the presence of saturated steam at atmospheric pressure. Calculate the heat transfer rate per unit width. [Ans 700 kw/m]

Solution

$$\text{Given } L = 2.8 \text{ m width} = 1 \text{ m } t_s = 54^\circ \text{C } t_{sat} = 100^\circ \text{C}$$

Heat transfer rate per unit width

To determine the film is laminar Or turbulent;

$$\text{Mean film temp.} = \frac{100+54}{2} = 77^\circ \text{C}$$

Properties of condensate at 77°C are

$$\mu_l = 365 \times 10^{-6} \text{Ns/m}^2 \quad k = 668 \times 10^{-3} \text{W/m}^\circ \text{C}, \quad \rho_l = 973.7 \text{kg/m}^3$$

Properties of saturated vapor at $100^\circ \text{C} = t_{sat}$;

$$\rho_v = 0.596 \text{ kg/m}^3, \quad h_{fg} = 2257 \text{ kJ/kg}$$

Assuming the flow to be turbulent the relevant equations are

$$Re = \frac{4\bar{h}L(t_{sat}-t_s)}{h_{fs}\mu_l}$$

$$\therefore \bar{h} = 0.0077 \left[\frac{\rho_l(\rho_l-\rho_v)k^2g}{\mu_l^2} \right]^{\frac{1}{3}} [Re]^{.4} \quad [\text{con}=9.39]$$

Eliminating $\bar{h}$ from the equals we get the condition that the flow will be turbulent if

$$Re = .00296 \left[\frac{\rho_l(\rho_l-\rho_v)k^3g(t_{sat}-t_s)^3L^3}{\mu_l^5(h_{fg})^3} \right]^{\frac{5}{9}} > 1800$$

Substituting the values

$$= .00296 \left[\frac{973.7(973.7-0.596)(668 \times 10^{-3})^3(100-54)^3 \times 9.81(2.8)^3}{(365 \times 10^{-6})^5(2257 \times 10^3)^3} \right]^{\frac{5}{9}}$$

$$= 2927.18 > 1800.$$

Thus, film is turbulent as assumed and $Re = 2927.1$

$$\bar{h} = .0077 \left[\frac{\rho_l(\rho_l-\rho_v)k^2g}{\mu_l^2} \right]^{\frac{1}{3}} (Re)^{.4}$$

$$= .0077 \left[\frac{973.7(973.7-0.596)(668 \times 10^{-3})^3 \times 9.81}{(365 \times 10^{-6})^2} \right]^{\frac{1}{3}} (2927)^{.4}$$

$$= 190.54 \times 24.35$$

$$= 4639.71 \text{ W/m}^2\text{ }^\circ \text{C}$$

Heat transfer rate per unit width

$$Q = \bar{h} A (t_{sat} - t_s) = 4639.71(2.8 \times 1)(100 - 54) \\ = 597594.64 \text{ W} = 597.59 \text{ kW/m (Ans)}$$

Q 9

A vertical tube of 50mm outside diameter 2m long is exposed to steam atmospheric pressure. The outer surface of the tube is maintained at temperature of 84 c by circulating cold water through the tube. Determine

i. rate of heat transfer to coolant.

ii. rate of condensation of steam.

$$[\text{Ans } 79 \text{ kW ii) } 28.6 \text{ kg/hr}]$$

Solution

$$\text{Given } D = 50 \text{ mm} = .05 \text{ m} \quad L = 2 \text{ m} \quad t_s = 84 \text{ c}$$

Assuming non condensable and steam absence

$$\text{Mean film temp.} = \frac{100 + 84}{2} = 92 \text{ c}$$

Properties of water at 92 c

$$\rho_l = 975 \text{ kg/m}^3, \mu_l = 375 \times 10^{-6} \text{ Ns/m}^2, k = .67 \text{ W/m c}$$

Properties of saturated vapor at 100 c = t sat.

$$\rho_v = .596 \text{ kg/m}^3 \quad h_{fg} = 2257 \text{ kJ/kg}$$

i) rate of heat transfer Q

The laminar condensation on a vertical surface

$$\bar{h} = 1.13 \left[\frac{\rho_l (\rho_l - \rho_v) k^3 g h_{fg}}{\mu L (t_{sat} - t_s)} \right]^{\frac{1}{4}} = 1.13 \left[\frac{975 (975 - .596) .67^3 9.81 (2257 \times 10^3)}{375 \times 10^{-6} \times 2 \times (100 - 84)} \right]^{\frac{1}{4}} \\ = 1.13 [525877087434075]^{.25}$$

$$=1.13 \times 4788.7 = 5411.28 \text{ W/m}^2 \text{ } ^\circ \text{C}$$

$$\begin{aligned} Q &= \bar{h} A (t_{sat} - t_s) = 5411.28 \times (\pi D L) (100 - 84) \\ &= 5411.28 \times (\pi \times 0.05 \times 2) \times 16 \\ &= 27194.9 \text{ W} \\ &= 27.194 \text{ kW Ans} \end{aligned}$$

ii) Rate of condensation of steam = m

$$\begin{aligned} \frac{Q}{h_{fg}} &= \frac{27194.9}{2257 \times 10^3} = 0.012049 \text{ kg/s} \\ &= 48.37 \text{ kg/hr} \end{aligned}$$

If we check whether film condensation is laminar

$$\begin{aligned} Re &= \frac{4m}{P\mu_L} = \frac{4 \times 0.012049}{(\pi D) 375 \times 10^{-6}} = \frac{4 \times 0.012049 \times 10^6}{\pi (0.05) \times 375} \quad (P \text{ is perimeter}) \\ &= \frac{4 \times 12049}{58.9} = 818.25 \end{aligned}$$

818.25 < 1800, hence the flow is laminar.

Q10

A horizontal tube of outer diameter 25 mm is exposed to dry steam at 100 °C. The tube surface temperature is maintained at

84 °C by circulating water through it. Calculate the rate of formation of condensation per meter length of the tube.

[Ans ; 21.94 kg/h r]

Solution

Given $D=25\text{mm}=0.025\text{m}$ $t_s=84^\circ\text{C}$ $t_{sat}=100^\circ\text{C}$

Mean film temp $=92^\circ\text{C}$

Properties at 92°C of saturated liquid

$$\rho_{liquid}=963.4\text{ kg/m}^3, \mu_{liquid}=306 \times 10^{-6}\text{Ns/m}^2$$

$$K=0.677\text{W/m}^\circ\text{C}$$

Properties of at 100°C of saturated vapor

$$\rho_v=0.596\text{ kg/m}^3 \quad h_{fg}=2257\text{ kJ/kg}$$

To find rate of condensation per meter length of the tube ‘m’

$\bar{h}$ = average heat transfer coefficient

$$\begin{aligned} &=0.725 \left[\frac{\rho_l(\rho_l-\rho_v)g\mu k^3 h_{fg}}{(t_{sat}-t_s)D\mu_L} \right]^{\frac{1}{4}} \\ &=0.725 \left[\frac{963.4(963.4-0.596) \times 9.81 (0.677)^3 (2257 \times 10^3)}{(306 \times 10^{-6}(100-84) \times 0.025)} \right]^{.25} \\ &=0.725 \left[\frac{6366562162931.3}{.0001224} \right]^{.25} \\ &=0.725 [52014396755974673.2]^{.25} \\ &=0.725 \times 15101.87 \\ &=10948.85\text{ W/m}^2\text{ }^\circ\text{C} \end{aligned}$$

$$\begin{aligned} \text{Heat transfer per unit length} &= \frac{Q}{L} = \bar{h} \times \pi D (t_{sat} - t_s) \\ &= 10948.85 \times \pi \times 0.025 \times (100 - 84) \\ &= 10948.85 \times \pi \times 0.025 \times 16 \\ &= 13758.73\text{ W/Unit length} \end{aligned}$$

Rate of formation of condensate /meter length of tube

$$\begin{aligned}
\frac{m}{L} &= \frac{Q/L}{h_{fg}} = \frac{13758.73}{2257 \times 10^3} \\
&= \frac{13758.73}{2257} \times 10^{-3} \\
&= 6.096 \times 10^{-3} \\
&= .006096 \text{ kg/s} \\
&= .006096 \times 3600 \text{ kg/h r} \\
&= 21.94 \text{ kg/h r Ans.}
\end{aligned}$$

Q11

A condenser is to be designed to condense 2250 kg /hr of dry and saturated steam at a pressure of 15 k pa. A square array of 400 tubes each of 6mm diameter is to be used. If the tube surface temperature is to be maintained at 26 c, calculate the heat transfer coefficient and the length of each tube, assessing single pass. [Ans 5205.3 w/m²°c, 1.35 m]

Solution

Given m=mass of dry and saturated steam 2250 kg/h r

D=Diameter of each tube =6mm=.006m

No of tube =400 no s

As per table of saturated steam pressure and temperature against 15 k p a is 53.97°c= t_{sat} , $\rho = 992.2 \text{ kg/m}^3$, $k=.628 \text{ w/m}$

$\mu = .653 \times 10^{-3}$ $h_{fg}=2373 \text{ kj/kg}$, $t_s=26 \text{ c}$, $h_f=226 \text{ kj/kg}$

$h_g=2599 \text{ kj/kg}$, $\rho_v = \frac{1}{v_g} = .0676 \text{ kg/m}^3$, $t_{film} = \frac{53.97+26}{2} = 40 \text{ c}$

As tubes are to be arranged in array, the number of horizontal tubes in vertical columns $N = \sqrt{400} = 20$.

$$\begin{aligned}
 \text{i) heat transfer coefficient } \bar{h} &= .725 \left[\frac{\rho_l (\rho_l - \rho_v) g k^3 h_{fg}}{N \mu_l (t_{sat} - t_s) D} \right]^{\frac{1}{4}} \\
 &= .725 \left[\frac{999.2(999.2 - .0676)(.628)^3 9.81(2373 \times 10^3)}{20 \times (.653 \times 10^{-3})(53.97 - 26) \times .006} \right]^{.25} \\
 &= .725 \left[\frac{5754304711000}{.002197292} \right]^{.25} \\
 &= .725 [2618816393542596.1]^{.25} \\
 &= .725 \times 7153.6 \\
 &= 5186.38 \text{ w/m}^2 \text{ } ^\circ \text{C} \quad \text{Ans}
 \end{aligned}$$

ii) the length of each tube assuming single pass to find

$$Q = \text{Heat transfer rate} = \bar{h} A_E (t_{sat} - t_s)$$

$$\text{Or } m x h_{fg} = \bar{h} (400 \times \pi D L) (t_{sat} - t_s)$$

$$\frac{2250}{3600} \times (2373 \times 10^3) = 5186.36 (400 \times \pi \times .006) \times L (53.97 - 26)$$

$$\begin{aligned}
 L &= \frac{1483125}{5186.36 \times 400 \times \pi \times .006 \times 27.97} = \frac{1483125}{1093745.4} \\
 &= 1.356 \text{ m (Ans)}
 \end{aligned}$$

Chapter 10

Heat Exchangers

Q 1

The flow rate of hot and cold-water streams running through a heat exchanger are 600 kg/h r and 1500 kg / h r respectively. The inlet temperature on the hot and cold side is 70°C and 25 °c respectively. The exit temperature of hot water is 50 ° c. If the individual heat transfer coefficient on both sides is 700 w/m² c. Calculate the area of heat exchanger. [Ans.1.38 m²c]

