THE LIFE WITH PHYSICS

A NARRATIVE BASED TEXTBOOK

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Preface

To anyone who has ever stared at physics equations & terms and thought ‘which material should I use to understand it easily and better?’ This book is for you. This Title not only provides approachable and understandable academic contents, but also sets a fine stage for learning.

When I started to be serious in learning Physics, many times I got myself tense and struggled to keep continuing. The reason I admitted was mainly because of not having a helpful learning environment.

The goal of this book is to address the issue and keep the readers electrified throughout the study by presenting the academic modules with the help of a novel.

THE LIFE WITH PHYSICS is an invitation to study in an engaging and interactive environment. In the pages that follow, you will meet six friends–Aditi, Aakash, Dolly, Kimaya, Arnold, and David–living in the coastal village of Kolty. Their adventures are your adventures; their questions are your questions.

By embedding high-quality 11th-grade physics concepts within a relatable, character-driven story, we hope to transform the concepts into an exciting lens through which you can understand it clearly.

I wish to thank my friend Animesh and elder brother Sandeep Patil for invaluable suggestions and critical review of the manuscript. I also wish to express my gratitude to my typist Gaurav Jee for their helpful cooperation.

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Chapter - 1
Dimensions

It was a cloudy day with cold air blowing through Kolty, a small coastal village that was completely different from the hustle-bustle of the nearest city.

Four residents of the village–Aditi, Kimaya, Dolly, and Aakash–were waiting eagerly near a huge mango tree for the private bus. They, along with two other guys, Arnold and David, had arranged last evening for transportation to the city, about 30 kms away. This was the first day of attending 11th-Grade class in their new institution, which was located in the centre of the city.

But suddenly, two female tourists were seen near the spot talking excitedly and very personally about their boyfriends. The talk was uncomfortable for the group, especially for Aditi. Aakash quickly walked over and requested them to keep their voices down as it was getting awkward for him and his friends. One of the tourists responded shyly, "Okay, dear, and sorry!"

Shortly after, the bus arrived and the other two guys were already on it. Now all of them were on the way. Some were talking about the Physics chapter 'Dimension' which was scheduled to start on that day, while others were praising the principal's approval that let them attend only the Physics class as they had requested.

They were allowed to do so in response to a letter that stated their willingness to attend only formal Physics class and their ability to score well in other subjects without traditional classroom instruction. The letter was signed by all the guys and their parents before Dolly submitted it to the institution's head.

At the same time, Aditi and Akash were laughing, putting hands over their mouths, probably remembering the awkward stuff.

Time passed quickly; the bus arrived at the destination. Dolly said quietly to the driver, "Park the bus on the left side, you can wait or explore the city until the class is over." She then told him when the class would finish.

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The Physics teacher, Gita, who must be in her 30s, appeared very professional and began the class after a brief personal introduction.

1.1 Unit

A unit is a quantity with a fixed and defined magnitude that is used as a standard to measure physical quantities of the same type. For example, suppose the length of a desk is measured to be 2 m. Here m is a unit, its magnitude is 1 and length is a physical quantity which is measured.

Fundamental Units: It is also known as base units. The units required to express or measure fundamental physical quantities are called fundamental units. These units are also used to give units of derived quantities.

For example Metre (m), Kilogram (kg), Second (s), Ampere (A), Kelvin (k), Mole (mol), Candella (cd).

Derived Units: The units of all other physical quantities that can be shown or expressed as a combination of the base units known as derived units.

For examples: speed - m/s, Acceleration - m/s^2.

1.2 System of Units

It is a collection of both fundamental and derived units to measure physical quantities. It is also called the system of units of measurement. There are different systems of units like CGS, MKS, and FPS systems, but the MKS system is the most widely used system of units.

Long ago, different scientists belonging to different nations used different systems of units for measurement like CGS FPS and MKS. The base units for Length, Mass and Time in these systems were as follows:

CGS system - centimetre, gram and second

FPS system - foot, pound and second

MKS system - metre, kilogram and second

However, at the general conference on weights and measures in 1971, a meeting was held to decide a system of units that was to be accepted internationally and it continues till now is known as the international system of units.

It is abbreviated as SI, MKS (metre kilogram second) is the SI units of measurement for length, mass and time respectively. Below are the base quantities and their SI units:

Base Quantity	SI Units
Length	Metre (M)
Mass	Kilogram (kg)
Time	Second (s)

Electric	Ampere (A)
Amount of substance	Mole (mol)
Luminous Intensity	Candela (cd)
Thermodynamic Temperature	Kelvin (K)

1.3 Significant Figures

Significant Figures or Significant Digits are the digits in a number that are important for indicating the precision of a calculation or measurement. We will go through some details to generalize rules to determine significant figures.

First, a choice of change of different units doesn't change the number of significant digits or figures in a measurement. For example, 4.304 kg has four significant figures. However, in a different unit, the same value can be written as 4304 g, 0.004304 t (t for ton). All these numbers have the same number of significant figures, that is four.

This example lead us to formulate many rules:

- All the non-zero digits are significant
- All the zeros between two non-zero digits are significant
- If the number is less than 1, the zero(s) on the right of the decimal point but to the left of the first non-zero digit are not significant. (In 0.002308, the underlined zeroes are not significant.)
- The terminal or trailing zero(s), in a number without a decimal point are not significant. (Thus 145m = 14500cm = 145000mm has three significant figures, the trailing zero(s) are not significant.)
- The trailing zero(s) in a number with a decimal point are significant. (The numbers 3.500 or 0.06900 have four significant figures each.)

Example 1.1 Match List - I with List - II

List I (Number)	List II (Significant Figure)
(a) 1003	(i) 3
(b) 040.1	(ii) 4
(c) 100.200	(iii) 5
(d) 0.0020010	(iv) 6

The correct matching is (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

However, the rules are not followed everywhere. For example, a length is reported to be 6.800m. Let us change the units; thus, 6.800m = 680.0cm =

6800mm = 0.006800km. According to the rules, 6800 mm should have two significant figures, but it actually has four significant figures.

It was researched that the best way to report every measurement is in Scientific Notation (in the power of 10).

In this notation, every number is expressed as $a \times 10^b$, where a is a number between 1 and 10 and b could be any positive or negative exponent. The power of 10 is irrelevant to the determination of significant figures. $6.800\ m = 6.800 \times 10^2 cm = 6.800 \times 10^3 mm = 6.800 \times 10^{-3}\ km$. The number of significant digits is 4 here.

Scientific Notation is ideal for reporting measurements.

The multiplying or dividing factors which are neither rounded numbers nor numbers representing measured values are exact, and they have an infinite number of significant digits. For example $r = \frac{s}{2}$, here the factor 2 is an exact number, and it can be written as 2.0, 2.00, or 2.000 as required.

The class was interrupted here, as a member of the non-teaching staff began knocking on the glass door. He had a medium-sized cart with numerous snack containers, each filled with French fries, samosa, paneer pastry and a sweet. It was a long-standing institution tradition to distribute such treats to HSC students on their first day.

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Dolly said quietly to Kimaya, “Oh, the pastry is very tasty, and look, it's a decent size too!” “That’s ok, but you know I am wondering, as we always assume 2 = 2.0 = 2.00 = 2.000 like this, then why do all these numbers have different numbers of significant figures, i.e., different properties?” Kimaya asked in a little confused state of mind.

Before she could speak anything, David said interestingly, “Dear Kimaya, while the numbers 2, 2.0, 2.00, 2.000 are mathematically equivalent, their representations carry different implications for precision when they represent measured values, so these numbers have different numbers of significant figures.”

Dolly: “Yeah, I guess so. We could ask the teacher to explain it again, but it would be pointless.” It appeared that they were convinced.

On the other hand, the teacher’s voice could be heard clearly even in the noisy environment, saying, “You could do it later; let me finish the class.” She was probably referring to the treats that many students had started eating while ignoring her lecture. After a while, however, she managed to silence everyone and restart the class.

Rules for Arithmetic Operations with Significant Figures

(I) In multiplication or division, the final result should retain as many significant figures as there are in the original number with the least significant figures.

For example: If mass of a material is 4.024 gm and volume over which it is distributed is 8.0004 cm^3. Find its density taking care of the significant figures.

Solution: $density = \frac{mass}{volume}$

$\Rightarrow \frac{4.024gm}{8.0004cm^3} = 0.5030\ gm/cm^3$

(II) In addition or subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.

For example, the sum of the numbers 136.32 g, 228.2 g and 0.605 g in the point of view of significant figures is 365.1 g

Example 1.2 If a substance of 6.546g occupies 1.24cm^3. Find its density taking care of its significant figures.

Solution: There are 4 significant figures in the measured mass and 3 significant figures in the measured volume. Hence the density should have 3 significant figures.

$$\text{Density} = \frac{6.546g}{1.24cm^3} = 5.28gcm^{-3}.$$

Rules for Determining the Uncertainty (error) in the Results of Arithmetic Calculations

The rules for determining the uncertainty (error) in the number/measured quantity in arithmetic operations can be understood from the following examples.

(I) If the length and breadth of a thin rectangular sheet are measured, using a metre scale as 16.2 cm and 10.1 cm respectively, there are three significant figures in each measurement. It means that the length l may be written as $l = 16.2 \pm 0.1\,cm = 16.2\,cm \pm 0.6\,\%$. Similarly, the breadth b may be written as $b = 10.1 \pm 0.1\,cm = 10.1\,cm \pm 1\,\%$.

Then, the error of the product of two (or more) experimental values, using the combination of errors rule, will be $l\,b = 163.62\,cm^2 \pm 1.6\% = 163.62 \pm 2.6\,cm^2$. This leads us to quote the final result as $l\,b = 164 \pm 3\,cm^2$. Here 3 cm^2 is the uncertainty or error in the estimation of the area of the rectangular sheet.

(II) If a set of experimental data is specified to n significant figures, a result obtained by combining the data will also be valid to n significant figures. However, if data are subtracted, the number of significant figures can be reduced. For example, $12.9\ g - 7.06\ g$, both specified to three significant figures, cannot properly be evaluated as $5.84\ g$, but only as $5.8\ g$, as uncertainties in subtraction or addition combine in a different fashion (based on the smallest number of decimal places rather than the number of significant figures in any of the number added or subtracted).

(III) The relative error of a value of number specified to n significant figures depends not only on n but also on the number itself. For example, the accuracy in measurement of mass $1.02\ g\ is\ \pm\ 0.01\ g$, whereas another measurement, $9.89\ g$, is also accurate to $\pm\ 0.01\ g$. The relative error in $1.02\ g$ is $= (\pm 0.01/1.02) \times 100\ \% = \pm 1\%$ Similarly, the relative error in $9.89\ g$ is $= (\pm\ 0.01/9.89) \times 100\% = \pm 0.1\%$. Finally, remember that intermediate results in a multi-step computation should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement. These should be justified by the data and then the arithmetic operations may be carried out; otherwise rounding errors can build up. For example, the reciprocal of 9.58, calculated (after rounding off) to the same number of significant figures (three) is 0.104, but the reciprocal of 0.104 calculated to three significant figures is 9.62. However, if we had written $1/9.58 = 0.1044$ and then taken the reciprocal to three significant figures, we would have retrieved the original value of 9.58. This example justifies the idea to retain one more extra digit (than the number of digits in the least precise measurement) in intermediate steps of the complex multi-step calculations in order to avoid additional errors in the process of rounding off the numbers.

The first day's lecture ended here.

The next session started with the topic 'Dimensions of Physical Quantities.'

1.4 Dimensions of Physical Quantities

The dimensions of a physical quantity are the powers to which the base quantities are raised to represent the quantity. Using [ ] square brackets around a quantity means we are working with the dimensions of the quantity.

For example, force, it is a product of mass and acceleration. It can be expressed as $Force = mass \times acceleration \left(\frac{length}{time^2}\right)$. Therefore, the dimensions of force are expressed as $[M] \times [L] \times [T]^{-2} = [MLT^{-2}]$. Hence, force has one dimension in mass, one dimension in length, and -2 dimensions in time. It has 0 (zero) dimension in all other base quantities. In mechanics, all of the physical quantities can be written in terms of the dimensions $[L]$, $[M]$ and $[T]$

1.5 Dimensional Formula and Dimensional Equations

The expression that explains how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula of the physical quantity. The dimensional formula of the volume is $[M^0L^3T^0]$.

An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity.

For example: $[V] = [M^0L^3T^0]$, $[F] = [MLT^{-2}]$

There was some gossip going on, among the first-row students while Gita was busy teaching the modules dimensional formula and dimensional equations. First, she warned them not to talk as she was getting disturbed delivering her lecture. However, after learning that they were struggling with the topic 'significant figure'. She became involved with them to address their concerns, instructing others to hold on for a bit and not be noisy.

On the other hand, Aditi sweetly asked Aakash if he could explain the difference between 'dimensions' and 'dimensional formula'. Aakash responded, "While for a physical quantity, we use the same notation to represent its dimensions and to write its dimensional formula (for instance, $[LT^{-1}]$ for velocity), there is a fundamental difference between these two terms. Dimensions of a physical quantity are simply the powers of base quantities, while the dimensional formula is combined of both, which means how powers are raised and of which base quantities the powers are raised for.

For better clarity, let's use words to understand the difference. Take velocity as an example, we would say it has 1 dimension in length and -1 dimension in time.

However, in the case of the dimensional formula, the expression itself is $[LT^{-1}]$, that is 'length to the power of one, time to the power of negative one'.

At the end Aakash said, "There is no need to delve into it further, that's enough in the context of our syllabus."

Aditi exclaimed, "Oh! Now I am feeling better about the topic!" She then looked at her own row attentively and realized Kimaya and Dolly were not there. She learned from Arnold that they had gone to a mall secretly from the back door because Kimaya's skirt had been damaged, making her feel very uncomfortable. That day, she was wearing a different type of dress, a tight skirt with a light-green top (as there was no particular dress code in the institution). However, she didn't have any card or cash but Arnold had offered his father's credit card. Aditi felt a little moody and her expression was just like, 'Oh my God!' She laughed, saying "She is a tragedy queen."

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1.6 Dimensional Analysis and Its Applications

It is the analysis of the relationship between different physical quantities by identifying their base quantities such as length, mass, time and electric current and units of measurement such as metres and grams and tracking these dimensions as calculations or comparisons are performed.

Applications

Used for checking the validity of equations: Ensure that scientific formulas and equations are consistent and valid.

Converting units: It converts units from one system to another.

Used for deriving equations by ensuring the dimensions on both sides are equal.

1.7 Checking the dimensional consistency of equations

The magnitudes of physical quantities can be added or subtracted together only if they have the same dimensions. This principle is called the principle of homogeneity of dimensions or dimensional consistency. It is extremely useful in checking the correctness of an equation.

For example,

Example 1.3 Check whether the equation $s = ut + \frac{1}{2}at^2$ is dimensionally correct or not (where s is distance).

Solution: Dimension of $s = [L]$ and therefore the dimension of the terms on the right side of the equation must be equal to $[L]$ as well.

$$[ut] = [LT^{-1}T] = [L]$$

$$\left[\left(\frac{1}{2}\right)at^2\right] = [LT^{-2}][T^2] = [L].$$

The dimension of each term on the right-hand side is the same, that is equal to that of length. And hence the equation is dimensionally correct.

Example 1.4 Consider an equation $ut = \left(\frac{1}{2}\right)at^2$. Check whether the equation is dimensionally correct or not, where u is the speed, t is time and a is the acceleration.

Solution: The dimensions of LHS are

$[LT^{-1}T^{1}] = [L]$

The dimensions of RHS are

$[LT^{-2}T^{2}] = [L]$

Therefore, the equation is dimensionally correct.

Example 1.5 The SI unit of energy is $J = kgm^2s^{-2}$; and the acceleration is ms^{-2}. Which of the formulae for potential energy (V) given below we can give on the basis of dimensional arguments.

(1) MGV
(2) MGH
(3) MLT

Ans - Correct option is (2)

1.8 Deducing relation among the physical quantities

The method of dimensions can be used to deduce relation among the physical quantities. Dimensional analysis is very useful in deducing relations among the interdependent physical quantities. Practice would help to develop this skill.

Some examples have been given below to understand it.

Example 1.6 If the velocity of light C , universal gravitational constant G and Planck's constant H are chosen as fundamental quantities. The dimensions of mass in the new system is :

Solution:

$$[M] = [G]^X[H]^Y[C]^Z$$

$$[M] = [M^{-1}L^3T^{-2}]^X[ML^2T^{-1}]^Y[LT^{-1}]^Z$$

$$[M^1L^0T^0] = [M^{-X+Y}][L^{3X+2Y+Z}][T^{-2X-Y-Z}]$$

$Y - X = 1$ ...(i)

$3X + 2Y + Z = 0$ ...(ii)

$-2X - Y - Z = 0$ ...(iii)

On solving we get $X = -\frac{1}{2}, Y = \frac{1}{2}$ & $Z = \frac{1}{2}$

So $m = \sqrt{\frac{HC}{G}}$ answer.

Example 1.7

In the equation $[x + \frac{a}{y^2}][y - b] = RT$, x is pressure and its SI unit is $kgm^{-1}s^{-2}$, y is volume, R is universal gas constant and T is temperature. (Given that SI units of energy, Momentum and Force are kgm^2s^{-2}, $kgms^{-1}$ and $kgms^{-2}$) respectively. Then the physical quantity equivalent to the ratio $\frac{a}{b}$ is:

Solution:

x and $\frac{a}{y^2}$ have the same dimensions and y and b have the same dimensions, let's analyze the dimensions of $\frac{a}{b}$.

Since x represents pressure, it has dimensions of $[ML^{-1}T^{-2}]$

Since x and a/y^2 have the same dimensions, we have :

$[a/y^2] = [ML^{-1}T^{-2}]$

Then the dimensions of a are : $[a] = [ML^{-1}T^{-2}]\,[y^2] = [ML^5T^{-2}]$

Now, since y and b have the same dimensions and y represents volume, we have: $[b] = [L^3]$

Now let's find the dimensions of the ratio a/b :

$[a]/[b] = [ML^5T^{-2}]/[L^3] = [ML^2T^{-2}]$

It corresponds to the dimensions of energy.

Example 1.8 The speed of a wave produced in water is given by $v = \lambda^a g^b \rho^c$. Where λ, g, ρ are wavelength of wave, acceleration due to gravity and density of water respectively. The values of a, b, c respectively are

Solution: $v = \lambda^a g^b \rho^c$

Using dimensional formula

$[M^0L^1T^{-1}] = [L^1]^a[L^1T^{-2}]^b[M^1L^{-3}]^c$

$[M^0L^1T^{-1}] = L^{a+b-3c}T^{-2b}M^c$

$\therefore\ c = 0,\ a + b - 3c = 1, -2b = -1, b = \frac{1}{2}$

Now $a + b + c\ =\ 1\ \Rightarrow\ a + \frac{1}{2} - 0\ =\ 1$

$\Rightarrow a = \frac{1}{2}\ \therefore\ a = \frac{1}{2}, b = \frac{1}{2}, c\ =\ 0.$

The class concluded with an announcement that the next physics class, on Friday, would be entirely for clearing doubts. Monday, Wednesday and

Friday were the days the Physics classes were being held. However, the guys were not interested in attending the doubt session, probably because they had already understood the topics.

The guys were heading back to the village and talking nicely among themselves.

"It's been quite chaotic since the day our Matriculation results were announced to choose an institution for further studies," David said, "Now we're going to have four days off. We should go for a short holiday!" Aakash suggested a trip to a hill station to experience its lifestyle during the break. The group agreed happily.

Exercises

1. Match List I with List II

List I (Number)	List II (Significant Figures)
(A) 145000	(I) 5
(B) 421.0	(II) 7
(C) 00.42500	(III) 4
(D) 0.0056045	(IV) 3

2. Fill in the blanks with suitable conversion of units.
   (a) 3.0 ms^{-2} = ________ kmh^{-2}
   (b) 1 $kgms^{-2}$ = ________ $gcms^{-2}$
   (c) 1 m = ________ mm
   (d) 1 kgm^2s^{-2} = ________ gcm^2s^{-2}
3. Let consider an equation $S = ut + \frac{1}{2}at^2$
   Where S is distance, u = speed, a = acceleration and t = time (S.I unit of acceleration is m/s^2). Check whether the equation is dimensionally correct or not.
4. What will be the new dimensional formula of F if $F = a \times t \times m^2$ (where F = force, a = acceleration, t = time and m = mass of the body)?
5. A person measures the mass of 3 different particles as $435.42g$, $226.3g$ and $0.125g$. According to the rules for arithmetic operations with significant figures, the addition of the masses of 3 particles will be ….. (ans $661.8\ g$)
6. For an experimental expression $y = \frac{32.3 \times 1125}{27.4}$, where all the digits are significant. Then to report the value of y we should write ….
7. What will be the dimensional formula of kinetic energy, if its SI unit is $KGMS^{-2}$?
8. Fill in the blanks

   Dimensional Formula **SI units**

   i. ML^{-1} ________

ii. MT^{-2} ________

iii. MLT ________

9. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: 1 Å = 10^{-10}m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms?
10. Which of the following is the most precise device for measuring length:
    (a) a vernier callipers with 20 divisions on the sliding scale
    (b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale
    (c) an optical instrument that can measure length to within a wavelength of light ?
11. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 3.01 cm respectively. Find the area and volume of the sheet taking care of significant figures?
12. The mass of a box measured by a grocer's balance is 2.30 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures ?
13. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of volumes:
14. The SI unit of a physical quantity is pascal-second. The dimensional formula of this quantity will be :
15. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density?
16. If a substance of 4.564g occupies 2.2 cm^3. Find its density taking care of its significant figures:

Answers

1. A - IV, B - III, D - II, C - I

2. (a) 38880 km/hr^2	(b) 10^5	(c) 10^3	(d) 10^7
4. $LT^{-1}M^2$	5. 661.8 g	6. 1330	7. MLT^{-2}
8. (i) kg/m	(ii) kg/m^2	(iii) $kgms$	9. $3 \times 10^{-7} m^3$
10. (c)	11. 4.255 m^2, 0.128 m^3		
12. (a) 2.34 kg	(b) 0.02 g	13. $[F^\circ L^3 T^\circ]$	14. $ML^{-1}T^{-1}$
15. $FL^{-4}T^2$	16. 2.1 g/cm^3		

Chapter - 2
Kinematics - Motion In A Straight Line

The trip started with horse riding and exploring the hill station on the first day. Later that evening, they attended a show organised by the town Mayor and performed by some local children. It was a beautiful show, a reflection of their culture and kindness.

The group's holiday was going pretty well, but an unfortunate incident occurred on the third day. Some local boys were playing football near their homestay. One shot hit the parked bus so hard that it broke the bus's front window. This made things difficult for the guys.

The bus couldn't continue the journey without a new front window. And, arranging the money for it during the holiday was proving difficult because the town's businesses were heavily reliant on cash. However, in the end, Kimaya sold the golden bracelet she was wearing at a jewellery shop located in the middle of the hill station market to replace the windshield.

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It took all four days that they hadn't planned for. They were back at their village on Monday instead of Sunday, arriving just a few minutes before the bus's usual departure time for the city.

Although they managed to enter the classroom almost on time, they were in a hurry. In the class, everyone was silent, looking like all of them were waiting for the teacher to come. The teacher suddenly entered and began the class with the basic topic, Frame of Reference.

2.1 Frame of Reference

It is a set of coordinates or a point of view from which the motion of an object is observed or measured, defining the position and orientation of an object in space and time. It acts like a coordinate system with a designated origin used to track the object's movement.

2.2 Motion

A body is said to be in motion when it changes its position continuously over a period of time. For example, suppose a body starts moving with velocity $4m/s$ at $t = 0s$ and continues to move until $t = 3s$. First, the body at $t = 0$ is at $x = 0$; at $t = \frac{1}{2}s$, it is at $x = 2m$; at $t = 1s$ it is at $4m$, and so on. Here, the body continuously changes its position until $t = 3s$;

hence, the body is in motion till $t = 3s$. After $t = 3s$, the body is no longer moving; therefore, at this point ($t > 3s$), it is not in motion.

2.3 Distance and Displacement

Distance: The total length of the path travelled by an object is called distance, irrespective of the direction in which it moves.

Displacement: The distance between initial position and final position along a straight line connecting them is called displacement. Displacement is the shortest distance between two points (final position and initial position).

Difference between distance and displacement

In Figure 2.1, you can see the distance between two points A and D depends on the path taken by the object. This means distance can vary according to the path. If an object travels via AOD, the distance is $8m$; if it follows $AO'D$, the distance is $9m$. However, in the case of displacement, the distance is always fixed between two points and is measured along a straight line connecting them. Here, the displacement is equal to $6m$.

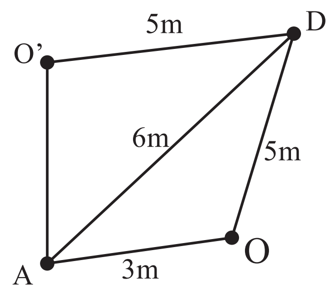


Fig. 2.1

2.4 Speed and Velocity

Speed: It is the distance travelled per unit time. Its SI unit is m/s.

Example 2.1 suppose if a vehicle travels $80m$ in 5 seconds then what will be its speed?

Solution: $Speed = \frac{distance\ travelled}{total\ time\ taken}$

$\frac{80m}{5s} = 16m/s$ answer.

Velocity: velocity is the displacement travelled per unit time. Its SI unit is the same as that of speed, that is m/s.

The teacher, Gita, acknowledged that the topic 'Frame of reference' had not been delivered clearly to the students. However, the group from the village didn't seem to be interested in discussing it even among themselves. Meanwhile, elsewhere in the class, a number of students were chatting on the topic, trying to understand it. Gita requested them to be quiet, and went on to explain it comprehensively.

We conceive three mutually perpendicular straight lines in space, known as the x, y and z axes to describe an object's position. These axes intersect at a common point called the origin. This combination of an origin and a set of axes forms a frame of reference.

This involves adopting the $x\ y\ z$ axes as reference for direction from the chosen origin O (0,0,0). For describing other phenomena, the choice of origin may vary. This system of an origin and coordinate axes is the fundamental concept behind a frame of reference.

There are some important points to note, as follows:

(I) **Reference of position:** The position of an object is expressed or measured relative to a specific point or origin in a coordinate system ($x\ y\ z$ axes). The point or origin is known as the reference of position of the object. For example, let 'a' be at $x = 3m$ and 'b' be at $x = 6m$ (with respect to an origin 'o' as shown in Figure 2.2). Then, the position of 'b' with reference to 'a' is $6m - 3m$ or $3m$.

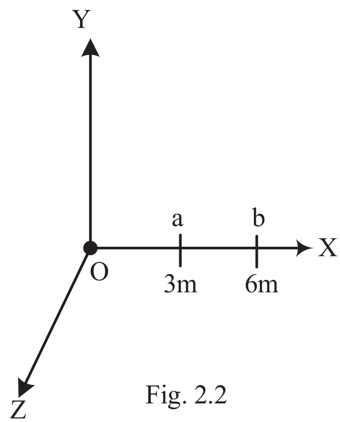


Fig. 2.2

(II) **Observer of motion:** The perception of an object's motion is always relative to a chosen frame of reference. For instance, consider an observer inside a moving train with a toy. From their perspective, the toy is at rest. In contrast, an observer outside the train, using the ground as their frame of reference, would see the toy is in motion because its position is continuously changing relative to them. This highlights that motion is a relative concept, entirely dependent on the observer's frame of reference.

(III) It is crucial to stay consistent with your chosen frame of reference throughout a problem to accurately interpret the results. For example, in a problem involving relative motion, you cannot mix measurements from two different reference frames without using the appropriate transformation rules.

Note: However, the motion path can be circular, parabola, hyperbola etc but we will be confined to motion in a straight line in this chapter.

2.5 Now we will read Uniform and Non-Uniform motion

Uniform Motion : The motion in which the velocity of an object does not vary at any point in time. This means the object moves at a constant speed along a straight line, covering equal distances in equal intervals of time, is called Uniform Motion.

Non-Uniform Motion : The motion in which velocity of an object varies over time. Variation can manifest in several ways: the object may have different speeds at different points in time in a straight line, may travel not in a straight line while maintaining a constant speed, or may experience variations in both speed and direction during its motion.

2.6 Average Speed and Instantaneous Velocity

Average Speed

When an object is in motion, does it necessarily have the same speed during the whole course of motion? The answer is obviously no. If someone travels some distance and, upon reaching the destination, is asked about their speed. It is ridiculous to give a detailed account of every speed change. It is unwieldy to state that his bike started with speed $50kph$, had speed $70kph$ when he was at $t = 20$seconds, speed was $80kph$ at $t = 25$ seconds, or between $t = 26$ seconds to $t = 30$ seconds speed was $75kph$ and so on. To ease this impracticality, we adopt the term known as average speed. It is defined as, 'the total distance traveled by an object divided by the total time taken to travel that distance.'

$$Average\ Speed\ = \frac{Total\ distance}{Total\ time}$$

For example, if a car travels $100km$ in 2 hours. Then its average speed is equal to $50km/hr$

Example 2.2 If a car travels 40 m between $t = 0s$ to $t = 4s$, and it travels $60m$ between $t = 4s$ to $t = 8s$. Find

(I) average speed between $t = 4s$ to $t = 8s$
(II) average speed during the whole course of the motion.
(I) **Solution:** $\frac{60m}{4s} = 15m/s$
(II) **Solution:** $\frac{100m}{8s} = 12.5\ m/s$

Example 2.3 A vehicle travels $4km$ with speed of $3km/h$ and another $4km$ with speed of $6km/h$. Find its average speed.

Solution: We know that average speed is equal to total distance covered/ total time taken

And total time = $(4\ km/3km/hr)\ +\ (\ 4\ km/6\ km/hr)$

$=$ 2hr, ∴ average speed = $\frac{(4km+4\ km)}{2hr} = 4\ km/hr$

Example 2.4 A man walks on a straight road from his home to a market 3 km away with a speed of 6 kmh^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 9 kmh^{-1}. What is the (a) magnitude of average velocity, and (b) average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min ?

Solution:

Time taken by the man to reach the market from home is $t_1 = \frac{3}{6} = \frac{1}{2} hr = 30\ min$.