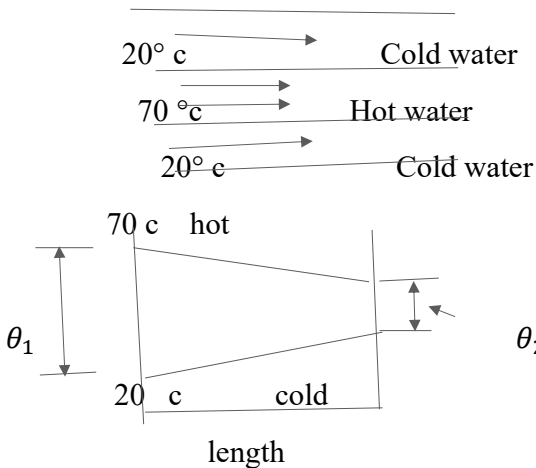
Solution

Given $\dot{m}_h = 600 \text{ kg/hr}$, $\dot{m}_c = 1500 \text{ kg/hr}$ $t_{hi} = 70^\circ \text{C}$, $t_{h2} = 50^\circ \text{C}$

$t_{ci} = 25^\circ \text{C}$, $h_1 = h_2 = 700 \text{ W/m}^2 \text{C}$

Area of heat exchanger A

Heat exchanger is diagrammatically shown below; -



i) to find area of heat exchanger

$$\begin{aligned}
 Q &= \text{Heat transfer rate} = m_h \times C_{ph} \times (t_{h1} - t_{h2}) \\
 &= .16 \times 4.187 \times (70 - 50) \\
 &= 13.3984 \text{ kJ / s}
 \end{aligned}$$

Heat lost by hot water = heat gained by cold water

$$\begin{aligned}
 13.3984 &= m_c \times C_{pc} \times (t_{c2} - t_{c1}) = .4 \times 4.187 \times (t_{c2} - 25) \\
 13.3984 &= 1.6748 \times (t_{c2} - 25) \\
 t_{c2} &= \frac{133984}{1.6748} + 25 = 8 + 25 = 33^\circ \text{ C}
 \end{aligned}$$

We have to find out (LMTD) θ_m (Logarithmic mean temp. diff)

$$\begin{aligned}
 &= \frac{\theta_1 - \theta_2}{\log \frac{\theta_1}{\theta_2}} \text{ (eq u; 10.9)} \\
 &= \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\log \left(\frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}} \right)} \\
 &= \frac{(70 - 25) - (50 - 33)}{\log \frac{70 - 25}{50 - 33}} = \frac{45 - 17}{\log \frac{45}{17}} \\
 &= \frac{28}{\log 2.64} = \frac{28}{.97} = 28.86^\circ \text{ C}
 \end{aligned}$$

Overall heat transfer coefficient U is arrived from the relation

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{700} + \frac{1}{700} = \frac{2}{700} = \frac{1}{350}$$

So $U = 350 \text{ W/m}^2 \text{ } ^\circ\text{C}$

$$Q = U A \theta_m$$

$$\begin{aligned}
 \text{So, } A &= \frac{Q}{U \theta_m} = \frac{13.3984 \times 1000}{350 \times 28.86} \\
 &= \frac{13398.4}{10101}
 \end{aligned}$$

$$= 1.326 \text{ m}^2 \text{ (Ans) Area of heat exchanger.}$$

Q 2

The following data relate to parallel heat exchanger in which air is heated by hot exhaust gases

- i) heat transfer /s=46.52 k j
- ii) inside heat transferred coefficient =116 w/m²c
- iii) outside heat transferred coefficient 186w/m²°c
- iv) inlet outlet temperature of hot fluid = 400° c and 150°c
- v) inlet and outlet temperature of cold fluid =50 ° c and 100 °c
- vi) inside and outside diameter of tube required 50mm and 60mm

calculate the length of the tube required for heat transfer assuming the tube resistance to be negligible

[Ans 25 .2 m]

Solution

Given Q=46 .52K J/S $h_{inlet}=116 \text{ w/m}^2 \text{ C}, h_{outside} =186\text{w/m}^2$

$$t_{h1}=400 \text{ c} \quad t_{h2}=150 \text{ c} \quad t_{c1}=50 \text{ c} \quad t_{c2}=100 \text{ c}$$

$$d_1=50\text{mm}=.05\text{m} \quad r_i=.025\text{m}$$

$$d_o=60\text{mm}=.06 \text{ m} \quad r_o=.030 \text{ m}$$

Length of tube L; ---

Logarithmic mean temperature difference

$$\theta_m = \frac{\theta_1 - \theta_2}{\log \frac{\theta_1}{\theta_2}} = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\log \frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}}} = \frac{(400 - 50) - (150 - 100)}{\log \frac{(350)}{(50)}} \\ = \frac{300}{\log 7} = \frac{300}{1.9459} = 154.17^\circ \text{ c} = \theta_m$$

Overall heat transfer coefficient = U

$$\frac{1}{U} = \frac{r_o}{r_1} X \frac{1}{h_1} + \frac{1}{h_2} = \frac{.03}{.025} X \frac{1}{116} + \frac{1}{186} = .0103 + .0053 = .0156$$

$$U = \frac{1}{.0156} = 64.1 \text{ W/m}^2 \text{ } ^\circ \text{C}$$

$$\text{Total heat transfer } = Q = U A \theta_m$$

$$46.5 \times 1000 = 64.1 \times \pi D_o L \times 154.17$$

$$= 64.1 \times \pi \times .06 \times L \times 154.17$$

$$L = \frac{46.5 \times 1000}{1962.7} = \frac{46500}{1862.7} = 24.96 \cong 25 \text{ m} = \text{length of tube.}$$

Q 4

A counter flow heat exchanger through which passes 11.3 kg/s of air to be cooled from 538 °C to 148 °C contains 4500 tubes each having diameter 25.4 mm. The inlet and outlet temperature of cooling water are 38 °C and 85 °C, if water side resistance to flow is negligible calculate tube length required for the duty.

For turbulence flow inside tube $Nu = 0.023 Re^{.8} Pr^{.4}$

Properties of air at average temperature are; $C_p = 1.0082 \text{ K J/kg } ^\circ \text{C}$ $\rho = 1.009 \text{ kg/m}^3$ $\mu = 2.075 \times 10^{-5} \text{ N s/m}^2$ $k = 3.003 \times 10^{-2} \text{ W/m } ^\circ \text{C}$

[Ans 2 m]

Solution

Given $\dot{m}_h = 11.3 \text{ kg/s}$ $t_{h1} = 538 \text{ } ^\circ \text{C}$ $t_{h2} = 148 \text{ } ^\circ \text{C}$

$t_{c1} = 38 \text{ } ^\circ \text{C}$ $t_{c2} = 85 \text{ } ^\circ \text{C}$, $N = 4500$, $d = 25.4 \text{ mm} = .0254 \text{ m}$

To find total length = L, N = number of tubes,

$$\text{Reynold number} = Re = \frac{\rho v d}{\mu}$$

$$\dot{m} = N A V \rho$$

$$\therefore \rho V = \frac{m}{NA} =$$

$$R_e = \frac{m d}{N A \mu} = \frac{11.3 \times 0.0254}{4500 \times \frac{\pi}{4} (0.0254)^2 \times 2.075 \times 10^{-5}}$$

$$= \frac{.28702}{.0001892} = 6068.07$$

$$\text{Prandtl number} = P_r = \frac{\mu C_P}{K} = \frac{2.075 \times 10^{-5} \times 1.0082 \times 10^3}{3.003 \times 10^{-2}}$$

$$= \frac{.0209}{.03003} = .69597$$

$$\text{Nusselt number} = Nu = \frac{h d}{k} = 0.023 R_e^{.8} \times P_r^{.4}$$

$$= .023 (6068.04)^{.8} \times (.6959)^{.4} = 0.023 \times 1062.77 \times .8650$$

$$= 21.14$$

$$= \frac{h d}{k} = 21.14$$

$$h = \frac{21.4 \times k}{d} = \frac{21.4 \times 3.003 \times 10^{-2}}{.0254} = 24.99$$

Since water side resistance to flow is negligible

$$\frac{1}{U} = \frac{1}{h} = \frac{1}{24.99}$$

$$U = 24.99 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

LMTD = Logarithmic mean temperature difference

$$= \theta_m = \frac{\theta_1 - \theta_2}{\log \frac{\theta_1}{\theta_2}} = \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{\log \frac{t_{h1} - t_{c2}}{t_{h2} - t_{c1}}}$$

$$= \frac{(538 - 85) - (148 - 38)}{\log \frac{538 - 85}{148 - 38}} = \frac{343}{\log 4.118}$$

$$= \frac{343}{1.415} = 242.4 \text{ } ^\circ\text{C}$$

Further rate of heat transfer $Q = \dot{m}_h \times C_{ph} \times (t_{h1} - t_{h2})$

$$= U A \theta_m = U (\pi D L)$$

$\times \theta_m$

$$L = \frac{11.3 \times 1.0082 \times 1000 (538 - 148)}{25 (4500 \times 0.0254) \times 242.4} = \frac{11.3 \times 1008.2 \times 390}{25 \times 114.3 \times 242.4}$$

$$= \frac{4443137.4}{2176049.28} = 2.04 \text{ m}$$

Hence tube length required = 2 m Ans

Q 5

16.67 kg/s of the product at 700 °C ($C_p = 3.6 \text{ kJ/kg}$) in a chemical plant are to be used to heat 20 kg/s of the incoming fluid from 100 °C ($C_p = 4.2 \text{ kJ/kg}^\circ\text{C}$). If the overall heat transfer coefficient is $1 \text{ kW/m}^2\text{C}$ and the installed heat transfer surface is 42 m^2 . Calculate the fluid outlet temperature for the counter flow and parallel flow arrangement.
[Ans $t_{h2} = 438^\circ\text{C}$ $t_{c2} = 286.8, 274.4^\circ\text{C}$]

Solution

Given $\dot{m}_h = 16.67 \text{ kg/s}$, $t_{h1} = 700^\circ\text{C}$, $C_{ph} = 3.6 \text{ kJ/kg}^\circ\text{C}$, $\dot{m}_c = 20 \text{ kg/s}$

$$t_{c1} = 100^\circ\text{C}, C_{pc} = 4.2 \text{ KJ/kg}^\circ\text{C}$$

, $U = \text{overall heat transfer coefficient} = 0.95 \text{ kW/m}^2\text{C}$

$$A = 42 \text{ m}^2$$

To find t_{h2} & t_{c2} for both counter and parallel flow arrangement

i) counter flow t_{h2} and t_{c2}

$$\text{thermal capacity of hot fluid} = C_h = \dot{m}_h \times C_{ph} = 16.67 \times 3.6$$

$$= 60.012 \text{ kW/K}$$

$$\text{Thermal capacity of cold fluid} = C_c = \dot{m}_c \times C_{pc} = 84 \text{ kW/K}$$

Cold fluid is maximum and hot fluid is minimum so

$$R = \frac{C_{MIN}}{C_{MAX}} = \frac{60.012}{84} = .71$$

$$\text{Number of transfer unit} = NTU = \frac{UA}{C_{min}} = \frac{.95 \times 42}{60.012} = .665$$

$$\begin{aligned} \text{Effectiveness of counter flow } \varepsilon &= \frac{1 - \exp[-NTU(1-R)]}{1 - R \exp[-NTU(1-R)]} \quad (\text{eq 10.5}) \\ &= \frac{1 - e[-.665(1-.71)]}{1 - .71e[-.665(1-.71)]} = \frac{1 - e^{-0.1928}}{1 - .71e^{-.1928}} = \frac{1 - .8246}{1 - .71 \times .8246} = \frac{.1754}{1 - .5854} \\ \varepsilon &= \frac{.1754}{.4146} = .423 \end{aligned}$$

$$\text{Further } \varepsilon = \frac{C_h(t_{h1} - t_{h2})}{C_{min}(t_{h1} - t_{c1})} \quad (\text{equ 10.38})$$

Because the hot fluid is also minimum, we have

$$\begin{aligned} \varepsilon &= \frac{t_{h1} - t_{h2}}{t_{h1} - t_{c1}} = \frac{700 - t_{h2}}{700 - 100} \\ \therefore .423 &= \frac{700 - t_{h2}}{600} \\ t_{h2} &= 700 - 600 \times .423 = 441.2^\circ \text{ C Ans} \end{aligned}$$

$$\text{Also, } \varepsilon = \frac{C_c(t_{c2} - t_{c1})}{t_{min}(t_{h1} - t_{c1})} = \frac{(t_{c2} - 100)84}{60 \times (700 - 100)} = .423$$

$$t_{c2} - 100 = \frac{15228}{84}$$

$$t_{c2} = 181.28 + 100 = 281.28^\circ \text{ C (Ans)}$$

ii) parallel flow arrangement

$$\begin{aligned} \text{value of } \varepsilon \text{ in parallel flow} &= \frac{1 - e(-NTU(1+R))}{1+R} \\ &= \frac{1 - e[-.665(1+.71)]}{1+.71} = \frac{1 - e^{-1.137}}{1.71} \\ &= \frac{1 - .320}{1.71} = .3976 \end{aligned}$$

$$\text{Also, } \varepsilon = \frac{C_c(t_{c2}-t_{c1})}{C_{min}(t_{h1}-t_{c1})} = \frac{84(t_{c2}-100)}{60(700-100)}$$

$$84(t_{c2} - 100) = 60 \times 600 \times .3976 = 14313.6$$

$$\therefore t_{c2} = \frac{14313.6}{84} + 100$$

$$= 170.4 + 100$$

$$= 270.4^\circ \text{ C (Ans)}$$

Q 19

Calculate the following cases, the surface area required for a heat exchanger which is required to cool 1 kg/s of benzene

($C_p = 1.74 \text{ kJ/kg}^\circ \text{C}$) from 75°C to 45°C . The cooling water ($C_p = 4.18 \text{ kJ/kg}$) at 15°C has a flow rate of 0.694 kg/s .

1. single pass counter flow

2. 1-4(exchanger) (1 shell pass and 4 tube pass)

3. cross flow single pass with water mixed and benzene unmixed.

For each configuration the overall heat transfer coefficient may be taken as $.3 \text{ kW/m}^2 \text{ }^\circ \text{C}$

$$[\text{Ans i) } 4.87 \text{ m}^2 \text{ ii) } 5.29 \text{ m}^2 \text{ iii) } 5.18 \text{ m}^2]$$

Solution

Given $\dot{m}_h = 1 \text{ kg/s}$, $\dot{m}_w = \dot{m}_c = 0.694 \text{ kg/s}$, $t_{c1} = 15^\circ \text{C}$, $t_{h1} = 75^\circ \text{C}$

$$t_{h2} = 45^\circ \text{C} \quad U = .3 \text{ kW/m}^2 \text{ }^\circ \text{C} = 300 \text{ W/m}^2$$

Surface Area required A

i) case

using energy balance on both fluids we have

$$=\dot{m}_h \times C_{ph} \times (t_{h1} - t_{h2}) = \dot{m}_c \times C_{pc} (t_{c2} - t_{c1})$$

$$1 \times 1.74 \times (75 - 45) = .694 \times 4.18 \times (t_{c2} - 15)$$

$$t_{c2} = \frac{1 \times 1.74 \times 30}{2.9 \times .694 \times 4.18} + 15 = 18 + 15 = 33^\circ \text{ c}$$

$$\begin{aligned} \text{Energy balance on hot fluid } Q &= \dot{m}_h \times C_{ph} \times (t_{h1} - t_{h2}) \\ &= 1 \times 1.74 \times (75 - 45) \\ &= 52.2 \text{ kw} \end{aligned}$$

a) super-pass-counter flow

$$\begin{aligned} \theta_m &= \frac{\theta_1 - \theta_2}{\log \frac{\theta_1}{\theta_2}} = \frac{t_{h1} - t_{c2} - (t_{h2} - t_{c1})}{\ln \frac{t_{h1} - t_{c2}}{t_{h2} - t_{c1}}} \\ &= \frac{(75 - 33) - (45 - 15)}{\ln \left(\frac{75 - 33}{45 - 15} \right)} = \frac{42 - 30}{\ln \frac{42}{30}} = \frac{12}{\ln 1.4} \\ &= \frac{12}{.336} = 35.714^\circ \text{ c} \end{aligned}$$

$$\text{Area of exchanger } A = \frac{Q}{U \theta_m} = \frac{52.2}{.3 \times 35.71} = 4.87 \text{ m}^2$$

b) 1 shell 4 tube pass exchanger

since the number of passes is more than one θ_m needs correction factor F. To know P and R numbers.

$$P = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} = \frac{33 - 15}{75 - 15} = \frac{18}{60} = .3$$

$$R = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} = \frac{75 - 45}{33 - 15} = \frac{30}{18} = 1.66$$

ii) From correction factor graph for P and R numbers for shell pass, we found both lines cut at approximately at .94. (graph 10.40, original book)

$$A = \frac{52.2}{.94 \times .3 \times 35.7} = 5.27 \text{ m}^2$$

iii) for cross flow single pass with mixed and benzene unmixed

graph (10.43) we find corresponding P and R lines cut at .95 which is the correction factor F.

$$\text{Accordingly, } A = \frac{52.2}{.95 \times .3 \times 35.7} = 5.16 \text{ m}^2$$

Q 20

It is required to design a shell and tube heat exchanger for heating 2.5 kg/s of water from 15 ° c to 85° c by hot engine oil

($C_p = 2.35 \text{ K J/kg c}$) flowing through the shell of heat exchanger

The oil makes a single pass entering at 150 ° c and leaving at 95 °

With an average heat trans coefficient of 400 w/m^2 .The water flows through 10 thin-walled tubes of 25 mm diameter with each tube making 8 passes through the shell. The heat transfer coefficient on the water side is 3000 w/m c . Calculate the length of the tube required for the heat exchanger to accomplish the water heating [Ans tube length =38,88m, Shell length =4.86m (8 passes)]

Solution

Given $\dot{m}_w = \dot{m}_c = 2.5 \text{ kg/s}$. $t_{c1} = 15 \text{ c}$, $t_{h1} = 150 \text{ c}$, pass=8.

$C_{ph} = 2.35 \text{ k j/kg}$ $t_{c2} = 85 \text{ C}$. $t_{h2} = 95 \text{ c}$, $d = 25 \text{ mm} = .025 \text{ m}$

$h_i = 3000 \text{ w/m}^2$ $N = 10$ thin-walled tube. $h_o = 400 \text{ w/m}^2 \text{ c}$

i) to find length of tube; - L

overall heat transfer coefficient; - neglecting thermal resistance of tube we get

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{3000} + \frac{1}{400} = \frac{3400}{12000}$$

$$U = \frac{12000}{3400} = 352.94 \text{ W/m}^2 \text{ c}$$

Parameter required to get the correction factor are

$$P = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} = \frac{85 - 15}{150 - 15} = .518,$$

$$R = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} = \frac{150 - 95}{85 - 15} = \frac{55}{70} = .7857$$

[from fig 10.40 of book]

Correction factor $F = .825$

From the conventional counter flow arrangement

$$\theta_1 = t_{h1} - t_{c2} = 150 - 85 = 65 \text{ } ^\circ\text{C}$$

$$\theta_2 = t_{h2} - t_{c1} = 95 - 15 = 80$$

$$\theta_m = \frac{\theta_1 - \theta_2}{\log \frac{\theta_1}{\theta_2}} = \frac{65 - 80}{\log \frac{65}{80}} = \frac{-15}{\log .8125} = \frac{15}{.2076} = 72.25^\circ \text{C}$$

Heat transfer rate $= Q = \dot{m}_c \times C_{pc} (t_{c2} - t_{c1})$

$$= 2.5 \times 4.18 \times 10^3 (85 - 15)$$

$$= 731500 \text{ W}$$

($C_{pc} = 4.18 \times 10^3$ not provided in the sum)

Also, $Q = F \times U \times A \times \theta_m$

$$\text{Or, } A = \frac{Q}{F U \theta_m} = \frac{731500}{.825 \times 352.94 \times 72.25} \text{ [A is heating surface]}$$

$$= \frac{731500}{21037.42}$$

$$= 34.77 \text{ m}^2$$

But $A = \pi \times d \times L \times N$

$$\therefore L = \frac{A}{\pi d N} = \frac{34.77}{\pi \times .025 \times 10} = \frac{34.77}{.7854} = 44.27 \text{ m (Ans)}$$

$$\text{Shell length} = \frac{44.27}{8} = 5.5 \text{ m (Ans)}$$

Q 24

In a large steam power plant shell and tube type condenser is employed which effect a heat exchange rate of 2200 MW. The following data pertains to this condenser

Number of shell pass = 1

Number of tube (this walled) = 32000 each executing 2 passes.

Diameter of each tube = 30mm

Mass flow of rate of water through the tube = $3.2 \times 10^4 \text{ kg/s}$

Condensation temperature of steam = 50° c

Condensation on the outer surface of the tube.

The heat transfer coefficient on the steam side = $11500 \text{ w/m}^2 \text{ C}$

The inlet temp = 20° c

Using (LMTD) correction factor method and NTU method

Calculate the outlet temperature of cooling water and length of tube per pass. Take the following properties of water

$t_{bulk} = 27^\circ \text{ c}$, $C_p = 4.18 \text{ K J /kg c}$, $\mu = 855 \times 10^{-6} \text{ Ns/m}^3$, $k = 0.613 \text{ w/m c}$
 $Pr = 5.83$

Thermal resistance of tube and fouling effect are neglected.