Time taken by the man to reach home from the market is $t_2 = \frac{3}{9} = \frac{1}{3} hr = 20\ min$.

i) 0 to 30 min

$$Average\ velocity = \frac{displacement}{time} = \frac{3}{\frac{1}{2}} = 6\ km/hr$$

$$Average\ speed = \frac{distance}{time} = \frac{3}{\frac{1}{2}} = 6\ km/hr$$

ii) 0 to 50 min

Time in $hr = \frac{50}{60} = \frac{5}{6}\ hr$

Net displacement during the period $= 0$

Total distance $= 3 + 3 = 6\ km$

$$Average\ velocity = \frac{displacement}{time} = \frac{0}{\frac{5}{6}} = 0$$

$$Average\ speed = \frac{distance}{time} = \frac{6}{\frac{5}{6}} = \frac{36}{5} = 7.2\ km/hr.$$

Example 2.5 A horse rider covers half the distance with $10m/s$ speed. The remaining part of the distance was travelled with speed $20m/s$ for half the time and with speed $35m/s$ for the other half of the time. The mean speed of the rider over the whole time of motion is _______ (Note: Mean speed is equal to average speed)

Solution: $t_1 = \frac{\frac{s}{2}}{10}$ …..(i)

Also, $\frac{s}{2} = \frac{20t_2}{2} + \frac{35t_2}{2}$

$\Rightarrow t_2 = \frac{s}{55}$ ….(ii)

$\Rightarrow$ Mean Speed $= s/(t_1 + t_2)$

$\Rightarrow \frac{s}{\left(\frac{s}{20}+\frac{s}{55}\right)} = \left(\frac{44}{3}\right) m/s$

Before we study kinematics further, some basic calculus needs to be known:

Differential calculus

(i) $\frac{d(x^n)}{dx} = nx^{n-1}$

(ii) $\frac{d(e^x)}{dx} = e^x$

(iii) $\frac{d(log\ x)}{dx} = \frac{1}{x}$

(iv) $\frac{d(a^x)}{dx} = a^x \log_e a$

(v) $\frac{d(sin\ x)}{dx} = cos\ x$

(vi) $\frac{d(cos\ x)}{dx} = -sin\ x$

(vii) $\frac{d(tan\ x)}{dx} = \sec^2 x$

(viii) $\frac{d(cot\ x)}{dx} = -\operatorname{cossc}^2 x$

(ix) $\frac{d(sec\ x)}{dx} = sec\ x\ tan\ x$

(x) $\frac{d(cosecx)}{dx} = -cosecx\ tanx$

Integral calculus

(i) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

(ii) $\int dx = x + c$

(iii) $\int \cos x\, dx = \sin x + c$

(iv) $\int \sin x\, dx = -\cos x + c$

(v) $\int \sec^2 x\ \ dx = tan\ x\ + c$

(vi) $\int \operatorname{cosec}^2 x\ \ dx = -\cot x\ + c$

(vii) $\int \sec x \tan x\, dx = \sec x + c$

(viii) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

2.7 Instantaneous velocity

Instantaneous velocity is the velocity of an object at a particular point in time. It is defined as the rate at which an object changes its position or displaces over an infinitesimally small period of time. Mathematically, it is expressed as $V(t) = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$, Δr is the displacement during infinitesimally small time interval Δt. The limit of the value $\frac{\Delta r}{\Delta t}$ as Δt approaches zero is known as called instantaneous velocity at a particular point 't' in time. This is also represented as $\frac{dr}{dt}$.

It can also be understood in terms of average velocity. First, let the average velocity of a particle between time t to $t + \Delta t$ be V_{avg}, this average velocity can be written as $V_{av} = \frac{\Delta r}{\Delta t}$ where Δr is the displacement in the time interval Δt. When we make Δt very infinitesimally small and find the limiting value of $\frac{\Delta r}{\Delta t}$. This value is called instantaneous velocity V of the particle at time t. Mathematically, it is expressed as: $V = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$. The representation '$\Delta t \to 0$' tells Δt tends to be zero.

- **Instantaneous speed:** It is just the magnitude of velocity at a particular point in time.

Example 2.6 If the distance of a particle is given by $D = 5t^3 + 2t^2 + 5$ along x axes.

(I) Find the instantaneous velocity at $t = 2s$.
(II) The average velocity during $t = 2s$ to $t = 4s$

Solution:

(I) We know that $v(t) = \frac{d(s)}{d(t)}$

$\frac{d(5t^3 + 2t^2 + 5)}{dt} = 15t^2 + 4t = 68\text{m/s}$.

(II) $V_{avg} = \frac{(D_4 - D_3)}{time\ taken}$

$D_4 = 5t^3 + 2t^2 + 5$

$= (5 \times 64) + (2 \times 16) + 5 = 357$

Similarly, $D_3 = 5 \times 2^3 + 2(2)^2 + 5 = 61m$

$V_{avg} = \frac{(357 - 61)m}{2s} = 148m/s.$

2.8 Average acceleration and Instantaneous acceleration

Average acceleration: Average acceleration of an object is defined as the change in its velocity divided by the time interval over which that change occurs. Its SI unit is m/s^2.

$$a = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

Instantaneous acceleration: It is defined analogously to instantaneous velocity. The change in velocity of an object divided by an infinitesimally small period of time over which that change occurs is known as instantaneous acceleration.

$$a = \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$$

We know that velocity V is the first derivative of position x with respect to time t. Therefore, acceleration a, being the first derivative of velocity with respect to time, can also be expressed as the second derivative of position with respect to time: $a = \frac{dV}{dt} = \left(\frac{dV}{dx}\right).\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$. This is also known as the second derivative of x with respect to t.

Example 2.7 Suppose the velocity of an object is 10m/s at starting and it has velocity equal to 60m/s at $t = 5s$. Find acceleration of the object during the motion.

Solution: $Acceleration = \frac{change\ in\ velocity}{time\ taken}$

$$= \frac{Vf - Vi}{total\ time\ taken} = \frac{(60m/s - 10m/s)}{5s}$$

$$= 10m/s^2 .$$

Example 2.8 A particle moves in a straight line so that its displacement x at any time t is given by $x^2 = 1 + t^2$. Its acceleration at any time t is x^{-n}. Then find the value of n.

Solution: First, let's find the first derivative of displacement with respect to time t, which gives velocity.

Differentiating $x^2 = 1 + t^2$ with respect to t, we get:

$$2x\left(\frac{dx}{dt}\right) = 2t$$

This simplifies to:

$$\frac{dx}{dt} = \frac{t}{x}$$

Now, let's differentiate the velocity to find the acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{d\left(\frac{t}{x}\right)}{dt}$$

To differentiate it, we will use the quotient rule:

$$\frac{d\left(\frac{t}{x}\right)}{dt} = \left(x.1 - t.\frac{dx}{dt}\right)\left(\frac{1}{x^2}\right)$$

Substitute $\frac{dx}{dt} = \frac{t}{x}$ into the equation

$$a = \frac{\left(x(1) - t\left(\frac{t}{x}\right)\right)}{x^2} = \frac{(x^2 - t^2)}{x^3}$$

Recalling that the displacement equation given was $x^2 = 1 + t^2$,substitute this into our expression for acceleration

$$a = \frac{(1 + t^2 - t^2)}{x^3} = \frac{1}{x^3}$$

Thus, the acceleration of the particle at any time t is x^{-3}, which means our value for n is 3.

The class was half the usual duration as it had been announced a day before that a Musician would come to the campus to entertain. However, the group found the show boring and soon they left, heading towards their residential compound.

It was a rainy day; the sky wasn't clear, looking as if the ocean had risen into the sky. The bus was moving slowly into the village.

Kimaya, Aditi, Dolly and Aakash were joking with each other. Dolly, with a sweet smile, said, " Walking on the house roof in the morning is good for health, right Aditi?". Aditi and Aakash, both wore expressions that suggested " oh my god, she has been watching us secretly". Aditi after a pause said shyly, "Sure, facing the rising sun is always good for health; I do it.'' Dolly : "I know Aakash and you both do it; it is lovely."

Dolly suspected a new close bond, beyond mere friendship, was developing between Aditi and Aakash.

However, Kimya and David also had romantic feelings for each other but they went unnoticed to anybody.

Meanwhile, David and Arnold were engaged in a serious discussion about the previous trip and planning to help Kimaya financially so she could purchase the same type of bracelet again.

Following the discussion, David said to Kimaya, "Here is some cash from me and Arnold, you keep it, it could be helpful covering many of your expenses or you could get a similar bracelet again. But Kimaya politely refused, suggesting they open a bank account, whose balance could be used for such incidents, which they were not ready for. David conceded, "This is a point to consider for sure."

*****

The next session, the class continued, starting with questions and solutions based on previously taught modules.

Example 2.9 If displacement of a particle $x = -3t^3 + 18t^2 + 16t$, find the velocity of the particle when its acceleration becomes 0.

Solution: We know $a = \frac{dv}{dt}$ and $v = \frac{dx}{dt}$

So, $v = \frac{dx}{dt} = \frac{d(-3t^3+18t^2+16t)}{dt}$

$\Rightarrow -9t^2 + 36t + 16$ ….. (i)

$a = \frac{dv}{dt} = \frac{d(-9t^2+36t+16)}{dt} = 0$

$\Rightarrow -18t + 36 = 0$

$\Rightarrow t = 2s$, by putting $t = 2$ in equation (i) we get $v = -9(4) + 36(4) + 16 = 124$ m/s.

Example 2.10 A particle initially at rest starts moving from reference point $x = 0$ along x-axis, with velocity v such that it varies as $v = 4\sqrt{x}$ m/s. The acceleration of the particle is

Solution: First we need to differentiate the velocity function with respect to time. The velocity function given is $V = 4\sqrt{x}$. However, the function gives the velocity as a function of position x, not as a function of time t. Since acceleration is the rate of change of velocity with respect to time, we'll need to use the chain rule to differentiate v with respect to t.

$a = \frac{dv}{dt} = \left(\frac{dv}{dx}\right).\left(\frac{dx}{dt}\right)$

$V = 4\sqrt{x} = 4x^{1/2}$

Differentiating this with respect to x, we get:

$\frac{dv}{dx} = 4.\left(\frac{1}{2}\right)x^{-1/2} = \frac{2}{\sqrt{x}}$

We have $v = 4\sqrt{x}$, therefore $\sqrt{x} = \frac{v}{4}$. Replacing

$\sqrt{x}$ as $\frac{v}{4}$

We get: $\frac{dv}{dx} = \frac{2}{\sqrt{x}} = \frac{8}{v}$

Now, $a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = \left(\frac{8}{v}\right)v$. Simplifying this we have now $a = 8\ ms^{-2}$. Thus the acceleration of the particle is $8\ ms^{-2}$.

Example 2.11 The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is

Solution : given $x^2 = at^2 + 2bt + c$ ...(i)

On differentiating eq (i) with respect to t, we get

$\Rightarrow 2xv = 2at + 2b$

$\Rightarrow v = \frac{at+b}{x}$ ...(ii)

Differentiating equation (ii) with respect to time

$\Rightarrow xa' + v^2 = a$

Here a' is acceleration

$$\Rightarrow a'x = a - v^2 = a - \left[\frac{at+b}{x}\right]^2$$

$$\Rightarrow a'x = \frac{ax^2 - (at+b)^2}{x^2}$$

$$\Rightarrow a' = \frac{a\left(at^2 + 2bt + c\right) - (at+b)^2}{x^2}$$

$$\Rightarrow a' = \frac{ac - b^2}{x^3}$$

$$\therefore\ a' \propto \frac{1}{x^3} \propto x^{-3}$$

$$\therefore n = 3.$$

Example 2.12 A particle moves in a straight line so that its displacement x at any time t is given by $1.5x^2 = 1 + t^2$. Its acceleration at any time t is $\left(\frac{4}{9}\right)x^{-n}$. Find the value of n

Solution: We need to find the acceleration to find the value of n. In the question, given that the displacement of the particle is equal to $1.5x^2 = 1 + t^2$ and the acceleration at any time t is $\left(\frac{4}{9}\right)x^{-n}$.

First, find the first derivative of displacement with respect to time, which gives the velocity. Differentiating $1.5x^2 = 1 + t^2$ with respect to t , we get

$$3x\left(\frac{dx}{dt}\right) = 2t\left(\frac{dt}{dx}\right)$$

$$\Rightarrow \left(\frac{dx}{dt}\right)\left(\frac{dx}{dt}\right) = \frac{2t}{3}$$

$$\Rightarrow \frac{d^2x}{dt^2} = \frac{2t}{3x}$$

To differentiate it we will use the quotient rule

$$a = \frac{d^2x}{dt^2} = \frac{2}{3}\frac{d\left(\frac{t}{x}\right)}{dt}$$

$$= \frac{\left(x.1 - t.\frac{dx}{dt}\right)2}{3x^2}$$

Substituting $\frac{dx}{dt} = \frac{2t}{3x}$

$$\frac{\left(x(1) - t.\left(\frac{2t}{3x}\right)\right)}{x^2}\frac{2}{3}$$

$$= \left(\frac{4}{9}\right) \times \frac{1}{x^3}$$

In the question, it is given that the acceleration is $\left(\frac{4}{9}\right)x^{-n}$. Comparing this with the result we can see that $n = 3$.

Note: one important fact $a = \frac{vdv}{dx}$

Example 2.13 A particle is moving in a straight line such that its velocity is increasing at $5m/s^{-1}$ per meter. The acceleration of the particle is ....ms^{-2} at a point where its velocity is $20ms^{-1}$.

Solution: Given $\frac{dv}{dx} = \frac{5ms^{-1}}{m}$

So acceleration of the particle, when $v = 20$m/s, is

$a = \frac{vdv}{dx} = 20 \times 5\ m/s^2 = 100m/s^2$.

2.9 Relative velocity:

Assume an object is moving with a velocity of 40 m/s, and a person is moving with a velocity of 25 m/s in the same direction. Then the person would perceive the velocity of the object as 15m/s. This velocity is called the relative velocity of the object with respect to the person. It is given as, actual velocity of the object - actual velocity of the person.

That is, $40m/s - 25m/s\ = 15m/s$.

It is also called velocity of the object with respect to the observer.

Example 2.14 If an object is moving with velocity 40 m/s along $+x$ axes then find velocity of the object relative to a person who (i) is moving with velocity 10m/s in $+x$ axes (ii) is moving with velocity 20m/s in $-x$ axes

i) we know

$relative\ velocity = velocity\ of\ the\ object - velocity\ of\ the\ observer$

= 40m/s - 10m/s

= 30m/s answer.

ii) is moving with velocity 20m/s in $-x$ axes

$relative\ velocity = velocity\ of\ the\ object\ -$
$velocity\ of\ the\ observer$

= 40m/s - (-20m/s)

= 60m/s. The person will observe the object is moving with speed 60m/s in +ve x axes.

Example 2.15 A jet aeroplane travelling at the speed of 500 km/hr ejects its products of combustion at the speed of 1500 km/hr relative to the jet plane. What is the speed of the latter with respect to an observer on the ground?

Solution: Speed of combustion relative to the jet plane = speed of combustion – speed of the jet plane

$\Rightarrow$ Speed of combustion - (-500km/hr) = 1500 km/hr

$\Rightarrow$Speed of combustion=1500 km/hr - 500 km/hr = 1000 km/hr.

2.10 Position - Time graph: Acceleration can be positive, negative or zero. If the change in velocity at a point in time is negative, the acceleration will be negative, Conversely, if change in velocity is positive, the acceleration will be positive. If there is no change in velocity, the acceleration will be zero.

i) Figure 2.3(a) shows position time graph with positive acceleration
ii) Figure 2.3(b) shows position time graph with negative acceleration
iii) Figure 2.3(c) position time graph with zero acceleration

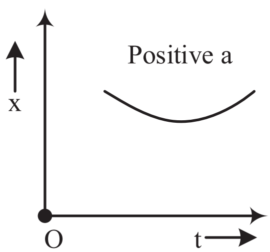


Fig. 2.3(a)

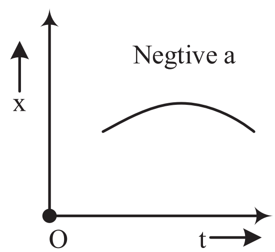


Fig. 2.3(b)

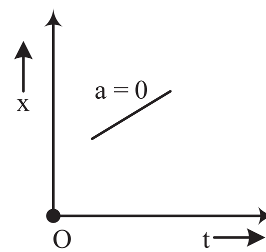


Fig. 2.3(c)

2.11 Velocity - Time graph

i) Figure 2.4(i) shows an object moving in a positive direction with positive acceleration.
ii) Figure 2.4(ii) shows an object moving in positive direction with a negative acceleration
iii) Figure 2.4(iii) shows an object moving in a negative direction with negative acceleration.
iv) Figure 2.4(iv) shows an object moving in positive direction till time t_1, and then turns back with the same negative acceleration.