[Ans 36.44° c , 4.77 m]

Solution, given $Q = 2200 \text{ MW}$, $d = 30 \text{ mm} = 0.03 \text{ m}$

$N_p = \text{no of tubes} = 32000$, $\dot{m}_w = \dot{m}_c = 3.2 \times 10^4 \text{ kg/s}$

$t_{h1} = t_{h2} = 50^\circ \text{ c}$, $h_0 = 11500 \text{ w/m}^2 \text{ c}$

i) water outlet temp = t_{c2}

to obtain we use overall energy balance

$$Q = \dot{m}_c X c_{pc} \times (t_{c2} - t_{c1})$$

$$2200 \times 10^6 = 3.2 \times 10^4 \times 4.18 \times 10^3 (t_{c2} - 20)$$

$$t_{c2} - 20 = \frac{22000}{133.76} = 16.44$$

$$t_{c2} = 16.44 + 20 = 36.44^\circ \text{C (Ans)}$$

ii) Length of tube/pass

a) LMTD correction factor method

total rate of heat transfer = $F A U \theta_m$

to find F we have to find P and R

$$P = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} = \frac{36.44 - 20}{50 - 20} = \frac{16.44}{30} = .548.$$

$$R = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} = \frac{50 - 50}{t_{c2} - t_{c1}} = 0$$

Since $R = 0$ $P = .548$, we get $F = 1$

To find value of U, h_i has to find out

$$\text{Mass flow rate in each tube} = \dot{m} = \frac{3.2 \times 10^4}{32000} = 1$$

$$\begin{aligned} \text{Reynold number} &= \frac{\rho d v}{\mu} = \frac{4 \dot{m}}{\pi d \mu} = \frac{4 \times 1}{\pi \times 0.03 \times 855 \times 10^{-6}} \\ &= \frac{4 \times 10^6}{\pi \times 0.03 \times 855} = 4.9 \times 10^4 \end{aligned}$$

$Re > 2300$ hence flow is turbulent

$$\text{So, } Nu = \frac{hd}{k} = .023 (Re^8) (Pr^4)$$

$$= .023 \times 5651 \times 2.02 = 262.54$$

$$\bar{h} = \frac{k}{d} \times Nu = \frac{.613}{.03} \times 262.54 = 5364.56 \text{ W/m}^2 \text{C}$$

$$\text{Now } \frac{1}{U} = \frac{1}{h_o} + \frac{1}{h_i}$$

$$U = \frac{h_i x h_o}{h_i + h_o} = \frac{5364.56 \times 11500}{5364.56 + 11500} = 3658.11 \text{ W/m}^2$$

Further LMTD is given by

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln \frac{\theta_1}{\theta_2}} = \frac{t_{h1} - t_{c1} - (t_{h2} - t_{c2})}{\ln \frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}}}$$

$$= \frac{50 - 20 - (50 - 36.44)}{\ln \frac{50 - 20}{50 - 36.44}} = \frac{30 - 13.56}{\ln \frac{30}{13.56}} = \frac{16.44}{\ln 2.21}$$

$$= \frac{16.44}{.79} = 20.8^\circ \text{ C}$$

Substituting the value, we get

$$Q = FUA\theta_m = F U (2N_p \pi d L) \theta_m$$

$$2200 \times 10^6 = 1 \times 3658.11 \times (2 \times 32000 \times \pi \times 0.03 \times L)$$

$$= 458956153.4 \text{ L}$$

$$\therefore L = \frac{2200 \times 10^6}{458956153.4} = 4.79 \text{ m (Ans)}$$

b) NTU method

Since heat exchanger is a condenser hence

$$C_h = C_{max} = \infty$$

$$\text{And } C_{min} = C_c = \dot{m}_c, \quad C_{pc} = (3.2 \times 10^4)(4.18 \times 10^3)$$

$$= 13376 \times 10^4$$

$$C_h > C_c \quad \text{hence}$$

$$\text{And } \varepsilon = \text{emissivity} = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} = \frac{36.44 - 20}{50 - 20} = \frac{16.44}{30} = .548$$

$$\text{Here } \frac{C_{min}}{C_{max}} = 0$$

$$\varepsilon = 1 - \exp(-NTU) \quad (\text{EQU 10.51})$$

$$.548 = 1 - e^{-NTU}$$

$$e^{-NTU} = 1 - .548 = .452$$

$$-NTU = \ln(.452) = -.794$$

$$NTU = .794$$

$$NTU = \frac{UA}{C_{min}} = \frac{U(2N_P \pi d L)}{C_{min}}$$

$$.794 = \frac{3658.11(2 \times 32000 \pi \times .03 \times L)}{13376 \times 10^4}$$

$$L = \frac{106205440}{22065199.68} = 4.81 \text{ m (Ans)}$$

NOTE; -- Since most of the sums in this chapter are very lengthy and depends on correction factors which are to be estimated on the basis various graphs provided in the original book So I have decided not to go for any more sums. Students may please note.

(chapter 9 has 11 sums which have taken same number of pages as 7 sums taken for chapter 10.)

Chapter 11

Thermal Radiation – Basic Relations

Q 1

The effective temperature of a body having an area of 0.1 m^2 is 627°C . Calculate i) total rate of energy emission

ii) the intensity of normal radiation and iii) the wave length of maximum monochromatic emissive power.

[Ans i) 3720 w ii) 11841.4 w/m² iii) 3.22 μm]

Solution

Given $A = 0.11 \text{ m}^2$, $T = 627 + 273 = 900 \text{ K}$, σ = Stefan Boltzman constant. $= 5.67 \times 10^{-8} \text{ w/m}^2 \text{K}^4$

$$\begin{aligned}\text{i) total rate of energy emission} &= E_b = \sigma \times A \times T^4 \quad (\text{eq 11.2a}) \\ &= 5.67 \times 10^{-8} \times 0.1 \times (900)^4 \\ &= 5.67 \times 0.1 \times (9)^4 \\ &= 3720 \text{ w (Ans)}\end{aligned}$$

ii) intensity of normal radiation = I_{bn}

$$\begin{aligned}&= \frac{\text{energy emission}}{\pi} \quad (\text{eq 11.23}) \\ &= \frac{E_b}{\pi} = \frac{5.67 \times \left(\frac{900}{100}\right)^4}{\pi} \quad (\text{where } E_b \text{ is w/m}^2 \text{K}^4) \\ &= 11841.4 \text{ w/m}^2 \text{.sr. (Ans)}\end{aligned}$$

iii) wave length of max monochromatic emissive power λ_{max} from Wein's displacement law $= \lambda_{\text{max}} \cdot T = 2898 \mu\text{m K}$ (eq 11.18)

$$\lambda_{\text{max}} = \frac{2898}{T} = \frac{2898}{900} = 3.216 \mu\text{m}$$

Q 2 Assuming the sun to be black body emitting radiation with maximum intensity at $\lambda = 0.5 \mu\text{m}$, calculate

i) surface temperature of the sun

ii) heat flux at its surface.

$$[\text{Ans } 5796 \text{ k}, 6.398 \times 10^7 \text{ W/m}^2]$$

i) surface temperature of sun T

According to Wein's displacement law

$$\lambda_{\text{max}} \times T = 2898 \mu\text{m k}$$

$$T = \frac{2898}{\lambda_{\text{max}}} = \frac{2898}{.5} = 5796 \text{ k (Ans)}$$

ii) heat flux at the surface (E_{SUN})

$$= \sigma T^4$$

$$= 5.67 \times 10^{-8} \times \left(\frac{T}{100}\right)^4$$

$$= 5.67 \times \left(\frac{5796}{100}\right)^4$$

$$= 5.67 \times (57.96)^4$$

$$= 63987709.4$$

$$= 6.39 \times 10^7 \text{ W/m}^2 \text{ (Ans)}$$

Q 3

Assuming the sun (diameter $1.4 \times 10^9 \text{m}$) as a black body having a surface temp 5800 k and a mean distance of $15 \times 10^{10} \text{m}$ from the earth (diameter $12.8 \times 10^6 \text{m}$) Estimate

i) total energy emitted by the sun

ii) emission received W/m^2 , just outside the atmosphere of the earth

iii) Total energy received by earth if no radiation is blocked by the atmosphere of the earth.

iv) the energy received by 1,5mx1.5m solar collector who's normal is inclined at 45° to the sun.

Energy loss through the atmosphere is 40% and diffuse radiation is 20% of direct radiation

[i) 3.95×10^{26} W ii) 1397 w iii) 1.797×10^{17} iv) 1598.9 W]

Solution

$$\text{Given radius of sun} = \frac{1.4 \times 10^9}{2} = .7 \times 10^9 \text{ m} = r_s$$

$$\text{Mean distance of the sun from earth} = 15 \times 10^{10} \text{ m}$$

$$\text{Radius of earth} = 6.4 \times 10^6 \text{ m}$$

$$\text{Surface temperature of sun} = 5800 \text{ K}$$

$$\begin{aligned} \text{i) total energy emitted by the sun } E_b &= \sigma A T^4 \\ &= (5.67 \times 10^{-8}) (4\pi r_s^2) (5800)^2 \\ &= .8 \times 10^{25} \text{ W} \\ &= 39.2 \times 10^{25} \text{ W (Ans)} \end{aligned}$$

ii) Emission received per m^2

sun may be regarded as a point source because of distance 15×10^{10} m from the earth. the mean area just outside the earth 's atmosphere over which radiation will fall

$$= 4\pi R^2 = 4\pi (15 \times 10^{10})^2$$

$$\text{SO, emission received per } m^2 = \frac{3.92 \times 10^{26}}{4\pi (15 \times 10^{10})^2} = 1379.35 \text{ W/m}^2 \text{ (Ans)}$$

iii) total energy received by earth assuming earth a spherical body, energy received by it will be proportional to the perpendicular projected area in that of a circle $= \pi \times r_e^2 = \pi \times (6.4 \times 10^6)^2$

$$\begin{aligned}\therefore \text{energy received by earth} &= 1379.35 \times \pi (6.4 \times 10^6)^2 \\ &= 177494.25 \times 10^{12} \\ &= 1.7 \times 10^{17} \text{ W (Ans)}\end{aligned}$$

iv) Energy received by solar collector area $1.5\text{m} \times 1.5\text{m}$; --

a) Direct energy reached the earth (40% less)

$$= (1 - 0.4) \times 1379.35 = 827.61 \text{ W}$$

b) by diffused radiation (20%) $= 827.61 \times 0.2 = 165.52 \text{ W}$

$$\text{total radiation energy received} = a) + b) = 993.13 \text{ W}$$

Projected area of solar collector $= A \times \cos \theta$

$$\begin{aligned}&= 1.5 \times 1.5 \cos 45^\circ \\ &= 2.25 \times \frac{1}{\sqrt{2}} \\ &= 1.6 \text{ m}^2\end{aligned}$$

Energy received by solar collector

$$= 993.13 \times 1.6 = 1589.0 \text{ W (Ans)}$$

Chapter 12

Radiation Exchange Between Surfaces

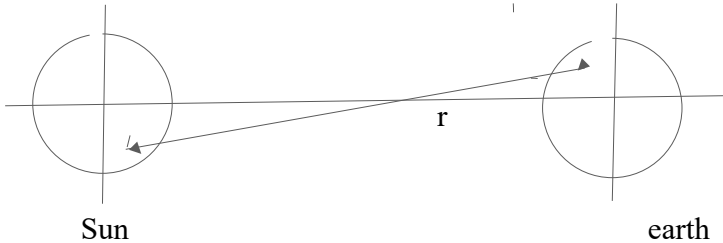
Q 1

Assuming the sun to radiate as a black body, calculate the temperature from the data given below;

Solar constant = 1400 W/m^2 , radius of sun = $6.97 \times 10^8 \text{ m}$

Distance between sun and earth = $14.96 \times 10^{10} \text{ m}$.

Surface temperature of the sun; --



Heat flow from small area dA_1 on the sun and small area dA_2

On the earth is given by;

$$dQ_{1-2} = \frac{I_{b1} \cos\theta_1 \cos\theta_2 dA_1 dA_2}{r^2} \quad [\text{eq n=12.2}]$$

$$\frac{d\theta_{1-2}}{dA_2 \cos\theta_2} = \frac{E_b}{\pi r^2} \times dA_1 \cos\theta_1 \quad \left[I_b = \frac{E_b}{\pi} \right]$$

Integrating both side

$$\int \frac{dQ_{1-2}}{dA_2 \cos \theta_2} = \frac{E_b}{\pi r^2} \int dA_1 \cos \theta_1]$$

$$\text{also, Solar constant} = \int \frac{dQ_{1-2}}{dA_2 \cos \theta_2}$$

$$\text{so} \quad = \frac{E_b}{\pi r^2} \int dA_1 \cos \theta_1 =$$

as sun is at long distance θ_1 is considered 0 so

$$\int dA_1 \cos \theta_1 = A_1 = \pi r_s^2$$

$$\text{solar constant} = \frac{E_b}{\pi r^2} \times \pi r_s^2 = 1400 \text{ w/m}^2$$

$$\text{but } E_b = \sigma T^4$$

$$\text{SO } \frac{\sigma T^4}{\pi r^2} \times \pi r_s^2 = 1400$$

$$\sigma T^4 = 1400 \times \frac{\pi r_s^2}{\pi r^2} = 1400 \times \left(\frac{r}{r_s}\right)^2$$

$$5.67 \left(\frac{T}{100}\right)^4 = 1400 \left(\frac{14.96 \times 10^{10}}{6.97 \times 10^8}\right)^2$$

$$\left(\frac{T}{100}\right)^4 = \frac{1400}{5.67} \left(\frac{14.96 \times 10^2}{6.97}\right)^2$$

$$= 246.9 \left(\frac{14.96}{6.97}\right)^2 \times 10^4 = 1088.8 \times 10^4$$

$$\frac{T}{100} = 5.744 \times 10 = 57.44$$

$$T = 5744^\circ \text{K} \quad \text{Ans.}$$

Q2

A small sphere (outside diameter 50mm) with surface temperature 277 °c is located at the geometric center of a large sphere (inside diameter =250mm) with an inner surface temperature of 7 °c. Calculate how much of emission from the inner surface of the large sphere is

incident upon the outer surface of small sphere assume that both sides approach black body behavior

What is the net interchange of heat between the 2 spheres?

[Ans 4% ,38W]

Solution

Given radius r_1 =small sphere =25mm=.025m

“ r_2 = large “ =125mm=.125 m

Since all radiations being emitted by the small sphere is incident upon and absorbed by the inner surface of the large sphere, therefore configuration factor between 1 and 2 is

$$F_{1-2} = 1$$

As per reciprocating theorem

$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$4\pi r_1^2 \times F_{1-2} = 4\pi r_2^2 F_{2-1}$$

$$F_{2-1} = F_{1-2} \left(\frac{r_1^2}{r_2^2} \right) = 1 \times \left(\frac{.025}{.125} \right)^2$$

$$= \left(\frac{1}{5} \right)^2 = (.2)^2 = .04 = .04 \times 100 = 4\%$$

Thus 4 % of emission from the inner surface of large sphere is incident upon the small sphere and absorbed by it.

Also $F_{2-1} + F_{2-2} = 1$

$$\text{OR, } F_{2-2} = 1 - F_{2-1} = 1 - .04 = .96 = 96\%$$

Thus 96% of the emission from the large sphere is absorbed by the inner surface of the sphere itself.

∴ the net interchange of heat between the 2 spheres is

$$Q_{net} = F_{1-2} \times A_1 \times \sigma \times (T_1^4 - T_2^4)$$

$$\begin{aligned}
 &= 1 \times 4\pi (.025)^2 \times 5.67 \left[\left(\frac{277+273}{100} \right)^4 - \left(\frac{7+273}{100} \right)^4 \right] \\
 &= 1 \times 4\pi \times .000625 \times 5.67 \times [915.06 - 81] \\
 &= 1 \times 4\pi \times .000625 \times 5.67 \times 834.06 \\
 &= 37.34 \text{ W (Ans)}
 \end{aligned}$$

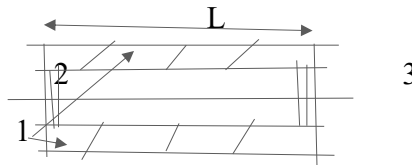
Q 3

A 60mm thick plate with a circular hole of 30mm diameter along the thickness is maintained at uniform temperature of 277°C. Find the loss of energy to the surroundings at 20°C, assuming that 2 ends of the hole to be as parallel disc and metallic surface have a black body characteristic.

[As 6.3 W]

Solution

Given



$$r_2 = r_3 = \frac{30}{2} = 15 \text{ mm} = 0.015 \text{ m} = \text{holes.}$$

$$L = 60 \text{ mm} = 0.06 \text{ m}$$

$$T_1 = 277 + 273 = 550 \text{ K}$$

$$T_{SUR} = 20 + 273 = 293 \text{ K}$$

Above fig (1) solid portions of steel plate

(2) and (3) are 2 parallel holes

$$\frac{L}{r_2} = \frac{0.06}{0.015} = 4$$

$$\frac{r_3}{L} = \frac{0.015}{0.06} = 0.25$$

With ref to fig 12.3 of graphs for parallel disc (from original book) r_2 and r_3 configuration factor F_{2-3} is .065 .

By summation rule $F_{2-1} + F_{2-2} + F_{2-3} = 1$

But $F_{2-2} = 0$

So $F_{2-1} = 1 - F_{2-3} = 1 - .065 = 0.935$

Also, by reciprocating theorem

$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$F_{1.2} = \frac{A_2 F_{2-1}}{A_1} = \frac{\pi (.015)^2 \times .935}{\pi \times \text{disc dia} \times L} = \frac{.00066}{\pi \times .030 \times .06}$$

$$= \frac{.00066}{0.00565} = .1168$$

So, by symmetry $F_{1-3} = F_{1-2} = .1168$

Total loss of energy = loss of heat by both ends

$$= A_1 F_{1-2} \sigma (T_1^4 - T_{SUR}^4) + A_1 F_{1-3} \sigma (T_1^4 - T_{SUR}^4)$$

(As $F_{1-2} = F_{1-3}$)

$$= 2 A_1 F_{1-2} \sigma (T_1^4 - T_{SUR}^4)$$

$$= 2 \times (\pi \times .030 \times .06) \times .1168 \times 5.67 \left[\left(\frac{550}{100} \right)^4 - \left(\frac{293}{100} \right)^4 \right]$$

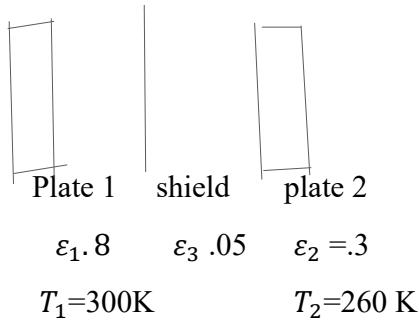
$$= 2 \times .00565 \times .1168 \times 5.67 \times 841.3$$

$$= 6.29 \text{ W (Ans)}$$

Q 5

A double walled flask may be considered equivalent to two infinite parallel plates. The emissivity of walls is 0.3 and 0.8. The space between the wall of the flask is evacuated Find the heat transfer per m^2 area when inner and outer surface temperature are 300 k and 260 k. to

reduce the heat flow a shield of polished Aluminum with $\epsilon = 0.05$ is inserted between the walls. Find the reduction in heat transfer. [Ans 51.2 W/m²]



Heat transfer per m²

In the absence of radiant shield, the heat flow between plate 1 and 2 is given by

$$\begin{aligned}
 Q_{1-2} &= \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \quad [\text{Eqn}=12.21] \\
 &= \frac{\sigma [(\frac{300}{100})^4 - (\frac{260}{100})^4]}{\frac{1}{.8} + \frac{1}{.3} - 1} \\
 &= \frac{5.67[81 - 45.69]}{1.25 + 3.33 - 1} \\
 &= \frac{200.15}{3.58} = 55.9 \text{ W/m}^2
 \end{aligned}$$

% age reduction in heat transfer when a shield is placed between them; --

$$\begin{aligned}
 \frac{A \sigma (T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} &= \frac{A \sigma (T_3^4 - T_2^4)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1} \\
 \text{OR, } \frac{(\frac{300}{100})^4 - (T_3)^4}{\frac{1}{.8} + \frac{1}{.05} - 1} &= \frac{(T_3)^4 - (\frac{260}{100})^4}{\frac{1}{.05} + \frac{1}{.3} - 1}
 \end{aligned}$$

$$\text{Or } \frac{81 - T_3^4}{20.25} = \frac{T_3^4 - 45.69}{22.33}$$

$$\text{Let } T_3 = X$$

$$81 \times 22.33 - 22.33 X^4 = 20.25 X^4 - 20.25 \times 45.69$$

$$42.58 X^4 = 2733.95$$

$$X^4 = 64.2$$

$$X = \sqrt[4]{64.2} = 2.83$$

$$\frac{T_3}{100} = 2.83$$

$$T_3 = 2.83 \times 100 = 283 \text{ K}$$

Heat transfer net

$$\begin{aligned} (Q_{1-3})_{\text{net}} &= \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{5.67 \left(\left(\frac{300}{100} \right)^4 - \left(\frac{283}{100} \right)^4 \right)}{\frac{1}{.8} + \frac{1}{.05} - 1} \\ &= \frac{5.67 (81 - 64.14)}{21.25} = \frac{95.59}{21.25} \\ &= 4.49 \text{ W} \end{aligned}$$

$$\text{Reduction of heat flow due to shield} = (Q_{1-2})_{\text{net}} - (Q_{1-3})_{\text{net}}$$

$$= 55.9 - 4.49 = 51.41 \text{ W/m}^2 \text{ [Ans]}$$

$$\% \text{age reduction} = \frac{51.41}{55.9} = .91 = 91 \% \text{ [Ans]}$$

Q 7

A steel rod of 25 mm diameter has been mounted axially in a heat treatment muffle furnace of inside diameter 200 mm.

The inside furnace temperature of the muffle is 1400 K and has an emissivity of 0.86 while emissivity of surface of the rod is .62. Find the time required to heat the rod from 800 K to 900 K assuming that it

occupies full length of the furnace. For the rod material take specific heat as $0.67 \text{ kJ/kg}^\circ \text{K}$ and density as 7850 kg/m^3 [Ans 28.65 s]'

Solution

Given $\varepsilon_1 = .62$ $\varepsilon_2 = .86$

The net radiant heat exchanger between steel rod and furnace

$$= Q_{12} = (F_g)_{1-2} A_1 (T_1^4 - T_2^4) \text{ [1 and 2 are steel rod \& furnace]}$$

For concentric cylinders of equal length $F_{1-2} = 1$

As per eqn-12.35

$$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{1-2}} + \frac{1-\varepsilon_2}{\varepsilon_2} \left(\frac{A_1}{A_2}\right)}$$

Considering $F_{1-2}=1$ and putting values we get

$$\frac{1}{\frac{1}{.62} + \frac{1-.86}{.86} \times \frac{25}{200}} = \frac{1}{1.61 + .162 \times .125} = \frac{1}{1.63} = .613$$

Putting values, we get

$$\begin{aligned} \text{So, } Q_{12} &= .613 \left\{ \pi \times \frac{25}{1000} \times 1 \text{ m (length)} \times 5.67 \left[\left\{ \frac{T_1}{100} \right\}^4 - \left\{ \frac{T_2}{100} \right\}^4 \right] \right\} \\ &= .27 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right] \end{aligned}$$

Heat exchanger is heating from 800 K , 900 K to 1400 K is

$$Q_{12} = 0.27 \left[\left(\frac{1400}{100} \right)^4 - \left(\frac{800}{100} \right)^4 \right] = 9266 \text{ W}$$

$$Q_{12} = 0.27 \left[\left(\frac{1400}{100} \right)^4 - \left(\frac{900}{100} \right)^4 \right] = 8600 \text{ W}$$

$$\text{Average heat exchange /unit length} = \frac{8600 + 9266}{2} = 8633 \text{ W}$$

Heat required to raise the temperature from 800 K to 900 K

$$Q = m \times C_p \times \Delta t = \left[\frac{\pi}{4} \times \left(\frac{25}{1000} \right)^2 \times 1 \times 7850 \right] \times .67 \times (900 - 800)$$

$$= 256.4 \text{ kJ}$$

Time required to heat the rod from 800K to 900 K

$$= \frac{256.4 \times 1000}{\text{average heat exchange per unit length}}$$

$$= \frac{256400}{8933} = 28.7 \text{ seconds. (Ans)}$$

Q 8

Find the heat lost by radiation per meter length of 60mm diameter pipe at 350°C if

i) it is located in a large room with red bricks, walls maintained at a temperature of 30°C

ii) it is enclosed in a 150mm diameter conduit of red bricks maintained at temperature of 35°C.