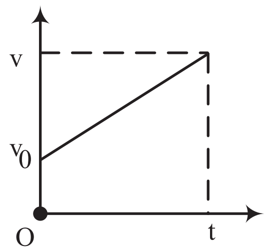


Fig. 2.4(i)

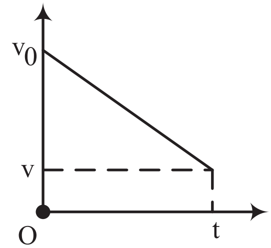


Fig. 2.4(ii)

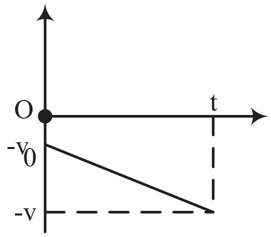


Fig. 2.4(iii)

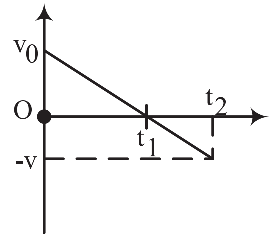
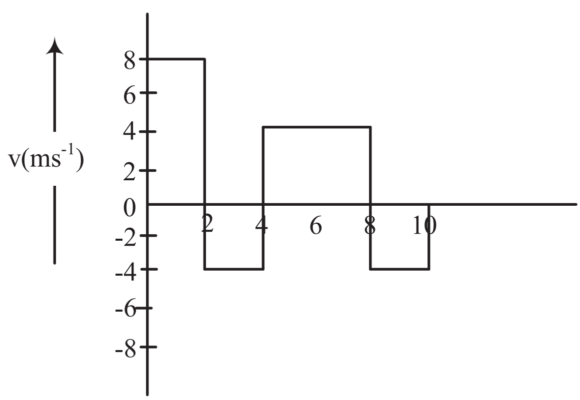


Fig. 2.4(iv)

Example 2.16 The velocity time graph of a body moving in a straight line is as shown in the figure. Find the ratio of displacement to distance traveled by the body in time 0 to 10s is

Solution: First we find displacement during time 0 to 10s



Total displacement = displacement traveled from 0 to 2s + displacement traveled during $t = 2s$ to $t = 4s$ + displacement during $t = 4s$ to $8s$ + displacement traveled during $t = 8s\ to\ t = 10s$

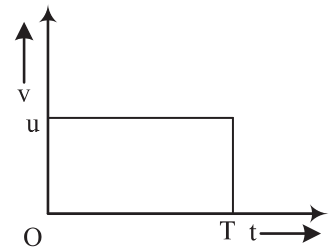
$$= 8 \times 2m + 2 \times (-4)m + 4 \times 4m + 4 \times (-2)m = 16m$$

Now total distance travelled during $t = 0s\ to\ t = 10s = 8 \times 2 + 4 \times 2 + 4 \times 4 + 4 \times 2 = 48m$

Therefore, ratio of displacement to distance traveled in $10\ s = \frac{16m}{48m} = 1:3$ ans

On being asked by Arnold for a re-explanation of the solution, the teacher replied "ok and pay attention closely to my words" and went on to explain it. This scene looked very professional.

"Displacement is the distance traveled along a straight line. And, any straight line always has two possible directions opposite to each other. For example, in the x axis there is $+ve\ x$ axis and $-ve\ x$ axis. Contrary to this, distance can't be positive or negative: it is just simply magnitude. Therefore, to find the total distance, we simply add all distances covered by the object at different intervals of time. However, when finding displacement, we have taken care of directions (i.e., sign convention) while adding."



Area under v – t curve
u × T (area under the curve)
= displacement in time T sec

- An important fact of a velocity - time graph for any moving object is that the area under the curve represents the displacement over a given time interval.

One point to keep in mind here is that we will read motion with constant acceleration (uniformly accelerated motion) only. However, we know motion with varied acceleration (non uniform accelerated motion) exists.

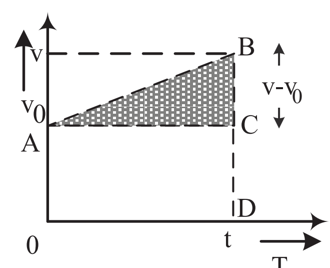


Fig. 2.5

Our chapter is confined with uniform acceleration only. Therefore, if the velocity of an object is v_0 at $t = 0$ and v at time t as shown in Figure 2.5, we have

$a = \frac{v-v_0}{t-0}$ or $v = v_0 + at$ (a=constant, during the whole motion therefore $a_{avg} = a_{inst}$)

2.12 Relations for uniformly accelerated motion

There are some important equations for uniformly accelerated motion (motion having constant acceleration) that relate different quantities involved in motion such as acceleration, final velocity, initial velocity, displacement and time. These equations are primarily known as kinematic equations for uniformly accelerated motion.

If the velocity of an object is v_0 at $t = 0$ and v at time t, we have

$a = \frac{v-v_0}{t-0}$ or $v = v_0 + at$ .....(i)

v = final velocity, v_0 = initial velocity and t = time taken by the object to reach velocity v from v_0.

We know that the area under the $v - t$ curve represents the displacement. Therefore the displacement x of the object = area between instants o and t = area of triangle ABC + area of rectangle $OACD$

$x = \frac{1}{2}(v - v_0)t + v_0 t$ ....(ii)

But $v - v_0 = at$ from Equation (i)

Therefore, $x = \frac{1}{2}(at^2) + v_0 t$

or, $x = v_0 t + \frac{1}{2}(at^2)$ ....(iii)

Equation (ii) can be also written as $(x) = \frac{(v + v_0)}{2}.t = v, t$ ...(iv)

Where $v, = \frac{v + v_0}{2}$ (for constant acceleration)

Equation (iii) and (iv) mean that the object has travelled displacement x with an average velocity equal to the arithmetic average of the initial and final velocities.

From Equation (i), $t = \frac{v - v_0}{a}$, substituting this in Equation (iii) we get $v^2 - v_0^2 = 2as$ .....(iv)

Thus, we obtain three important equations, that are as:

(1) $v = v_0 + at$
(2) $x = v_0 t + \frac{1}{2}(at^2)$
(3) $v^2 - v_0^2 = 2as$

All these three equations of motion are called kinematic equations of motion.

Example 2.17 A bus travelling along a straight highway at a speed of 72 km/h comes to a stop within 4 seconds after the brakes are applied. The distance travelled by the bus during this time (Assume the retardation is uniform) is ________.

Solution: First, convert the initial speed from km/h to m/s:

$U = 72 \times \frac{5}{18} = 20\ m/s$

Given: initial speed (u) = 20m/s

Final speed (v) = 0m/s

Time (t) = 4s, using the first equation of motion: $v = u + at$

$0 = 20 + 4a\ a = -5\ m/s^2$.

Therefore $s = 20(4)\ + \frac{1}{2}(-5)(4^2)$ (as $s = ut + \frac{1}{2}at^2$)

$= \ 40\ m.$

Example 2.18 A tennis ball is dropped on to the floor from a height of 9.8 m. It rebounds to a height of 5.0 m. The ball comes in contact with the floor for 0.2s. The average acceleration during contact is ……..m/s^2. (given g=10m/s^2)

Solution: The speed of the ball just before the collision with ground is

$U = \sqrt{2gh} = \sqrt{2 \times 10 \times 9.8} = 14$m/sec (downwards)

The speed of the ball just after the collision is

$V = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} =$10m/s (upwards)

So, $a = \frac{\Delta v}{\Delta t} = \frac{10-(-14)}{0.2} = \frac{10+14}{0.2} = 120$m/s^2.

Example 2.19 A ball is thrown vertically upwards with a velocity of $19.6\ ms^{-1}$ from the top of a tower. The ball strikes the ground after 6s. The height from the ground up to which the ball can rise will be (K/5)m. The value of k is …… (use $g = 9.8m/s^2$)

Solution: $v = 19.6m/s, t = 6s$

Time taken in upward motion above tower = 2s Time taken from top most point to ground = 4s

$= \sqrt{\frac{2h}{g}} = 4$

$h = \frac{16\times9.8}{2} = 8 \times 9.8$

$K = 8 \times 9.8 \times 5 = 392$

Example 2.20 A ball of mass 0.5kg is dropped from the height of 10 m. The height, at which the magnitude of velocity becomes equal to the magnitude of acceleration due to gravity, is ……..m. (use $g = 10\ m/s^2$)

Solution: Let assume height h where magnitude of velocity equal to acceleration due to gravity.

Then $v^2 -\ u^2 = 2\,(-10) \times\ (-h)$

$= \ v^2 = 20\ h$

$h = 5m$

$h' = 10\ - 5 = 5\ m.$

*****

It was a pleasurable evening at the beach. A beautiful sunset, gentle waves, and the group gathered there, seated on a long red bed sheet spread over the sand. After a short conversation with Kimaya and Aditi, Dolly said, "we have a strong bond among ourselves; this bond could be transformed into another beautiful relationship for sure. But along with that, keep in mind we need to deal with all academic modules and exams smartly." She concluded her statement in a lecturing style. It looked like Dolly was very serious academically.

It was quite clear that even she was not against the blooming of the new type of relationship: romantical, among the guys. But she didn't want them to be academically diverted due to any kind of stuff.

She had finished her statement but everyone looked hesitant to talk. Arnold finally broke the ice. "I have a concern regarding kinematic equations of motion," Arnold said. "Okay, let's go and get it cleared," Aakash said with a low voice, and they all headed to Arnold's house. Arnold: "I'm wondering if in many of the numerical problems only speed is given, direction is not mentioned, then why are we applying kinematic equations of motion (as kinematic equations of motion work for velocity not for speed). Dolly exclaimed, "Oh! See, we assume every object is moving in a straight line by default unless the problem provides more detail (even the chapter title is 'Motion in a Straight Line'). Therefore, we can apply."

Following the discussion they started setting some questions on their own and solving it .

Example 2.21 A small toy starts moving from the position of rest under a constant acceleration. If it travels a distance of 10 m in $t\ s$, the distance travelled by the toy in the next $t\ s$ will be:

Solution: We have $\frac{1}{2}\ at^2 = 10\ m$

Next, we calculate the total distance traveled in $2t$ seconds : $\frac{1}{2}$ a $(2t)^2 = 2at^2$

Since we know it from the initial condition that $\frac{1}{2}at^2 = 40\ m$

Thus, the additional distance traveled in the next t seconds is : total distance after $2t$ seconds - distance already traveled in t seconds

$= 40\ m - 10\ m = 30\ m.$

Example 2.22 Two spherical balls having equal masses with radius of 4 cm each are thrown upwards along the same vertical direction at an interval of 2s with the same initial velocity of 20 m/s, then these balls collide at a height of .... m. (Take g=10 m/s^2)...

Solution: when both balls collide , $y_1 = y_2$

$$\Rightarrow\ 20t - \frac{1}{2} \times 10 \times t^2 = 20\,(\,t-2) - \frac{1}{2} \times 10 \times (t-2)^2$$

$$\Rightarrow\ 20t - 5t^2 - 20t = 20t - 40 - 5$$

$$\Rightarrow\ 5t^2 = 45$$

$$t = 3s.$$

∴ Height at which balls will collide

$$h = ut - \frac{1}{2}\, at^2$$

$$\Rightarrow\ 20 \times 3 - \left(\frac{1}{2} \times 10 \times 9\right)$$

$$\Rightarrow\ 15\ m.$$

Example 2.23 From the top of a tower, a ball is thrown vertically upward which reaches the ground in 6 s. A second ball thrown vertically downward from the same position with the same speed reaches the ground in 1.5 s. A third ball released, from the rest from the same location, will reach the ground in ____________ s.

Solution: Based on the situation

$h = -ut_1 + \frac{1}{2} g t_1^2$ ….throwing up...(i)

$h = ut_2 + \frac{1}{2} g t_2^2$ ….throwing down…(ii)

$h = \frac{1}{2}\, gt^2$ ….dropping…(iii)

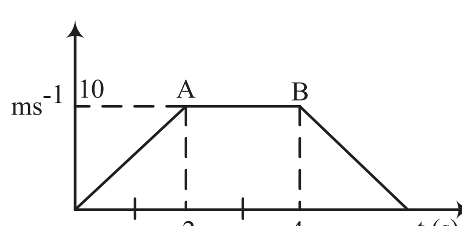
And $0 = u(t_1 - t_2) - \frac{1}{2} g\,(t_1 - t_2)^2$ …(iv)

Solving all the equations we get $t = \sqrt{t_1 t_2}$

$\sqrt{6} \times 1.5 = 3s$

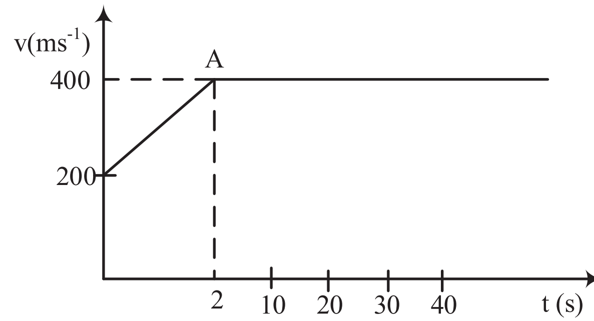
Exercises

1. The velocity-time graph of an object moving along a straight line is shown in the figure. What is the distance covered



by the object between $t = 0$ to $t = 4s$?

2. The motion of an airplane is represented by a velocity-time graph as shown in the figure. The distance covered by the airplane in the first 30.5 seconds is ……km.



3. A man walks on a straight road from his home to a market $3km$ away with a speed of $6kmh^{-1}$. Finding the market closed, he instantly turns and walks back home with a speed of $9kmh^{-1}$. What is the (a) magnitude of average velocity, and (b) average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min ?
4. Two cars are travelling towards each other at a speed of $20m/s$ each. When the cars are $400m$ apart, both the drivers apply brakes and the cars retard at the rate of $2m/s^{-2}$. The distance between them when they come to rest is :
5. A particle covers half the distance with $5m/s$ speed. The remaining part of the distance was travelled with speed $10m/s$ for half the time and with speed $15m/s$ for the other half of the time. The mean speed of the rider over the whole time of motion is……
6. A car moving along a straight highway with speed of $40m/s$ is brought to a stop within a distance of $200m$. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?
7. A particle moves in a straight line so that its displacement x at any time t is given by $x^2 = 4 + t^2$. Its acceleration at any time t is x^{-n}. Find the value of n
8. A player throws a ball upwards with an initial speed of $29.4m/s$.
   (a) What is the direction of acceleration during the upward motion of the ball ?
   (b) What are the velocity and acceleration of the ball at the highest point of its motion ?
   (c) To what height does the ball rise and after how long does the ball return to the player's hands? (Take $g = 9.8\, m/s^2$ and neglect air resistance).
9. Given below are two statements

   Statement I: Area under velocity- time graph gives the distance travelled by the body in a given time.

Statement II: Area under acceleration- time graph is equal to the change in velocity- in the given time.

In the light of given statements, choose the correct answer from the options given below.