Take ϵ (steel rod) = .8, (conduit of brick) ϵ = .92.

Calculate reduction in heat loss due to case ii).

[Ans i) 1215.96W ii) 1183 W iii) 32.96W]

Solution

ii) Given $d = 150 \text{ mm} = .15 \text{ m}$

$$T_1 = 350 + 273 = 623 \text{ K}, T_2 = 35 + 273 = 308 \text{ K}$$

$$\epsilon_1 = .8 \quad \epsilon_2 = .92 \quad d = .06 \text{ m}$$

Rate of heat loss

$$Q = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \left(\frac{1}{\epsilon_2} - 1\right) \frac{A_1}{A_2}}$$

$$A_1 = \pi d L = \pi \times .06 \times 1 = .188, \frac{A_1}{A_2} = \frac{\pi \times .06 \times L}{\pi \times .15 \times L} = .4$$

$$A_2 = P L = \pi \times .15 \text{ m}^2$$

$$Q = \frac{.188 \times 5.67 \left[\left(\frac{623}{100} \right)^4 - \left(\frac{308}{100} \right)^4 \right]}{\frac{1}{.8} + \left(\frac{1}{.92} - 1 \right)}$$

$$= \frac{.188 \times 5.67 (1506.44 - 90)}{1.25 + .032} = \frac{.188 \times 5.67 \times 1416.44}{1.282}$$

$$= 1177.69 \text{ W}$$

i) using the for parallel walls we have

$$q = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Taking mean temperature and mean emissivity of heated room

$$T_2 = \frac{350 + 30}{2} = 190^\circ\text{C} = 190 + 273 = 463 \text{ K}$$

$$\epsilon_2 = \frac{\epsilon_{\text{pipe}} + \epsilon_{\text{brk}}}{2} = \frac{.8 + .92}{2} = .86$$

$$\text{Putting values} = \frac{5.67 \left[\left(\frac{623}{100} \right)^4 - \left(\frac{463}{100} \right)^4 \right]}{\frac{1}{.8} + \frac{1}{.86} - 1}$$

$$= \frac{5.67 (1506 - 459)}{1.25 + 1.16 - 1}$$

$$= \frac{5.67 \times 1041}{1.41}$$

$$= \frac{590247}{1.41}$$

$$= 4186 \text{ W}$$

Note for student for i) part of my assumption may be correct or may be wrong as this part answer is not matching.

Student may try for matching the answer if think so.

Q 9

A 25cm x25cm ingot casting 1.5 m high and at 1225 k temperature is stripped from the mold the casting is made to stand on end on the floor of a large foundry whose wall, floor and roof can be assumed to be at 300 k temperature. If emissivity of casting material is .85, calculate rate of radiant heat interchanging between the casting and the room. [Ans 171.12kw]

Solution

Given

Rate of radiant heat exchange between ingot (1) and room (2)

$$Q_{12} = A_1 \sigma (T_1^4 - T_2^4)$$

$$\text{Area of ingot} = .25 \times 25 + 4 \times .25 \times 1.5 = 1.5625 \text{ m}^2$$

$A_1 \ll A_2$ $F_{1-2} = 1$ (since ingot is very small and enclosed compared to the room)

$$\begin{aligned} \text{Hence } (F_g)_{1-2} &= \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{1-2}} + \frac{1-\epsilon_2}{\epsilon_2} \left(\frac{A_1}{A_2}\right)} \\ &= \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + 1 + 0} = \epsilon_1 = .85 \end{aligned}$$

$$\begin{aligned} \therefore Q_{12} &= 0.85 \times 1.5625 \times 5.67 \left[\left(\frac{1225}{100}\right)^4 - \left(\frac{300}{100}\right)^4 \right] \\ &= 0.85 \times 1.5625 \times 5.67 [(12.25)^4 - (3)^4] \\ &= 0.85 \times 1.5625 \times 5.67 \times 22437.75 \\ &= 168966.77 \text{ W} \\ &= 168.966 \text{ KW. (Ans)} \end{aligned}$$

Q 10

A thermocouple is used to measure the temperature of a hot gas flowing through a pipe which is exposed to surrounding at 650 k. The convection heat transfer coefficient from the gas to thermocouple bead is 42 w/m²k. The thermocouple indicates is temperature of 750 k and it is so arranged that the heat transfer by conduction along the wires is negligible the emissivity of thermocouple material is .80. find the true gas temperature.

[Ans 898.9 k]

Solution

Given $T_{wall}=650$ k $T_{COUPLE} = 750$ K, $T_{GAS} = ?$

$h_{con.}=42$ w /m² k, $\epsilon=.8$, $A=AREA$, $\sigma=5.67$.

To find true gas temperature $=T_{gas}$

Heat generated by thermocouple by convection

= heat lost by thermocouple by radiation.

A x convection heat transfer coefficient from the gas x

$$(T_{gas} - T_{thermoc.}) = A \times \epsilon \times \sigma \times (T_{COUPL}^4 - T_{WALL}^4)$$

$$\text{Or, } A \times 42 \times (T_{gas} - 750) = A \times .8 \times 5.67 \left[\left(\frac{750}{100}\right)^4 - \left(\frac{650}{100}\right)^4 \right]$$

$$(T_{gas} - 750) = \frac{.8 \times 5.67}{42} [3164 - 1785]$$

$$=.108 \times 1379$$

$$T_{gas} = 148.9 + 750 = 898.9 \text{ k}$$

True gas temperature = 898 .9 ° K (Ans)

Chapter 13

Mass Transfer

Q 1

The molecular weight of the two components A and B of gas mixer are 20 and 40 respectively the molecular weight of the gas mixture is found to be 25. If the mass concentration of the mixture of 1kg/m^3 , determine the following

- i) molar fraction
 - ii) mass fraction
 - iii) total pressure if the temperature of the mixture is 27°C
- [i) .75, .25 ii) .6, .4 iii) 99.8 kPa. }

Solution

Given universal gas constant = $8.314\text{ kJ/kg} = G$

Molecular weight $M_A = 20, M_B = 40, M_{\text{gas mix}} = 25$

Mass concentration of mix of $1\text{kg/m}^3 = \rho$

Temperature of mixture = $27^\circ\text{C} = 27 + 273 = 300\text{ K}$

Molecular concentration of mixture = $C = \frac{\rho}{M} = \frac{1}{25} = .04$

$$C_A + C_B = C \quad [\text{EQ 13.5}]$$

$$= .04 \text{ ----- } 1$$

$$\rho_A = M_A C_A = 20 C_A$$

$$\rho_B = M_B C_B = 40 C_B$$

Adding

$$\rho_A + \rho_B = \rho$$

$$20C_A + 40C_B = \rho = 1 \text{ -----II)}$$

From i)

$$20 C_A + 20 C_B = .8 \text{ -- I)}$$

$$20C_A + 40C_B = 1 \text{ -- II)}$$

Subtracting we get

$$---- 20 C_B = --- .20$$

$$C_B = .01$$

$$\therefore C_A = .03$$

$$\therefore \rho_A = 20 C_A = 20 \times .03 = .6$$

$$\rho_B = 40 C_B = 40 \times .01 = .4$$

Now i) molar fraction $= x_A, X_B$

$$x_A = \frac{C_A}{C} = \frac{.03}{.04} = .75 \text{ Ans}$$

$$x_B = \frac{C_B}{C} = \frac{.01}{.04} = .25 \text{ Ans}$$

ii) Mass Fraction

$$m_A^* = \frac{\rho_A}{\rho} = \frac{.6}{1} = .6 \text{ Ans}$$

$$m_B^* = \frac{\rho_B}{\rho} = \frac{.4}{1} = .4 \text{ Ans}$$

iii) total pressure when temperature 27 C

Using gas equation

$$PV = m R T,$$

$$P = \frac{mRT}{V} = \left(\frac{m}{V}\right) \times R \times T$$

$$= \rho \times \left(\frac{R}{M}\right) \times T$$

$$\begin{aligned}
 &= 1 \times \frac{8.314}{25} \times (27^\circ\text{C} + 273) \\
 &= \frac{1 \times 8.314 \times 300}{25} \\
 &= 99.768 \text{ kPa (Ans)}
 \end{aligned}$$

Q2

A vessel contains a binary mixture of oxygen and nitrogen with partial pressure in ratio of .21 and .79 at 27°C . The total pressure of mixture is 1 bar. Determine

- i) molar concentration ii) Mass density iii) mass fraction
iv) molar fraction of each species.

$$\begin{aligned}
 &[\text{i) } .00842 \text{ kg mole}/\text{m}^3 \text{ ii) } .03167 \text{ kg mole}/\text{m}^3, .269 \text{ kg}/\text{m}^3 \\
 &\quad , .8848 \text{ kg}/\text{m}^3 \text{ iii) } .233, .767 \text{ iv) } .21, .79]
 \end{aligned}$$

Solution

$$\text{Given } T = 27 + 273 = 300 \text{ K}, P = 1 \text{ bar} = 1 \times 10^5 \text{ N}/\text{m}^2$$

$$\text{Gas constant} = 8314 \text{ J}/\text{kg mole K}$$

$$\text{Partial pressure ratio, oxygen} = .21 \times 1 \times 10^5$$

$$\text{--- do --- nitrogen} = .79 \times 1 \times 10^5$$

$$\text{Molar concentration } C_{O_2} \quad C_{N_2}$$

$$C_{O_2} = \frac{PO_2}{RT} = \frac{.21 \times 1 \times 10^5}{8314 \times 300} = \frac{21000}{2494200} = .00841 \text{ kg mole}/\text{m}^3 \text{ Ans}$$

$$C_{N_2} = \frac{PN_2}{RT} = \frac{.79 \times 1 \times 10^5}{8314 \times 300} = \frac{79000}{2494200} = .0316 \text{ kg mole}/\text{m}^3 \text{ Ans}$$

$$\text{ii) mass density } \rho_{O_2} = M_{O_2} \times C_{O_2}$$

$$[\text{EQ 13.1 } \rho = M, M = \text{mol wt}, C = \text{molar concentration}]$$

$$= 32 \times .00841$$

$$=.2688 \text{ kg/m}^3 (\text{Ans})$$

$$\rho_{N_2} = M_{N_2} \times C_{N_2}$$

$$= 28 \times .0316$$

$$=.8848 \text{ kg/m}^3 (\text{Ans})$$

iii) Mass fraction x_{O_2} , x_{N_2}

$$x_{O_2} = \frac{\rho_0}{\rho} = \frac{.2688}{1.1536} = .2330 (\text{Ans})$$

$$x_{N_2} = \frac{\rho_{N_2}}{\rho} = \frac{.8848}{1.1536} = .7669 \quad \text{Ans}$$

iv) molar fraction of each species $m_{O_2}^*$, $m_{N_2}^*$

$$C = C_{O_2} + C_{N_2} = .00841 + .0316 = .04001$$

$$m_{O_2}^* = \frac{C_{O_2}}{C} = \frac{.00841}{.04001} = .2101 \quad \{\text{Ans}\}$$

$$m_{N_2}^* = \frac{C_{N_2}}{C} = \frac{.0316}{.04001} = .7898 \quad \{\text{Ans}\}$$

Q 3

A steel rectangular container having 15mm thick is used to store gaseous hydrogen at elevated pressure. The molar concentration of hydrogen in the steel at the inside and outside surface are 1 kg mole/ m^3 and 0 respectively. Assuming diffusion coefficient for hydrogen in steel as $.25 \times 10^{-12} m^2/s$.