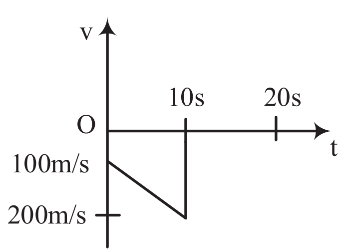
Option

(a) statement I is correct but statement II is incorrect
(b) statement I is incorrect but statement II is correct
(c) both statements are incorrect
(d) both statements are correct

10. The position of a particle related to time is given by $x = (6t^2 - 4t + 8)m$. The magnitude of velocity of the particle at $t = 3s$ will be :
11. The distance travelled by an object in time t is given by $S = (3.5\ )t^3$. The instantaneous speed of the object at $t = 4s$ will be :
12. A particle starts with an initial velocity of $10m/s$ along x direction and accelerates uniformly at the rate of $2\ m/s^{-2}$. The time taken by the particle to reach the velocity $80m/s$ is :
13. An object moves with speed v_1 v_2 and v_3 along a line segment AB, BC and CD respectively as shown in the figure. Where $AB = BC$ and $AD = 3AB$, then average speed of the object will be: (Hint - use the formula $Average\ speed\ = \frac{distance}{time}$)
14. A car is travelling on a straight highway at a speed of $30\ m/s$. The driver sees a deer ahead and applies the brakes, causing the car to decelerate uniformly at a rate of $5m/s^2$. How long does it take for the car to a complete stop?
15. A particle moving in a straight line covers half the distance with speed $6m/s$. The other half is covered in two equal time intervals with speed $9m/s$ and $15m/s$ respectively. The average speed of the particle during the motion is :
16. A bullet is shot vertically downwards with an initial velocity of $100m/s$ from a cartoon height. Within 10s, the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity-time graph for total time $t = 20s$ will be:
17. A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next $\frac{3}{2}x$ distance. The average velocity in this motion is $\frac{50}{7}m/s$. If v_1 is $5m/s$ then $v_2 =$ ______m/s.
18. A tennis ball is dropped on to the floor from a height of $9.8m$ it rebounds to a height $5.0m$. Ball comes in contact with the floor for $0.1s$. The average acceleration during contact is __________ m/s^2. (Given $g = 10m/s^2$)

19. An object starts moving from the position of rest with a constant acceleration. If it travels a distance of 20m in $2.5ts$, the distance travelled by the object in the next $2.5ts$ will be:
20. A ball is dropped from a height of $5m$ onto a sandy floor and penetrates the sand upto 10cm before coming to rest. Find the retardation of the ball in sand assuming it to be uniform. (take $g = 10m/s^2$)
21. A car travelling at 60 km/h overtakes another car travelling at 32km/h. Considering each car to be 1m long, find the time taken during the overtake and the total road distance used for the overtake.

Answers

1. $30m$	2. $12km$	3.(a)(i)6 km/hr (ii) 0 (iii) $\frac{15}{4} km/hr$	(b) (i) 6 km/hr (ii) $\frac{36}{5}$ km/hr (iii) $\frac{27}{4} km/hr$
4. $200m$	5. $\left(\frac{50}{7}\right) m/s$	6. $4m/s^2, 10s$	7. 3
8.(a) downwards (b) $0, 9.8m/s^2$ (c) $32m$	9. (d)	10. $32m/s$	11. $168m/s$
12. $35s$	13. $\frac{(3v_1v_2v_3)}{(v_1v_2+v_2v_3+v_3v_1)}$	14. 6s	15. 8m/s
16. 		17. 10	18. 240
19. 60m		20. 500 m/s^2	21. 0.275s, 4.28m

Chapter - 3
Motion in two dimensions

Students were gossiping, their chatter sounding like birds chirping in the class. The teacher was looking at her notebook, a type of diary she used as reference for teaching. Just then the group entered the classroom. The teacher looked at them and said kindly, “Be on time, please,” as they were a little bit late. The class had approximately 80 students; it was a completely new institution for almost all of them. It could be observed that some were slowly adapting to the new environment while others struggled. However, the guys seemed to be on another planet: confident, fearless and unified. That day the middle row was not empty–where they usually sat–forcing them to adjust in the last row. The sitting arrangement was like stairs, going from bottom to top just like in a cinema hall.

Gita, the teacher, stated, “Motion in two dimensions is considered to be very important on its own, and acts as a foundation for a number of other modules.” She then began the class more traditionally.

3.1 Vector and Scalar

Before delving into ‘motion in a plane’ it is necessary to be familiar with some terms like Vector and Scalar.

Note: Motion in a plane is also called motion in two dimensions.

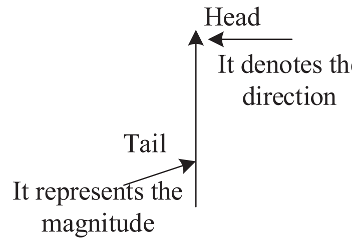
Vector: It is a physical quantity which possesses both magnitude as well as direction, and obeys the laws of vector addition. For example, Velocity, Displacement, Force and Momentum.

Suppose two cars a and b start moving with velocity $4m/s$ in horizontal direction and at $t = 2$sec, both cars have the same velocity $4m/s$ in the horizontal direction. But, at $t = 4s$, car b changed its direction by 90 degree from the horizontal direction, but car a continues to move in the same direction. However, still both cars' speeds are the same.

Here $V_{at2} = V_{bt2}$, but $V_{at4} \neq V_{bt4}$. Although the magnitude of V_{at4} is still the same as V_{bt4} $(4m/s)$, i.e., $|V_{at4}| = |V_{bt4}|$, they are not equal because of having different directions. Two vector quantities are equal only when their magnitudes and directions are the same. In mathematical notation, vectors are often denoted by Boldface Letters, for example $\mathbf{V_{at2}}$, $\mathbf{V_{bt2}}$ ($\mathbf{V_{at2}}$ denotes velocity of car a at $t = 2s$, $\mathbf{V_{bt2}}$ denotes velocity of car b at $t = 2s$)

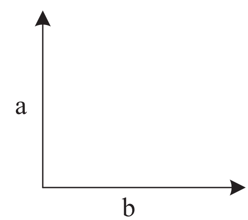
Scalar Quantity: A physical quantity which has only magnitude and no direction is known as scalar quantity. For example, Mass, Distance, Time and Temperature.

Representation of vectors



One of the famous representations of vectors is by arrows. The length of the arrow signifies the magnitude of the vector, and the direction the arrow points indicates the vector's direction.

Suppose velocity of an object is $4m/s$ in $+ve\ y$ direction. Then it could be represented by the diagram shown above.



When multiple vectors are involved in a problem, we represent the vector quantities by having a common tail end for all the vectors. For example, as shown for vectors **a** and **b**.

3.2 (I) Addition of vector quantities

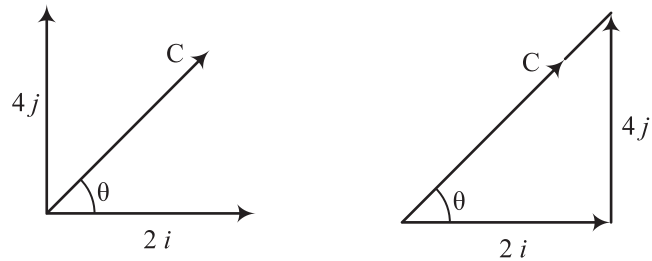
How vector quantities are added? If we try to understand this using a practical example. Assume a football is placed on the ground. Two players then try to impart velocity to it in two different directions (say, x and y), by simultaneously kicking it with different magnitudes V_x and V_y. In such a scenario, the football moves in neither of the original directions in which it was kicked, instead it moves in a direction which lies between x and y with a net magnitude V_c given by the formula: $V_c = \sqrt{V_x^2 + V_y^2 + 2V_xV_y\cos\theta}$ ….(3.1), where θ is the angle between x and y.

If the two vectors are at right angles to each other, we can find the resultant vector direction by simple trigonometry. To understand it better, some examples, 3.1, 3.2, are given.

V_c is the resultant vector of the sum of vectors V_x and V_y. V_x is known as the projection (or component) of V_c over x and V_y is the projection (component) of V_c over Y. We can find V_x simply by multiplying $cos\ \theta_1$ with V_c, where θ_1 is the angle V_c makes with V_x or with x axis. We can find V_y by multiplying $\sin\theta_1$ with V_c.

Note: I, J and K are considered as unit vectors along x, y and z axes respectively.

Example 3.1 If an object's velocity is $2i + 4j$ units, find magnitude and the direction of the velocity.



Solution: Given velocity $= 2i + 4j$ (we can assume the question is telling that the object is intending to move 2 units in x direction and 4 units in y direction simultaneously or simply the object has velocity 2 units in x direction and 4 units in y direction). Suppose $C = 2i + 4j$

The magnitude of C will be

$= \sqrt{(2)^2 + (4)^2 + 2(2)(4)\, cos\, cos\, 90°}$

$= (20)^{1/2} = 2(5)^{1/2}$

We know $cos\,\theta = \frac{base}{hypotenuse}$ (θ is the angle, the vector C makes with the vector in x direction)

$\Rightarrow cos\,\theta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\Rightarrow \theta = \left(\frac{1}{\sqrt{5}}\right) = 63.43°$, therefore the vector C is at $63.43°$ with the x axis.

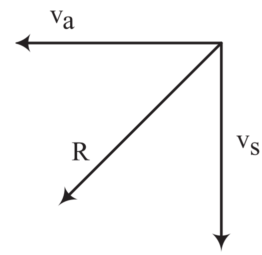
(Note: The vector along the x direction, which is $2i$, is known as the x component of the vector C. We can get it simply by multiplying magnitude of C with cosine of the angle ($cos\,\theta$), it makes with the x-axis, i.e. $(2\sqrt{5})\left(\frac{1}{\sqrt{5}}\right) = 2$, if the resultant vector is not given in component form. However, here it is already given in the component form $(2i + 4j)$)

(II) Multiplication of a vector by real number

(i) $2 \times 4i = 8i$ (ii) $4 \times 7j = 28j$ (iii)$3(2i + 5j) = 6i + 15j$

Example 3.2 A naughty child throws a stone from the top of a roof with speed $20m/s$ downwards and with the same speed air is flowing from east to west direction. Fortunately the man has an umbrella in his hand. In which direction the man should hold the umbrella to protect himself from the stone (Note - Neglect the effect of gravity due to Earth)

Solution: Let us represent the velocity of the stone and the wind by the vectors v_s and v_a respectively as shown in the figure. R is the resultant vector of v_s and v_a and its direction must lie between vectors v_s and v_a. And to find the direction θ that R makes with the vertical is given by



$$tan\ \theta\ = \frac{v_a}{v_s} = \frac{20}{20} = 1 \qquad so\ \theta = 45°.$$

Therefore, the boy should hold his umbrella at an angle of 45° west of the vertical.

Example 3.3 A motorboat is running towards south at 40 km/hr and the water current in the region is 30 km/hr in the direction of 60° north of east. Then what will be the resultant velocity of the boat?

Solution: Let the vector v_b representing the velocity of the motorboat and the vector v_c representing the water current as in the directions specified in the question. We can obtain the magnitude of resultant vector R as follow

$$R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 150°}$$

$$= \sqrt{1600 + 900 + 2(1200) \times (-\sqrt{3}/2)}$$

$$= 20.53\ km/hr.$$

Example 3.4 The velocity of a particle is given as $v = (4ti\ +\ t^2 j\ )$ unit, where t is in second and velocity is in meters. Find acceleration (a) of the particle at $t = 0$ and $t = 2s$.

Solution: $a(t) = \frac{d(v)}{dt} = \frac{d(4ti + t^2 j\ )}{dt}$

$$=\ 4i\ +\ 2tj\ , at\ t = 0, a = 4i$$

That is $4m/s^2$ in x direction.

At $t = 2\ , a = 4i\ +\ 2tj$

$\Rightarrow\ \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}\ m/s^2$.

Example 3.5 A particle starts from origin at $t = 0$ with a velocity $12.0i\ m/s$ and moves in $x - y$ plane under action of a force which produces a constant acceleration of $(4.0i\ + 2.0j)m/s^2$. What is the y-coordinate of the particle at the instant its x-coordinate is $32m$?

Sol: We know $s = ut + \frac{1}{2}\, at^2$

$\Rightarrow\ 32 = 12t + \frac{1}{2}\, 4t^2$

$\Rightarrow\ 2t^2 + 12t - 32 = 0$

$\Rightarrow\ 2t^2 + 16t - 4t - 32 = 0$

$\Rightarrow\ t = 2s$

Therefore, y coordinate $= u_y t + \frac{1}{2}\, a_y t^2$

$$\Rightarrow \frac{1}{2} \times 2 \times 4 = 4m.$$

Note: All of the above examples are the case of motion in a plane (two-dimensions). Motion in a plane can be treated as movement of an object along two perpendicular axes (typically x and y axes) simultaneously.

Gita finished the class with the statement, "That's all for today, I hope you enjoyed the module; however it acts as a challenge for several students. But my approach to help you understand it is effective."

*****

Over a month had passed, and no one remembered the driver's fees and the driver himself was hesitating to ask for that. However, the issue came to light at the moment when the bus was low on fuel while they were going to the village after attending the class. The bus stopped at a fuel station to refuel. David asked the driver for his bank account details on the spot so he could transfer his monthly fees, and the money was transferred instantly from the group's collective fund.

At the fuel station, Arnold saw a small dog in a wounded and very helpless state from the bus window. He stepped outside, took the dog in his arms, and started moving his hand gently over it in a comforting way. However, after a moment, a man looking to be in his 50s approached on his bike and said, "Nice to meet you, it's my dog." Arnold: oh! But it is in a bad state. "Yes, but don't worry I am taking it to a doctor right now," he responded with a hint of embarrassment.

The man appeared a little odd to Arnold. However, Arnold later learned from the bus driver that the man was the owner of the fuel station and was deeply concerned about his son's career, especially after his matriculation result day, as he had failed two subjects. This was a cause for deep concern for the owner, as he had hoped his son would receive a good education and expand the family business. Knowing it, Arnold expressed his desire to the driver to meet the owner's son. The driver replied gently, "I will try to arrange it."

*****

The next session started with 'Projectile Motion'

3.3 Projectile Motion

It is an important example of motion in a plane. An object that is in flight (i.e. object is in air under the influence of gravity) after being thrown or projected is called a projectile. The motion described by the object during this scenario is called projectile motion.

For example, an object from point o is thrown by making an angle θ_0 from x axes as shown in Figure 3.1. Its motion can be completely described in two axes x and y.

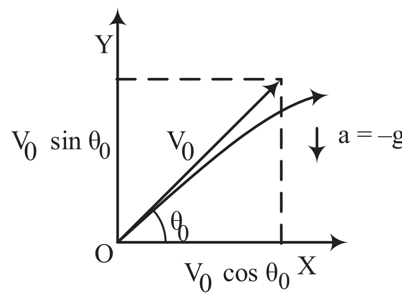


Fig. 3.1

- The acceleration of the object is only due to the gravity $a = -gJ$ (g is vertically downwards)

  or $a_y = -g$ and $a_x = 0$

- The components of initial velocity v_0 are $v_{ox} = v_0 cos\, \theta_0$ along x axis, $v_{oy} = v_0 sin\, \theta_0$ along y axis. (v_{ox} denotes component of v_0 along x direction, similarly v_{oy} denotes v_0 along y direction)

For convenience we take the initial position as the origin of the reference frame.

The initial position are $x_0 = 0, y_0 = 0$

- The displacement along x and y directions at time t: $x = v_{ox}t = (v_0 cos\, \theta_0)t$ ….(3.2) and $y = (v_0 sin\, \theta_0)t - \left(\frac{1}{2}\right) gt^2$….(3.3)
- The x component of velocity at time $t, v_x = v_{ox} = v_0 cos\, \theta_0$ (Velocity during whole motion will be constant, equal to $v_0 cos\, \theta_0$ at any time as $a_x = 0$).
- The y component of velocity at time $t, v_y = v_0 sin\, \theta_0 - gt$ (since a is vertically downwards)

(I)Equation of path of a projectile

One question strikes, what is the shape of the path followed by the projectile. We can get this by eliminating the time between the expressions for x and y as given in Equation (3.2) and (3.3).