Calculate the molar diffusion flux for hydrogen through the steel.

$$[\text{Ans } 16.66 \times 10^{-12} \text{ kg mole/s.m}^2]$$

Solution

$$\text{Given} \quad L = 15 \text{ mm} = .015 \text{ m} \quad C_{A1} = 1 \text{ kg mole/m}^3$$

$$C_{A2}=0, D_A=.25 \times 10^{-12} m^2/s$$

To find molar diffusion flux rate N_A :-

Assuming one dimensional and steady state condition the molar diffusion flux rate through the steel is given by Fick's Law of difference

$$N_A = \frac{m_A}{A} = D_A \left(\frac{C_A - C_{A2}}{L} \right) \quad [\text{Eq 13.34}]$$

$$= .25 \times 10^{-12} \left[\frac{1-0}{.015} \right]$$

$$= \frac{1}{.015} \times .25 \times 10^{-12}$$

$$= 16.66 \times 10^{-12} \text{ kg mole/s.m}^2$$

Q 4

A plastic membrane 0.25 mm thick has hydrogen gas maintained at pressure of 2.5 bar and 1 bar on its opposite sides

The binary diffusion coefficient of hydrogen in the plastic is

$8.5 \times 10^{-8} m^2/s$ and solubility of hydrogen in membrane is

$.0015 \text{ kg mole /m}^2 \cdot \text{bar}$. Under the uniform condition of 25°C

Calculate i) the molar concentration of hydrogen at opposite

Faces of membrane

ii) the molar and mass diffusion flux of hydrogen through the membrane.

$$[\text{i}) 0.00375 \text{ and } .0015 \text{ kg mole/m}^3, \text{ ii) } 76.5 \times 10^{-8} \text{ kg mole/s.m}^2$$

$$153 \times 10^{-8} \text{ kg/s.m}^2]$$

Solution

Given $L = .25 \text{ mm} = .00025 \text{ m}$, pressure = 2.5 bar and 1 bar

$S = \text{solubility} = .0015 \text{ kg mole /m}^2 \cdot \text{bar}$

D = Binary diffusion coefficient = $8.5 \times 10^{-8} \text{ m}^2/\text{s}$

M_n = molecular weight of hydrogen.

i) Molar concentration of hydrogen C_{h1} C_{h2}

concentration pressure and solubility are related as follows

$$C = S \cdot P$$

$$C_{h1} = S \cdot P = .0015 \times 2.5 = .00375 \text{ kg mole/m}^3$$

$$C_{h2} = S \cdot P = .0015 \times 1 = .0015 \text{ kg mole/m}^3$$

ii) Molar and mass diffusion flux: --

the molar diffusion flux of hydrogen through the membrane is calculated from the relation

$$\begin{aligned} N_h \frac{m_h}{A} &= \frac{D(C_{h1} - C_{h2})}{(x_2 - x_1)} = \frac{D(C_{h1} - C_{h2})}{L} \\ &= \frac{8.5 \times 10^{-8} (.00375 - .0015)}{.00025} \\ &= 76.5 \times 10^{-8} \text{ kg mole/s-m}^2 (\text{Ans}) \end{aligned}$$

Q 7

An open tank 5mm diameter contain 1 mm deep layer of benzene (molecular weight=78) at its bottom. The vapor pressure of benzene in the tank is 14 k pa. Diffusion of benzene takes place only through air film 3mm thick, the operating pressure and temperature of the system are 1 atm and 20°C and at this condition the diffusivity of benzene is $8.5 \times 10^{-6} \text{ m}^2/\text{s}$.

Calculate the time taken for entire benzene to evaporate Take density of benzene as 880 kg/m^3 . [Ans 10.73 min]

[k pa=kl pascal]

Solution

Given let suffixes b and a refers benzene and air respectively.

The partial pressure of benzene and air at inlet and outlet of air film are

$$p_{b1} = 14 \text{ kpa} , p_{b2} = 0.1 \text{ atm} = 1 \text{ atm} \cdot \text{pr} = 1.013 \text{ bar}$$

$p_{a1} = p - p_{b1} = 1.013 - .14 = .873 \text{ bar}$, $p_{b2} = 0$, because beyond air film there is no diffusion. Diffusivity of benzene $D = 8.5 \times 10^{-6} \text{ m}^2/\text{s}$.

$$\begin{aligned} m_b = \text{mass of benzene} &= \frac{D A M_b}{G J} \times \frac{p}{x_2 - x_1} \times \ln \left(\frac{p_{a2}}{p_{a1}} \right) \\ &= \frac{8.5 \times 10^{-6} \times \left(\frac{\pi}{4} \times 5^2 \right) \times 78 \times 1.013 \times 10^5}{8314 \times (20 + 273) (3 \times 10^{-3})} \times \ln \left(\frac{1.03}{.873} \right) \\ &= \frac{.85 \times 19.63 \times 78 \times 11.013 \times .148}{8314 \times .293 \times 3} \\ &= \frac{195.12}{7308.006} = .0266 \text{ kg/s} \end{aligned}$$

The mass of benzene in 5m diameter tank

$$\begin{aligned} &= \frac{\pi}{4} \times \frac{5^2 \times 1}{1000} \times 880 \\ &= 19.63 \times .88 \\ &= 17.27 \text{ kg} \end{aligned}$$

Hence time taken to entire benzene to evaporate

$$= \frac{17.2744}{.0266} = 648.28 \text{ sec} = 10.8 \text{ min (Ans)}$$

Q 8

Air at 25° c and 1 atm containing small quantities of iodine

($V=15.5 \times 10^{-6} \text{ m}^2/\text{s}$) ($D=0.82 \times 10^{-5} \text{ m}^2/\text{s}$) flows with a velocity of 5,25 m/s inside a 30 mm diameter tube. Calculate mass transfer coefficient for iodine [Ans 0.0176m/s]

Solution

Given $v=15.5 \times 10^{-6} \text{ m}^2/\text{s}$ $D=.82 \times 10^{-5} \text{ m}^2/\text{s}$

$U = 5.25 \text{ m/s}$, $d=\text{diameter of tube} = 30\text{mm}=.03 \text{ m}$

Mass transfer coefficient h_m

$$\text{Schmidt no} = S_c = \frac{\mu}{\rho D} = \frac{V}{D} = \frac{15.5 \times 10^{-6}}{.82 \times 10^{-5}} = \frac{1.55}{.82} = 1.89$$

$$\begin{aligned} \text{Reynold no} = R_e &= \frac{\rho L U}{\mu} = \frac{d U}{V} = \frac{.03 \times 5.25}{15.5 \times 10^{-6}} = \frac{30 \times 5250}{15.5} \\ &= 10161.29 \end{aligned}$$

Obviously, the flow is turbulent and thus

$$\begin{aligned} \text{Sherwood no} = S_h &= 0.023(R_e)^{.83}(S_c^{.44}) \\ &= 0.023(10161)^{.83}(1.89)^{.44} \\ &= 0.023 \times 2117.17 \times 1.323 \\ &= 64.42 \end{aligned}$$

$$\text{Sherwood no} = \frac{h_m d}{D}$$

$$\begin{aligned} \therefore \text{Mass transfer coefficient} = h_m &= \frac{S_h D}{d} \\ &= \frac{64.42 \times .82 \times 10^{-5}}{.03} \\ &= \frac{.000528}{.03} = .0176 \text{ m/s} \end{aligned}$$

Q 9

Air at 20°C flows over a tray (length = 0.3m x width = 0.4m)

Full of water velocity of 2.8m/s. The total pressure of moving air is 1 atm and partial pressure of water present in air is .007 bar. If the temperature on the water surface is 15°C, calculate evaporation rate of water. The thermophysical properties of air. $\rho = 1.205 \text{ kg/m}^3$, $\nu = 15.06 \times 10^{-6} \text{ m}^2/\text{s}$ and $D = 4.166 \times 10^{-5} \text{ m}^2/\text{s}$.

D = diffusion coefficient [Ans .84kg/h r]

Solution

Given $U = 2.8 \text{ m/s}$ (since it is not provided, we have considered from our side).

$L = .3 \text{ m}$, width = .4m $\rho = 1.205 \text{ kg/m}^3$, $\nu = 15.06 \times 10^{-6} \text{ m}^2/\text{s}$

$D = 4.166 \times 10^{-5} \text{ m}^2/\text{s}$ (binary diffusion coefficient)

$T_a = 20^\circ\text{C}$ air temperature., $P_a = 1 \text{ atm} = 1.013 \text{ bar}$ (air pressure)

$T_w = 15^\circ\text{C}$ = temp. of water surface, $P_{rw} = .007 \text{ bar}$ (partial pr of water) $A = .3 \times .4 = .12 \text{ m}^2$

To find type of flow; --

$$Re = \frac{\rho L U}{\mu} = \frac{L U}{\nu} = \frac{.3 \times 2.8}{15.06 \times 10^{-6}} = .557 \times 10^5, < 5 \times 10^5$$

Hence flow laminar.

$$\text{Now Sherwood no } = S_h = \frac{h_m \times L}{D} = 0.664 (Re)^{.5} \times (Sc)^{.33}$$

$$Sc = \text{Schmidt no} = \frac{\nu}{D} = \frac{15.06 \times 10^{-6}}{4.16 \times 10^{-5}} = 0.3615$$

$$\frac{h_m \times .3}{4.166 \times 10^{-5}} = 0.664 (.557 \times 10^5)^{.5} \times (.3615)^{.33} = 112.11$$

$$\therefore h_m = \frac{112.11 \times 4.166 \times 10^{-5}}{.3} = .0155 \text{ m/s} = \text{mass transfer coefficient}$$

Partial pressure of water over the water surface

P_{W1} is saturation pr of water at $15^\circ = .017 \text{ bar}$

p_{w2} is partial pr of water in the air $= .007 \text{ bar}$

So, mass transfer coefficient based on presser difference

$$\begin{aligned} = h_{mp} &= \frac{h_m}{R T} = \frac{.0155}{287 \times (15 + 273)} \text{ \{e qn 13.72\}} \\ &= \frac{.0155}{287 \times 288} = \frac{.0155}{82656} = 1.87 \times 10^{-7} \text{ m/s} \end{aligned}$$

Mass diffusion of water is given by

$$\begin{aligned} m_w &= h_{mp} \times A \times (P_{W1} - P_{W2}) \\ &= 1.87 \times 10^{-7} \times (.3 \times .4) \times (.017 - .007) \\ &= 1.87 \times .12 \times .01 \times 10^{-7} \times 10^5 \\ &= 0.00002244 \text{ kg/s} \\ &= 0.00002244 \times 3600 \text{ kg/hr} \\ &= 0.081 \text{ kg/h r (Ans)} \end{aligned}$$

Q 10

Hydrogen gas at 47°C and 3 atm flows through a rubber tubing of 1.5 cm inner radius and 2.5 cm outer radius The diffusivity of hydrogen through rubber is stated to be

$0.792 \times 10^{-4} \text{ m}^2/\text{s}$ (*) of solubility of hydrogel 0.06 m^3 of hydrogen / m^3 of rubber at 1 atm If the gas constant is

4160 kJ/kg and concentration of hydrogen at the outer surface of the tubing is negligible calculate the diffusion flux rate of hydrogen /m length of rubber tubing. [Ans; $0.395 \times 10^{-4} \text{ kg/h-m}$]

(*) considered as $.792 \times 10^{-4} m^2/hr$ to match the answer, student may please note.