We obtain $y = (tan\ \theta_0\ )x - \frac{gx^2}{2(v_0 cos\ \theta_0\ )^2}$ ...(3.4)

This is the equation of a parabola, i.e. the path of the projectile is a parabola as shown in Figure 3.2

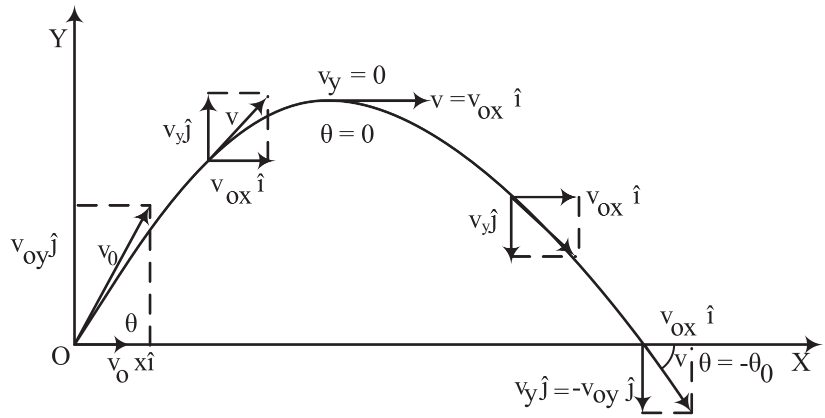


Fig. 3.2

Example 3.6 A man stands on the top of a roof $500m$ above the ground and throws a ball horizontally with an initial speed of $10ms^{-1}$. Neglecting air resistance, what will be the time taken by the ball to reach the ground, and the speed with which it hits the ground. (Take $g = 10\ m/s^2$).

Solution: $S_y = \frac{1}{2} \times -10 \times t^2$

$-500 = -5t^2$ (given that $S_y = -500m$)

$\Rightarrow\ t = 10s$

Now to find speed at which the ball hits the ground

$V_x = u_i + at$

$V_x = 10\ m/s$ ($a = 0$ in x direction)

$V_y = u_y + gt$ ($u_y = 0$ in y direction)

$V_y = -100\ m/s$ (–tells us the direction is in $-ve\ y$ direction)

Therefore, the net velocity when it hits the ground $(v_n\sqrt{(10)^2 + (-100)^2} = 100.5\ m/s$.

Example 3.7 If the initial velocity in horizontal direction of a projectile is unit vector i and the equation of trajectory is $y = 8x(1 - x)$. The y component vector of the initial velocity is ….. j.

Solution: Given that the trajectory equation is $y = 8x(1 - x)$

On expanding it we get $y = 8x - 8x^2$

The general form of a projectile's trajectory is $y = (tan\,\theta_0\,)x - \frac{gx^2}{2(v_0 cos\,\theta_0\,)^2}$

Comparing the given equation with the general form, it is clear that: $tan\,\theta_0\; = \;8$

Given that the initial velocity component u_x is 1 (unit vector i), we can find u_y as follows: $u_y = tan\,\theta_0 u_x \; = \; 8 \times 1 = 8j$.

(II) Time at which Maximum height attained by Projectile

Let us assume the time at which maximum height attained is t_m.

At this point $v_y = 0$,

$v_y = v_0 \sin\theta_0 - gt_m$ (from the kinematics equation of motion $v = u + at$)

($v_y = 0$ at the maximum height)

Therefore $t_m = \frac{v_0 sin\,\theta_0}{g}$

The total time t_f during which the projectile is in flight can be obtained by putting $y = 0$ in equation (3.2)

$v_0 sin\,\theta_0\; t_f - \frac{1}{2} g t_f^2 = 0$

$\Rightarrow v_0 sin\theta_0\; t_f = \frac{1}{2} g t_f^2$

$\Rightarrow t_f = \frac{(2v_0 sin\,\theta_0)}{g}$ (t_f is known as the time of flight of the projectile)

(III) Maximum height achieved by a projectile

It can be calculated by placing $t = t_m$ in Equation (3.2) ($y = v_0 sin\,\theta_0\; t - \frac{1}{2} g t^2$)

We have $t_m = \frac{v_0 sin\,\theta_0}{g}$,

therefore $y = h_m = v_0 \sin\theta_0 \left(\frac{v_0 sin\,\theta_0}{g}\right) - \frac{g(v_0^2 \sin^2\theta_0\,)}{2g^2}$

Or $h_m = \frac{v_0^2 \sin^2\theta_0}{2g}$.

(IV) Horizontal range of a projectile

R is the horizontal distance traveled during the time of flight t_f. It is also called the range of the projectile.

$R(d) = ut + \frac{1}{2}(at^2) = v_0 cos\, \theta_0\, t_f$ (as $a = 0$ towards horizontal direction)

$= \frac{(v_0 cos\, \theta_0\,)(2v_0 sin\, \theta_0\,)}{g}$

or $R = \frac{v_0^2 sin 2\theta_0}{g}$ $(sin\, 2\theta_0\; = 2cos\, \theta_0\, sin\, \theta_0\,)$

Note: When two bodies are projected at the same speed at different angles θ_1 and θ_2 and they achieve the same range (horizontal), then the sum of θ_1 and θ_2 always becomes $90°, \theta_1 + \theta_2 = 90°$.

Example 3.8 A boy throws a ball with speed 10 km/h vertically while running with speed 36 km/hr on a road . The Range of the ball will be …………m.

Solution: We know that horizontal range R of an object

$R = v_{horizontal} \times t_{total}$ (t_{total} is the time of flight (t_f), it means the total time during which the ball is in the air)

The time to reach the maximum height t_m is given by

$t_m = \frac{v_{initial}}{g}$

$\Rightarrow 2.78/9.8 = .287s\; (10\; km/h = 2.78\; m/s)$

And note, the time of flight is always twice the time to reach the maximum height.

Therefore $t_{total} = 2\; \times\; 0.287 = 0.57s$.

$R = v_{horizontal}\; \times t_{total}$

$\Rightarrow 10\; \times\; .57 = 5.7\; m\;\; (36km/h = 10\; m/s)$

Example 3.9 The maximum height reached by a projectile is 72 m. If the initial velocity is halved, the new maximum height of the projectile is _______ m

Solution: Using the formula for maximum height $H_m = \frac{(v_0^2 \sin^2 \theta_0\,)}{2g}$ where v_0 is the initial velocity of the projectile, θ_0 is the angle of projection of the projectile from the horizontal direction and g is the acceleration due to gravity.

$72 = \frac{(v_0^2 \sin^2 \theta_0\,)}{2g}$ ……….(i)

The new maximum height if the initial velocity is halved. Let's denote the new initial velocity as $v, = \frac{v_0}{2}$. Using the formula for maximum height again, we get:

$$H, = \frac{v_,^2 \sin^2 \theta_0}{2g}$$

Substituting $v, = \frac{v_0}{2}$ into this equation:

$$H, = \frac{\left(\frac{v_0}{2}\right)^2 \sin^2 \theta_0}{2g} = \frac{1}{4}\left(\frac{v_0^2 \sin^2 \theta_0}{2g}\right)$$

Since we know the original height:

$$72 = \frac{(v_0^2 \sin^2 \theta_0\,)}{2g}$$

Substituting this value into our equation for

$$H, = \frac{1}{4} \times 72 = 18\,m$$

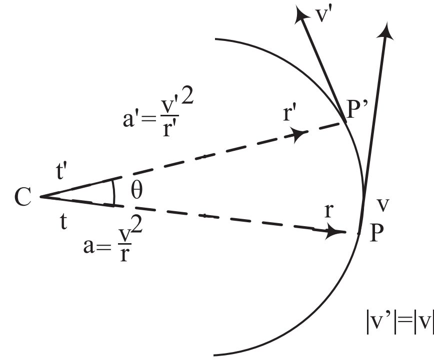
Therefore, if the initial velocity of the projectile is halved, the new maximum height reached by the projectile would be $18m$

3.4 Uniform Circular Motion

If a particle moves in a circle with uniform speed, the motion described by the particle is called uniform circular motion. Here, the velocity keeps changing because its direction is constantly changing, even though its magnitude (speed) remains constant, therefore, there must be an acceleration during the motion.

This acceleration is responsible for changing the direction of the velocity, and its direction is always towards the center. The magnitude of this acceleration is equal to $\frac{v^2}{r}$.

As it is circular motion, there must be some circular terms associated with it. So, let's know these terms and their relations with linear motion terms.



As the object moves from p to p' in time Δt $(\Delta t = t' - t)$, the line cp turns through an angle θ, as shown in Figure 3.3. $\Delta\theta$ is called angular displacement.

We define the angular speed ω (omega) as the time rate of change of angular displacement:

$$\omega = \frac{\theta}{\Delta t}$$

Now, if the distance travelled by the object during the time Δt is Δs, ($\Delta s = pp'$), then:

$$V = \frac{\Delta s}{\Delta t}$$

But $\Delta s = r\Delta\theta$. Therefore, $V = \frac{r\Delta\theta}{\Delta t} = r\omega$

$$V = r\omega$$

We can express centripetal acceleration a_c in terms of angular speed: $a_c = \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$

$$a_c = \omega^2 r$$

Frequency (v): The time taken by an object to make one revolution is known as its time period T. And the number of revolutions made in one second is called its frequency f $\left(= \frac{1}{T}\right)$, its SI unit is Hertz ($Hz$). (where T is measured in seconds)

Example 3.10 A stone tied to the end of a string 1.4 m long is whirled in a horizontal circle with a constant speed. If the stone makes 20 revolutions in 20 s, what is the magnitude and direction of acceleration of the stone?

Solution: Length of the string = 1.4 m

No of revolutions = 20

Time taken = 20s

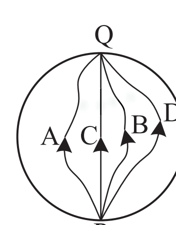
Frequency $f = \frac{No.of\ revolutions}{time\ taken} = \left(\frac{20}{20}\right) = 1Hz$

Angular velocity, $\omega = \frac{2\pi}{T} = 2 \times \left(\frac{22}{7}\right) = \frac{44}{7}\ rad/s$

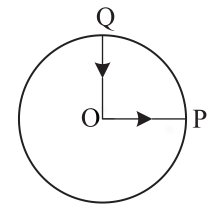
Centripetal acceleration $a_c = \omega^2 r = \frac{44}{7} \times \frac{44}{7} \times 1.4 = 55.31\ m/s^2$.

Exercises

1. Four boys running on a circular ice ground of diameter $300m$ start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in the fig. What is the magnitude of the displacement vector for each?



2. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in figure. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



3. A particle is projected at an angle of 30° from horizontal at a speed of 80 m/s. The height attained by the particle in the first second is h_1 and height attained in the last second, before it reaches the maximum height, is h_2. Find the height h_1 and h_2. (Take $g = 10m/s^2$)
4. A particle starts from origin at $t = 0$ with a velocity 5im/s and moves in $x - y$ plane under action of a force which produces a constant acceleration of $(3i + 2j)m/s^2$. If the x coordinate of the particle at that instant is 84 m. then the speed of the particle at this time is $\sqrt{\alpha}$ m/s. Find the value of $\sqrt{\alpha}$.
5. A cricket ball is thrown at a speed of 28 ms^{-1} in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.
6. A projectile fired at 30° to the ground is observed to be at the same height at time 3 s and 5 s after projection, during its flight. The speed of projection of the projectile is
7. A man can throw a ball to a maximum horizontal distance of 60 m. How high above the ground can the man throw the ball ?
8. A stone tied to the end of a string .7 m long is whirled in a horizontal circle with a constant speed. If the stone makes 20 revolutions in 20 s, what is the magnitude and direction of acceleration of the stone?
9. Two objects are projected from ground with same speeds $40m/s$ at two different angles with respect to horizontal. The bodies were found to have the same range. If one of the body was projected at an angle of 60° with horizontal then ratio of the maximum heights, attained by the two projectiles, is
10. An object is projected in the air with initial velocity u at an angle θ The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of the first object. The initial velocity remains the same in both cases. The value of the angle of projection, at which the second object is projected, will be ……….degree.
11. If the initial velocity in horizontal direction of a projectile is unit vector i and the equation of trajectory is $y = 4 \times (1 - x)$. The y component vector of the initial velocity is …..j. (Take $g = 10\ m/s^2$)

12. A body is projected from the ground at an angle of $45°$ with the horizontal. Its velocity after $2s$ is $20\ m/s$ The maximum height reached by the body during its motion is __________ m. (use $g = 10\ m/s^2$)
13. A ball of mass m is thrown vertically upward. Another ball of mass $2m$ is thrown at an angle θ with the vertical. Both the balls stay in the air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is _____________.
14. If the initial velocity in horizontal direction of a projectile is unit vector i and the equation of trajectory is $y = 5 \times (1 - x)$. The y component vector of the initial velocity is __ j (Take $g = 10m/s^2$)
15. Two balls with same mass and initial velocity, are projected at different angles in such a way that maximum height reached by the first ball is 8 times higher than that of the second ball. $T1$ and $T2$ are the total flying times of first and second ball, respectively, then the ratio of $T1$ and $T2$ is ……
16. A helicopter flying horizontally with a speed of $360km/h$ at an altitude of $2km$, drops an object at an instant. The object hits the ground at a point O, 20 s after it is dropped. Displacement of 'O' from the position of helicopter where the object was released is : (use acceleration due to gravity $g = 10\ m/s^2$ and neglect air resistance)
17. The position vector of a moving body at any instant of time is given as $r = (5t^2 i\ -\ 5tj)m$. The magnitude and direction of velocity at $t = 2s$ is:
18. A motor car is going due north at a speed of 50 km/h. It makes a 90° left turn without changing the speed. The change in the velocity of the car is about
19. A stone is released from an elevator going up with an acceleration a. The acceleration of the stone after the release is

    (a) a upward (b) $(g - a)$ upward
    (c) $(g - a)$ downward (d) g downward.
20. In a projectile motion the velocity
    (a) is always perpendicular to the acceleration
    (b) is never perpendicular to the acceleration
    (c) is perpendicular to the acceleration for one instant only
    (d) is perpendicular to the acceleration for two instants.

Answers

(1) 300	(2) (a) zero (b) zero (c) 21.4km/h	(3)35 m, 5m	(4) 673
(5) (a) 10m (b) 2.86s (c) 69.28m)	(6) 78.4 m/s	(7) 30 m	(8) 27.63m/s^2, along the radius, towards the

			centre of the circle
(9) 3:1	(10)15°	(11) 5	(12) 20m
(13)1	(14)5	(15) $2\sqrt{2}:1$	(16) $2\sqrt{2}km$
(17) $\frac{5\sqrt{17}m}{s}$, $\tan^{-1}4$ $with$ $-ve\ yaxis$	(18) 70 km/h towards south-west	(19) d	(20) c

Chapter - 4
Laws of Motion

This session, class commenced with 'Newton's First law of Motion.'

4.1 Newton's First law of Motion: An object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.

This tendency of an object to resist or oppose changes in its state of motion is called inertia. Newton's first law of motion is known as the law of inertia.

4.2 Newton's Second law of Motion: It states that the acceleration of an object is directly proportional to the net force applied to it and inversely proportional to its mass.

$F = ma$ ……(4.1)

In SI units, a newton (N) is defined as the force that causes an acceleration of 1 ms^{-2} to a mass of 1 kg. This unit is known as the Newton (N). 1 $N = 1\ kg\ ms^{-2}$

Example 4.1 A bullet of mass $.04\ kg$ moves with a speed of $60ms^{-1}$ enters a wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet ?

Solution: The retardation 'a' of the bullet (assumed constant) is given by $v^2 - u^2 = 2as$. (retardation: acceleration in opposite direction of motion is called retardation)

$$0^2 - u^2 = 2 \times a \times 0.6\ m$$

$$a = -\frac{(60ms^{-1})2}{2\times 0.6m} = 3000ms^{-2}$$

The retarding force is $F = ma$

$$\Rightarrow .04kg \times 3000ms^{-2} = 120\ N$$

We have assumed the resistive force during retardation of the bullet is uniform.

4.3 Momentum

Momentum (P) of a particle is defined as a product of its mass (m) and velocity (v).

$P = mv$ ….(4.3)

It is a vector quantity, possessing both magnitude and direction. (Momentum direction is in the direction of velocity)

In the international system of units (SI), the unit of measurement of momentum is the Kilogram metre per second (Kg m/s).

The rate of change of momentum of a body is directly proportional to the applied force

$$F = \frac{dp}{dt} = ma \qquad \text{....(4.4)}$$

4.4 Impulse

There are numerous cases where a large force acts on a body for a small period of time during collision. For example, when a ball bounces back after hitting a wall, the force exerted by the wall on the ball is known as impulsive force. This force may or may not be constant during the brief period of collision.

The product of force and the time period of collision is called impulse. It is equal to the change in momentum of the body and is denoted by J.

$$J = F_{avg}\Delta t = change\ in\ momentum \qquad \text{.... (4.5)}$$

4.5 Conservation of Momentum :

The momentum of a particle or a system of particles is conserved unless an external force acts on it. This fact is known as the law of conservation of momentum. For a single particle, momentum $P = MV$. For a system of particles, the total momentum P is the vector sum of individual moments: $P = P_1 + P_2 + P_3 + \ ... = M_1V_1 + M_2V_2 + M_3V_3 + \ ....$

For example, consider a bullet-gun system (which is a system of particles - the bullet and gun are assumed particles).

When a bullet is fired from a gun M_1V_1 is the momentum of the bullet and M_2V_2 is the recoil momentum of the gun.

Before firing the initial momentum of the system was 0. Therefore the final momentum $(M_1V_1 + M_2V_2)$ after firing must also be 0. Mathematically, this is expressed as $M_1V_1 + M_2V_2 = 0$

$$M_1V_1 = -M_2V_2 \qquad \text{..... (4.6)}$$

Equation 4.6 tells both are equal in magnitude and opposite in direction.

Example 4.2 A bullet of mass $100\ gm$ is fired from a gun of mass $500\ gm$. If velocity of bullet after firing is $100\ m/s$. What will be the recoil velocity of the gun ?

Solution: Let's assume mass of the gun $= \ M_1$

Recoil velocity of the gun = V_1

Mass of bullet = M_2 & velocity of bullet = V_2

By law of conservation of momentum, we have $M_1V_1 = -M_2V_2 = .1\,kg \times 100\,m/s$

$\Rightarrow v_1 = -20m/s$ ($-ve$ sign tells the direction of the gun is opposite to the velocity of bullet)

Example 4.3 A person hit a ball on earth's surface. if the point of contact between earth and ball is 2 seconds and force that acts on ball by earth varies as $2t + 4$. Then find the impulse by earth during the point of contact.

Solution: Impulse is equal to $\int_{t1}^{t2} F\Delta t$ (given $\Delta t = 2sec$)

$$\int_{t_1}^{t_2}(2t + 4)\Delta t = \int_{t_1}^{t_2}(2t + 4)\, 2s \; [\, t_2 = 2s, t_1 = 0s \,]$$

$$\left[\frac{2t^2}{2} + 4t\right]_0^2 = 12\ Ns\ .$$

4.6 Newton's third law of Motion

It states, there is always a reaction for every action which is equal in magnitude and opposite in direction.

It means if a body A exerts a force F on another body B, then B exerts a force $-F$ on A.

There are some important points about Newton's third law of motion.

(I) It occurs on a pair of bodies, meaning it always takes place on two bodies, one experiences action while another body faces reaction.

(II) Action and Reaction means force and nothing else, and both forces are equal in magnitude and opposite in direction.

(III) Action and Reaction occur on two different bodies. Consider two bodies A and B, if body A exerts force on B then there is also a force on B by A which is equal in magnitude and opposite in direction. Mathematically we write F_{BA} (force on B due to A) = $-F_{AB}$ (force on A due to B)

- **Inertial and uniformly accelerated frames of reference.**

Inertial frame of reference

It is the frame of reference which is either at rest or moves with constant velocity and thus Newton's laws of motion applies here directly. All the cases we have encountered in this chapter are of the Inertial frame of reference.

Uniformly accelerated frame of reference.

It is the frame of reference which moves with constant acceleration with respect to other inertial frames.

Here we can't directly apply Newton's laws of motion. To hold Newton's laws here we need to add an additional force F_s to any object of mass m in non-inertial frame of reference such as $F_s = -ma$ where a is acceleration of the inertial frame of reference to an inertial frame. F_s always acts opposite to the acceleration (or motion) of the Non-inertial frame of reference.

4.7 Equilibrium of a particle:

If a particle is in equilibrium, the net external force on it is zero. According to Newton's First Law, the particle is either in rest or in uniform motion. If three concurrent forces F_1, F_2, and F_3 are acting on a particle p and the particle is in equilibrium, then $F_1 + F_2 + F_3 = 0$

Example 4.4 A mass of 8 kg is suspended by a rope of length 2 m from the ceiling as shown in Figure 4.1(a). A force of 80 N in the horizontal direction is applied at the midpoint P of the rope. What is the angle the rope makes with the vertical in equilibrium? (Take $g = 10ms^{-2}$). Neglect the mass of the rope.

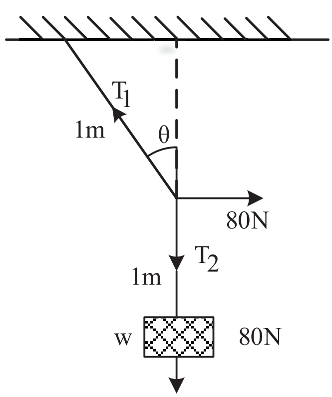


Fig 4.1(a)

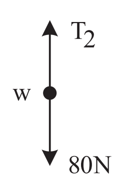


Fig 4.1(b)

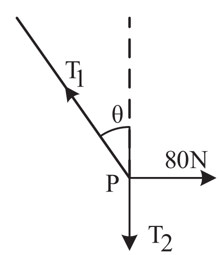


Fig 4.1(c)

Solution: Figures 4.1(b) and 4.1(c) are known as free-body diagrams. Figure 4.1(b) is the free-body diagram of W and Figure 4.8(c) is the free-body diagram of point P. Consider the equilibrium of the weight W. Clearly, $T_2 = 8 \times 10 = 80\ N$. Consider the equilibrium of the point P under the action of three forces - the tensions T_1 and T_2, and the horizontal force 80 N. The horizontal and vertical components of the resultant force must vanish separately:

$$T_1 cos\,\theta \; = T_2 = 80\ N$$

$$T_1 sin\,\theta \; = 80\ N$$

It gives us $tan\ \theta\ = \frac{8}{8} = 1 = 45°$.

4.8 Friction

When a body is in contact with a surface, the surface always exerts a force on the body. This force is always perpendicular to the surface and is called the normal force (N) (Figure 4.2).

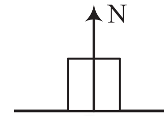


Fig 4.2

In the figure, there is a normal force acting vertically upwards on the body. Yet it is at rest. Why? The answer is because of gravitational force that acts downwards on the body and is equal in magnitude to the normal force. Thus, the net force is 0, and hence the body is at rest.

However, our main motive in discussing normal force is to understand friction because normal force is a crucial component in friction, so let's now know about friction.

If a body is kept on a surface, a force acts on a body by the surface when the body wants to move. This force is parallel to the surface and opposite to the direction in which the body moves or wants to move and the force is called frictional force.

Frictional force is of two types: (I) Static frictional force (II) Kinetic frictional force

(I) Static frictional force (f_s)**:** It is the force exerted by a surface on a body in contact when external force acts on the body, attempting to initiate motion. f_s always opposes the tendency of motion so it acts in the opposite direction of the external force. An interesting fact is that it varies according to the force applied on it. More the applied force, more will be the f_s. However, it has limit upto which it can increase, and is given as $(f_s)max = u_s N$, where u_s is a constant of proportionality and is called the coefficient of static friction.

It depends on the nature of the surface in contact. The law may be expressed mathematically as $f_s \leq u_s N$ (Figure 4.3 (a)).

(II) Kinetic frictional force (f_k)**:** What if the applied force exceeds (f_s) max. Then the body begins to move on the surface and still there is a frictional force, which continues to act in the opposite direction of the motion.

However, it doesn't vary and is constant. This type of frictional force is called kinetic friction.

$f_k = u_k N, u_k$ is the coefficient of kinetic friction.

Experimentally, it is found the magnitude of kinetic friction is less than (f_s) max (Figure 4.3 (b)).

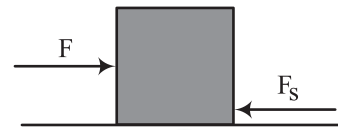


Fig 4.3(a) The body is trying to move

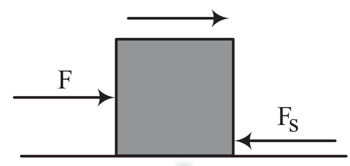


Fig 4.3(b) The body is moving

As the class ended, three of the fellows were involved in a deep conversation with the teacher in the conference room, probably clearing their doubts of the friction module. Arnold was having a taste of vanilla ice cream a few meters away from the parking space, while two of them, Aakash & Aditi, were sharing a light moment with each other on the bus. After a bit, Aditi said to Aakash in a low- pitched voice, "see they are coming."

*****

When the bus was passing the ice cream stall, Dolly wondered "why is the spot so crowded?" Arnold said, 'There is a boxing match to be held here this Sunday, and tickets are being sold on the black market.'

Aakash and Dolly suddenly shouted, 'stop the bus!' Aakash then got out of the bus and started running towards the crowd in a frantic manner to buy some tickets. At the same time, he was a little disappointed with Arnold, saying to himself expressively, "What the hell is this? He should have told us earlier about it."

However, he managed to get tickets for all of them. Upon entering, he exclaimed, "I got the tickets! I hope to see you all at boxing night!" Most of them felt thankful to Aakash for this gesture.

On the way to the colony, Dolly was a little stressed about the questions she had to solve that were given as homework. Later it was finalized that first they would gather at Aditi's second house to work on numericals together, especially those on Newton's laws and friction, and then would head towards the city on the same day (Sunday).

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Around 3pm they started solving problems one by one.

Example 4.5 What is the maximum acceleration of a box lying on the floor of a bus will remain stationary? Given that the coefficient of static friction between the box and the bus floor is 0.30.

Solution: The acceleration of the box is due to the static friction because friction opposes relative motion. Therefore, the friction provides force in the direction of the motion of the bus to keep it at rest with respect to the bus.

Hence, $ma = f_s \leq u_s N$ (m is mass of the box)

$ma_{max} = u_s mg$

$\therefore a_{max} = u_s g = .3 \times 10\, ms^{-2}$.

The solution is understandable to most but Kimaya had a hard time grasping it. She insisted on explaining it again in more detail. David said, "Okay," and began detailing the solution.

"When the bus is accelerating forwards, the box tends to move backward due inertia. As we know, friction opposes relative motion. To oppose this tendency of motion, friction provides acceleration in the forward direction to the box, so that there will be no relative motion. However, the frictional force has a limit up to which it can provide acceleration to keep the box at rest with respect to the bus.

That is $a_{max} = u_s g$ ($F_{max} = u_s N = u_s mg$, where friction is provided by the surface of the bus)."

Example 4.6 What is the tension of the string shown in Figure 4.4 ? Given the coefficient of kinetic friction between the trolley and the surface is .05, neglect the mass of the string.

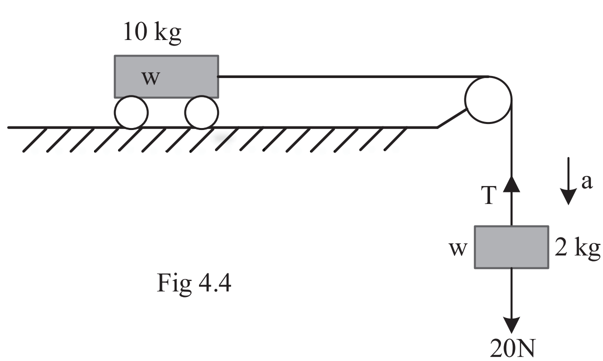
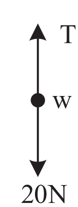


Fig 4.4

Solution: When a string is tight, it always has a force known as tension and it exerts this force to the body attached to the string.

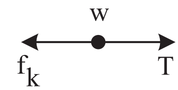


As the pulley is smooth, both the block and trolley have the same magnitude of acceleration.

Applying Newton’s second law of motion to the block

$20 - T = 2a$ ..(I)

Applying Newton’s second law to motion of the trolley



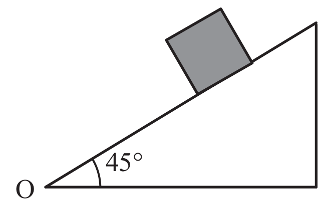
$T - u_k N = 10a,$

$T - (.05 \times 10 \times 10) = 10a \Rightarrow t - 5 = 10a$ …. (II)

By adding equation (I) and (II) we get, $12a = 15$, therefore $a = 1.25ms^{-2}$

By putting value of a in equation (I)

We get $T = 17.5\ N$.

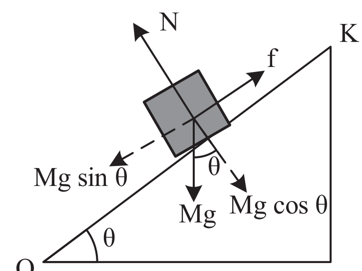


Example 4.7 Find the value of friction coefficient between block and the inclined for the body to just start sliding, if the angle of inclination is 45°.

Solution: $N = Mg\ cos\ cos\ 45°$ …(I)

$Mg\ \sin 45° = uN$ …(II)

(putting the value of N from eq (I) in eq (II)), we get $u = 1$.



(θ = 45°given)

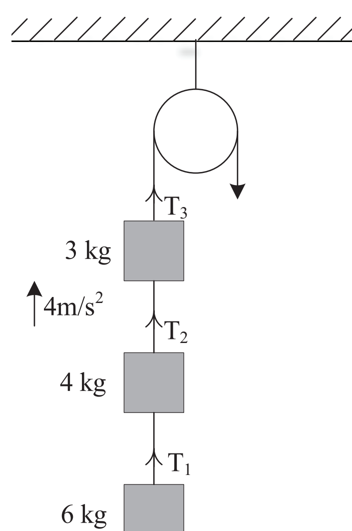
Example 4.8 There is a pulley mass system, find tension in the string as shown in the figure.

Solution: Acceleration is due to tension and is upwards in the left of the pulley and all the blocks will have the same acceleration. Therefore,

$T_1 - 60 = 24 \Rightarrow T_1 = 84N$

$T_2 - (40 + 84) = 16 \Rightarrow T_2 = 140N$

$T_3 - (T_2 + 30) = 12 \Rightarrow T_3 = 182N$



While everyone was putting their efforts into solving problems, even Aakash and Aditi duo, David and Kimaya were in a world of their own. Kimaya was eating Dairy Milk chocolate and repeatedly looking at David's face, and David simply enjoying her gaze. When Dolly noticed it, she conveyed through her expression to Kimaya, ‘what is going on?’ Both of them got shy and tried to concentrate on the study going on, but it was not easy for them.

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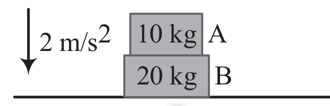
Example 4.9 There is a two-block system placed on a platform which is moving downward with an acceleration of $2m/s^2$. Then find the normal force on block by the platform.