Solution $D =$ diffusivity of hydrogen $= 0.792 \times 10^{-4} m^2/h$ r. (*)

Given $\rho = 3 \times 10^5 N/m^2$ $T = 47 + 273 = 320 K$,

$R = 4160 J/kg = 4160 \times 10^3 J/kg$

$r_1 = 1.5 cm = .015 m$, $r_2 = 2.5 cm = .025 m$

$S =$ solubility of hydrogen

$= .06 m^3 \times 3 = .18 m^3$ rubber tubing $= V$ Vol of hydrogen in m^3

From perfect gas law

$$PV = m RT$$

$$m = \frac{PV}{RT} = \text{constant} = C$$

$$= \frac{3 \times 10^5 \times .18}{(4160 \times 1000 \times 320)} = 4.05 kg/m^3 \text{ rubber tubing}$$

At the inner surface of pipe.

$$= C_{h1}$$

And $C_{h2} = 0$ as resistance to diffusion is negligible.

Now diffusion flux through cylinder

$$m = \frac{D(C_{h1} - C_{h2})}{\Delta x} \times A_m$$

Where $\Delta x = (r_2 - r_1) = .025 - .015 = .010 m$

$$\begin{aligned} A_m &= \frac{2\pi L(r_2 - r_1)}{\ln \frac{r_2}{r_1}} = \frac{2\pi(.025 - .015)}{\ln \frac{.025}{.015}} \\ &= \frac{2\pi \times .01}{\ln 1.666} = \frac{.0628}{.510} \\ &= .1232 m^2 \end{aligned}$$

$$\begin{aligned}
 \text{Diffusion flux } m &= \frac{.792 \times 10^{-4} (4.05 - 0)}{.010} \times .1232 \\
 &= \frac{.792 \times 4.05 \times 10^{-4}}{.010} \times .1232 \\
 &= .0000395 \text{ kg/h r-m} \\
 &= .395 \times 10^{-4} \text{ kg/h r-m (Ans)}
 \end{aligned}$$

Q 11

Air is contained in a tire tube of surface area 0.5 m^2 and wall thickness 1 cm

The pressure of air drop from 2 bar to 1.99 bar in a period of 5 days the solubility of air in the rubber is $.07 \text{ m}^3$ of air per m^3 of rubber at 1 bar Determine the diffusivity of air rubber at the operating temperature of 27°C if the volume of air in tube is $.025 \text{ m}^3$. [$0.55 \times 10^{-10} \text{ m}^2/\text{s}$]

Solution

Given surface area $= .5 \text{ m}^2$, $=A$, L =thickness $= .01\text{m}$

$P_i = P_1 = 2 \text{ bar}$. $P_f = P_2 = 1.99 \text{ bar}$, $t = 5 \text{ days}$. R =gas constant
 $= 287$.

Solubility of air $= .07 \text{ m}^3/\text{m}^3$, $T = 27 + 273 = 300 \text{ K}$

Volume of air in tube $= .025 \text{ m}^3$

To find diffusivity $= ? D$

Initial mass of air in tube $= m_i = \frac{P_1 V}{R T} = \frac{2 \times 10^5 \times .025}{287 \times 300} = .0580 \text{ kg}$

Final mass of air $= m_f = \frac{P_f V}{R T} = \frac{1.99 \times 10^5 \times .025}{287 \times 300} = .0577 \text{ kg}$

Mass of air escaped $= m_i - m_f = m_a = .0580 - .0577 = .0003 \text{ kg}$

Mass flux of air escaped $= N_a = \frac{m_a}{A \times \text{time elapsed}}$

$$= \frac{.0003}{24 \times 5 \times 3600 \times .5} = 1.43 \times 10^{-9}$$

S_2 = Solubility of air at mean specific pressure

$$(\text{Mean specific pressure } P_m = \frac{2+1.99}{2} = 1.995 \text{ bar})$$

= s x means specific pressure

$$= .07 \times 1,995 = .13965 \text{ m}^3/\text{m}^3$$

Air which escapes to atmosphere will be at 1 bar and its solubility will remain at $.07 \text{ m}^3$ of air per m^3 rubber.

Corresponding mass concentration at the inner and outer surfaces of tube from characteristic gas are calculated are calculated as follows

$$C_{a1} = \frac{P_m V_1 S_2}{RT_1} = \frac{1995 \times 10^5 \times .13965}{287 \times 300} = .3255 \text{ kg/m}^3$$

$$C_{a2} = \frac{P_2 V_2}{RT_2} = \frac{1 \times 10^5 \times .07}{287 \times 300} = \frac{7000}{86100} = .0813 \text{ kg/m}^3$$

The diffusion flux rate of air through the rubber is given by

$$N_a = \frac{mxA}{A} = \frac{D(C_{h1} - C_{h2})}{X_2 - X_1} = \frac{D(.3235 - .0813)}{.L} = \frac{DX.2422}{.01}$$

$$1.35 \times 10^{-9} = \frac{.2422 D}{.01}$$

$$D = \frac{1.35 \times 10^{-9} \times .01}{.2422} = .055 \times 10^{-9} = .55 \times 10^{-10} \text{ m}^2/\text{s} (\text{Ans})$$

= Diffusivity of air.

Q 12

Due to accidental opening of a valve the water has been split out on the floor of an industrial plant The water level 1 mm and temperature 20°C and 1 atm Assuming $D = 0.26 \times 10^{-4} \text{ m}^2/\text{s}$

And evaporation takes place due to molecular diffusion through an air film 5mm thick determine the time required to evaporate complete water [Ans 2.56 hour]

Solution

Given D = Diffusivity coefficient = $0.26 \times 10^{-4} m^2/s$

Film thickness = 6mm = .006 m = $(x_2 - x_1)$

W = specific humidity of air = 1.8 g/kg of dry air

Water level = 1.2mm = .0012m

$T = 20 + 273 = 293$ K, $P_a = 1$ bar at 20 c

p_{w1} = saturation pr at 20 c = 0.03169 bar. (not given in Q)

To find time required to evaporate water completely; ---

$w = \frac{0.622 p_{w2}}{p - p_{w2}}$ this expression obtained from specific humidity

$$1.8 \times 10^{-3} = \frac{.622 \times P_{W2}}{1 - P_{W2}}$$

We get $1.8 \times 10^{-3} = 1.8 \times 10^{-3} P_{W2} = .622 P_{W2}$

$$P_{W2} = \frac{.0018}{.622 + .0018} = .00288 \text{ bar}$$

Using (eq n 13.56) we get

m_W = total mass transport of water vapor

$$= \frac{D A M_W}{G T} x \frac{P}{X_2 - X_1} x \ln \left(\frac{p - p_{w2}}{p - p_{w1}} \right)$$

[G = gas constant = 8314, M_W = Mol w t of water = 18]

$$= \frac{.26 \times 10^{-4} \times 1 \times 18}{8314 \times 293} x \frac{1 \times 10^5}{.005} x \ln \frac{1 - .00288}{1 - .03169}$$

$$= \frac{.000026 \times 18 \times 10^5}{12387.86} x \ln \frac{.997}{.968} = \frac{2.6 \times 18 \times .029}{12387.86}$$

$$= \frac{1.3572}{12387.86} = .0001095 = 10.95 \times 10^{-5} \text{ kg/s-m}^2$$

Total amount of water to be evaporated $/m^2$

$$= 1 \times \frac{1mm}{1000} \times 1000 = 1 \text{ kg}$$

$$\text{Time required to evaporate} = \frac{1kg}{m_w)_{total}}$$

$$= \frac{1}{10.95 \times 10^{-5}} \text{ sec}$$

$$= \frac{10^5}{10.95 \times 3600} \text{ hrs}$$

$$= 2.536 \text{ hours (Ans)}$$

Q 13

Air at a temperature of 25°C and 40% of RH (relative humidity) flows over a water surface at a value of 1.5 m/s length parallel to flow is 180 mm. Average surface temperature is 16°C . Estimate the amount of water evaporated per hour per m^2 of surface area. Partial pressure of water vapor at 20°C and 40% RH is 0.011 bar at 16°C and saturated the vapor pressure is 0.017 bar. Viscosity and density of air are $18.38 \times 10^{-6} \text{ kg/m-s}$ and 1.22 kg/m^3 respectively. Assume $D = \text{diffusivity} = 0.256 \times 10^{-4} \text{ m}^2/\text{s}$.

$$[\text{Ans } 0.176 \text{ kg/h/m}^2]$$

Solution

Given $A = 1 \text{ m}^2$ $G = \text{gas constant} = 8314 \text{ J/kg mole K}$, $U = 1.5 \text{ m/s}$

$$L = 180 \text{ mm} = 0.18 \text{ m}, \mu = 18.38 \times 10^{-6} \text{ kg/m s}, \rho = 1.22 \text{ kg/m}^3$$

$$V = \frac{\mu}{\rho} = \frac{18.38 \times 10^{-6}}{1.22} = 1.507 \times 10^{-5} \text{ m}^2/\text{s}, D = 0.256 \times 10^{-4} \text{ m}^2/\text{s}$$

$$R_{H_2O} = \frac{G}{18} = \frac{8314}{18} = 461.9 \text{ J/KG K}, K_a = 25^\circ \text{C}, t_{ws} = 16^\circ \text{C}$$

$$P_{wa} = 0.011 \text{ bar}, P_{ws} = 0.017 \text{ bar}.$$

i) amount of water evaporated $m_w \text{ kg/h/m}^2$

$$R_e = \text{Reynold no} = \frac{UL}{\nu} = \frac{1.5 \times .18}{1.507 \times 10^{-5}} = \frac{27000}{1.507} = 17916.39$$

$$= 1.7916 \times 10^4$$

Since $R_e < 5 \times 10^5$ so flow is laminar.

ii) mass transfer coefficient

in case of laminar flow over surface we have (equ.13.77)

$$\text{Sherwood no} = S_h = \frac{h_m L}{D} = 0.664 (R_e)^{.5} (S_c)^{.33}$$

$$= .664 (1.7916 \times 10^4)^{.5} \left(\frac{\nu}{D} \right)^{.33}$$

$$= .664 \times 133.85 \times \left(\frac{1.507 \times 10^{-5}}{.256 \times 10^{-4}} \right)^{.33}$$

$$= .664 \times 133.85 \times .838 = 74.47 = S_c$$

Also, from above equation

Mass transfer coefficient based on concentration

$$h_m = \frac{S_h D}{L} = \frac{74.47 \times .256 \times 10^{-4}}{.18} = \frac{19.06 \times 10^{-4}}{.18} = 105.91 \times 10^{-4}$$

$$= .01059 \text{ m/s}$$

iii) concentration (densities) at temperature $t_a^\circ \text{C}$

[a-partial pr = saturated pr]

$$C_{wa} = \frac{P_{wa}}{R_{H_2O} T_a} = \frac{.011 \times 10^5}{461.9 \times (25 + 273)} = \frac{1100}{137646.2} = .00799 \text{ kg/m}^3$$

Concentration (density) at temperature t_s°

$$C_{ws} = \frac{P_{ws}}{R_{H_2O} T_s} = \frac{.017 \times 10^5}{461.9 \times (16 + 273)} = \frac{1700}{133489.1} = .01273 \text{ kg/m}^3$$

Mass diffusion of water is given by

$$m_w = h_m A (C_{ws} - C_{wa})$$

$$=.01059 \times 1(.01273 - .00799)$$

$$=.01059 \times 1 \times .00474$$

$$=.00005019$$

$$=5.019 \times 10^{-5} \text{ kg/sm}^2$$

$$=5.019 \times 10^{-5} \times 3600 \text{ kg/h per m}^2$$

$$=.1806 \text{ kg/h per m}^2(\text{Ans})$$