Solution: The whole system has acceleration of $2m/s^2$,

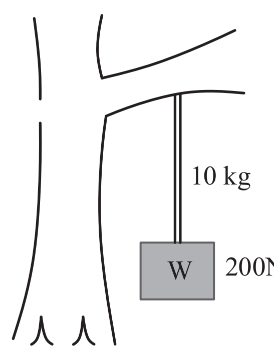
So, $100 - N_1 = 20,\ N_1 = 80$

$200 + N_1 - N_2 = 40 \Rightarrow N_2 = 240$



Aditi explained it thoroughly: “On the 10kg block, the gravitational force ($m_A g$) acts downwards, and the normal force (N_1) is exerted upwards by the surface of the 20kg block. On the 20kg block, its own weight ($m_B g$) acts downwards, and the normal force (N_1) from the 10kg block also acts downwards. The latter (N_1 downwards on the 20kg block) is according to Newton’s third law of motion, as the 10 kg block exerts an equal and opposite force on the 20 kg block and N_1 force also in downwards according to Newton’s 3rd law of motion.”

Example 4.10 There is a block of weight $200N$ which is hanging from a chain of mass $10kg$ uniformly distributed over it which is connected with a tree from top. Find the tension at the midpoint of the chain.



Solution: At middle the total force acting downward is $200\,N + 50\,N$ (as the mass is uniformly distributed)

This is tension in the upward direction at the point which is balancing downward force. Hence, $T = 250\,N$.

Ending the session, they were preparing to head off for the city. It was 6:30pm, and the match was to start at 7:30pm. After a while, all of them were on the bus. The driver was speeding the vehicle excessively. Actually, he was drunk this time. The whole way there were yells and funny sounds, such as ‘uncle calm down!’, ‘I am calm, baby’

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On reaching the destination, they first took him to pay and use a public restroom. After bathing him, they leaned him on the bus's front seat to help him regain consciousness. David then took the bus key.

However, an electrifying vibe there permeated the atmosphere: solid punches and captivating boxing drew their attention. Gita was also there, watching the show. When she saw the guys, she came closer to Aakash and, with a small smile asked "Hello, how are you doing?" 'I am fine, what about you?' Aakash responded. Mrs Gita: "Me too. You know, the principal has a special interest in you and your group. He frequently asks how interactive they are in the class and whether they are attending well or not."

Aakash: It is probably because we have not chosen a fully formal type of class. However, it is a compliment for us! Gita: Ok! How is your preparation going for the annual exam? Aakash: Good, but we can't take the pressure of anyone's expectation. Still, we are working almost everyday and will deal with it in the best possible way we can.

The next day would be Wednesday, and Monday, Wednesday and Friday were the days for the physics class. They were not in a mood to skip the upcoming class. So, just after the match, they saw the driver's condition and told him to depart for the village.

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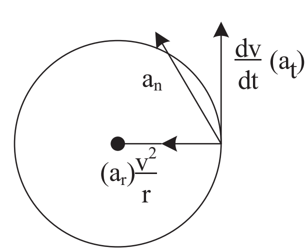
4.9 Circular Motion

We know the acceleration of a body moving in a circle of radius r with uniform speed v is v^2/r towards the center.

The force f_c providing this acceleration is $f_c = \frac{mv^2}{r}$.

Where m is the mass of the body and the force is always directed towards the center. Therefore it is called centripetal force.

Non-uniform Circular Motion



If the speed of the object in a circle is not constant, then the net acceleration has both the radial and tangential acceleration

And $a_t = \frac{dv}{dt}$

And the magnitude of the net acceleration is

$$a_n = \sqrt{a_r^2 + a_t^2}$$

$$= \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

A particle moves in a circle of radius $10cm$- its linear speed is given by $v = 2t + 2$, where, t is in second and v in meters/second. Find the radial and tangential acceleration at $T = 4s$.

Solution: The linear speed at $t = 4s$ is

$$V = 2(4) + 2 = 10m/s$$

The radial acceleration at $t = 3s$ is

$$a_r = \frac{v^2}{r} = \frac{100m^2/s^2}{.1m} = 1000m/s^2$$

The tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{d(2t+2)}{dt} = 2m/s^2$$

There are several practical examples in our daily life where we can observe circular motion and its different related terms like a_c (centripetal acceleration), centripetal acceleration is also known as radial acceleration sometimes denoted by a_r and centripetal force denoted by f_c.

(I) When we move a stone attached to a string with a constant velocity, the motion produced in this case is circular.

Here velocity direction at any instant is always perpendicular to the line connecting from the centre (radius). The acceleration on the stone is provided by tension in the string and is equal to $\frac{v^2}{r}$.

(II) When a car moves in a circular path on a flat road of a fixed radius (as shown in Figure a). The motion produced in this case is also known as circular motion on a level road. Here the centripetal force is provided by friction of the surface and this friction is static in nature because it opposes the impending motion of the car: trying to move away from the circle.

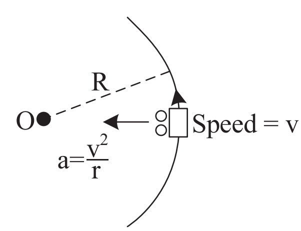


Figure (a)

We have $f = \frac{mv^2}{R} \leq u_s N$ ....(i)

Therefore, $v^2 \leq \frac{u_s RN}{m} = u_s Rg$ (by multiplying equation (i) by $\frac{R}{m}$)

Hence, $v_{max} = \sqrt{u_s Rg}$. (v_{max} to follow circular path of radius R)

(III) Motion of a car on a banked road

See in Figure b, the horizontal component of friction ($f cos\,\theta$) + the horizontal component of normal ($N sin\,\theta$) is equal to the centripetal force $\left(\frac{mv^2}{R}\right)$, $f cos\,\theta + N sin\,\theta = \frac{mv^2}{R}$ ....(i)

The Vertical component of normal force ($N\cos\theta$) is equal to the horizontal component of friction ($f\sin\theta$) + the force due to gravity (mg),

$N\cos\theta = f\sin\theta + mg$ ….(ii)

But $f \leq u_s N$ ….(iii)

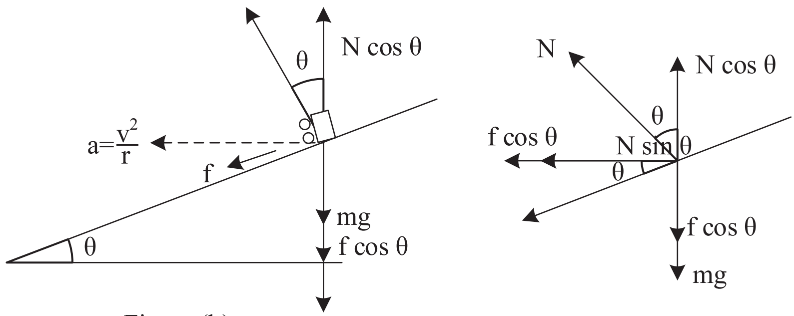


Figure (b)

Thus to obtain v_{max}, we put $f = u_s N$

Then equation (i) & (ii) become

$N\cos\theta = mg + u_s N\sin\theta$ …..(iv)

$N\sin\theta + u_s N\cos\theta = \frac{mv^2}{R}$ …..(v)

From equation (iv), we obtain $N = \frac{mg}{(\cos\theta - u_s\sin\theta)}$

Substituting value of N in equation (v), we get

$$\frac{mg(\sin\theta + u_s\cos\theta)}{(\cos\theta - u_s\sin\theta)} = \frac{(mv_{max}^2)}{R}$$

Or $v_{max} = \left(\frac{Rg(u_s + \tan\theta)}{(1 - u_s\tan\theta)}\right)^{1/2}$ ….(vi)

For $u_s = 0$ in equation (vi)

$v_0 = (Rg\tan\theta)^{1/2}$ …(vii)

At this speed, frictional force is not needed to provide the necessary centripetal force; only component of N provide the centripetal force. Driving at this speed on a banked road will cause little wear and tear of the tyres.

Equation (vii) also tells us that for $v < v_0$, frictional force will be up the slope and a car could be parked only if $\tan\theta \leq u_s$.

Example 4.11 A cyclist speeding at 5 km/h on a level road takes a sharp circular turn of radius $2m$ without reducing the speed. The coefficient of static friction between the tyres and the road is 0.8. Will the cyclist slip while taking the turn?

Solution: On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given by the equation: $v^2 \leq u_s Rg$

Now, $R = 2m$, $g = 9.8ms^{-2}$, $u_s = .8$, so $u_s Rg = 15.36\, m^2/s^2$.

$v = 9\, km/h = 2.5\, m/s, v^2 = 6.25\, m^2/s^2$. Since $v^2(6.25\, m^2/s^2) \leq u_s Rg(15.36\, m^2/s^2)$, the condition is obeyed. Therefore, the cyclist will not sleep while taking the turn.

Example 4.12 A circular racetrack of radius 100 m is banked at an angle of 30°. If the coefficient of friction between the wheels of a race-car and the road is 0.1, what is the (a) minimum speed of the racecar to avoid wear and tear on its tyres, and (b) maximum allowed speed to avoid slipping?

Solution: On a banked road, the horizontal component of the normal force and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the normal reaction's component is enough to provide the needed centripetal force. Therefore the frictional force is not needed. The optimum speed v_o is given by the equation: $v_o = (Rg\tan\theta\,)^{1/2}$. Here $R = 100m, \theta = 30°, g = 9.8\, ms^{-2}$. Hence, we have $v_o = \sqrt{558.6} = 23.63\, m/s$.

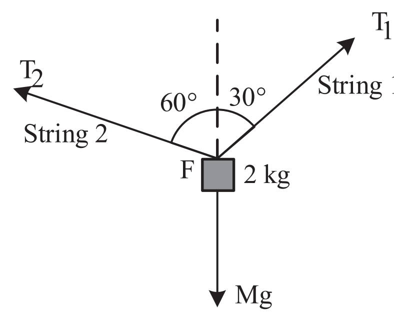
The maximum allowed speed v_{max} is $v_{max} = \left(\frac{Rg(u_x + \tan\theta\,)}{(1 - u_s \tan\theta\,)}\right)^{1/2} = 26.43m/s$.

Exercises

1. A small spherical object of mass $400gm$ is thrown vertically upwards. Give the direction and magnitude of the net force on the object, (a) during its upward motion, (b) during its downward motion, (c) at the highest point where it is at rest for a bit. (Neglect air resistance) (take $g = 10m/s^2$)
2. If the same spherical object is thrown at an angle of 60° with the vertical direction. Then does the direction and magnitude of the net force acting on the object change or not, (a) during its upward motion, (b) during its downward motion?
3. A constant force acting on a body of mass 4.0 kg, causing change in its speed from 2.0 ms^{-1} to 3.5 ms^{-1} in 15s. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force?
4. The driver of a bus moving with a speed of 18 km/h sees a lady standing in the road and brings his vehicle to rest in 5.0s just in time to

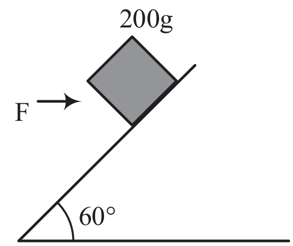
save the lady. What is the average retarding force on the vehicle ? The mass of the bus is 500 kg and the mass of the driver is 50 kg.

5. A rocket with a lift-off mass 10,000 kg is blasted upwards with an initial acceleration of 4.0 ms^{-2}. Calculate the initial thrust (force) of the blast.
6. Two masses 8kg and 12kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses, and the tension in the string when the masses are released.
7. Two bodies of equal masses 10 kg kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force $F = 400\ N$ is applied to (i) A, (ii) B along the direction of the string. What is the tension in the string in each case?
8. A body of mass 2 kg moving with velocity of $v_{in}\ 3i + \ 4j\ ms^{-1}$ enters into a constant force field of $6N$ directed along the positive z −axis. If the body remains in the field for a period of $\frac{5}{3}$ seconds, then velocity of the body when it emerges from the force field is …..
9. An object with mass 500 g moves along the x −axis with speed $v = 4\sqrt{x}\ m/s$. The force acting on the object is:
10. A body of mass 2 kg is suspended with the help of two strings making angles as shown in figure. Magnitudes of tensions T_1 and T_2 are : (take $g = 10m/s^2$)



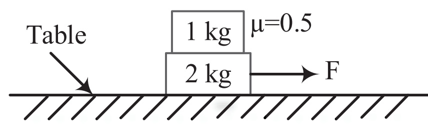
11. A player caught a cricket ball of mass 200gm moving at a speed of 40 m/s If the catching process is completed in 0.2 sec, the magnitude of force exerted by the ball on the hand of the player is:
12. A stone tied to a 180 cm long string at its end is making 28 revolutions in a horizontal circle every minute. The magnitude of acceleration of stone is $\frac{1936}{x}\frac{m}{s^2}$. The value of x is …… (Take $\pi = \frac{22}{7}$) ($\omega = 2\pi n$)
13. A horizontal force of 15 N is applied to a 3 kg block initially at rest on a rough horizontal surface. If the coefficient of kinetic friction between the block and the surface is 0.2. What is the acceleration of the block? Use $g = 10m/s^2$.
14. What is the minimum force required to start a 5kg block moving across a horizontal floor if the coefficient of static friction is 0.4?

15. A block of mass 200 g is kept stationary on a smooth inclined plane by applying a minimum horizontal force $F = \sqrt{x}\,N$ as shown in the figure. The value of x is ……



16. A force on an object of mass $100g$ is $(10i + 5j)N$. The position of that object at $t = 2s$ is $(ai\ +\ bj)m$ after starting from rest. The value of ab will be ___________.

17. The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is .... N. (Take $g = 10m/s^2$)



18. A body of mass 'm' is launched up on a rough inclined plane making an angle of 30 with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is _

Answers

1. (a) direction of the force is downwards and magnitude is 4 N (b) direction of the force is downwards and magnitude is 4 N (c) Same as in case a and b	2. No, it would be same as in case of question 1	3. 0.4 N, The net force acting on the body is in the direction of its motion	4. $-550N$
5. $1.38 \times 10^5 N$	6. $2m/s^2$	7. 200 N	8. $3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$
9. $4N$	10. $10\sqrt{3}$, 10	11. 40	12. 125
13. $3m/s^2$	14. 19.6 N	15. 12	16. 20000
17. 15	18. 3		

Chapter - 5
Work Energy and Power

The teacher called Aakash this morning to inform him that the class would be delayed by 2 hours and asked him to relay the information to the village classmates. ‘The weather is changing, take care of your health, see you soon,’ Gita added sweetly as she finished the call.

Aakash called Aditi, informing her and requesting to pass the information to the rest, as he was busy in the kitchen making tomato soup. Aditi called Aakash back shortly after and reported, “I informed everyone except Arnold, as he was not picking the call.” She added that she had seen him looking busy with a guest last night.

“Okay, I'll just leave a message on the fax machine as I can’t go to his home. There is no one in my house currently,” he said and hung up his landline phone.

*****

Everyone reached the class except Arnold. The teacher glanced around the middle row and simply launched into the day’s lesson.

5.1 Scalar Product

Before delving into the chapter deeply, we need to be aware of the term ‘Scalar Product’. We know how the same type of vectors are added, subtracted like displacement and displacement, velocity and velocity, but we have not come across how two different types of vectors are multiplied.

Here a term “Scalar product” is introduced : When two different types of vectors are multiplied and the resultant is a scalar quantity. The resultant is called a scalar product (or dot product). It is represented by . (dot), Like,

Force . Displacement = Work, we will say work is a scalar product of force and displacement. It has its own rule of multiplication.

Suppose there are two vectors A and B and θ is the angle between them. Then, their scalar product is $\boldsymbol{A}.\boldsymbol{B} = ABcos\theta = BAcos\theta$ …(5.1)

Equation (5.1) shows, the scalar product obeys the commutative law as $\boldsymbol{A}.\boldsymbol{B} = \boldsymbol{B}.\boldsymbol{A}$

It obeys the distributive law. $A.(B + C) = A.B + A.C$

For unit vectors I, J, K (Magnitude of 1 in the direction of x y and z respectively). We have $I.I = J.J = K.K = 1$, $I.J = J.K = K.I = 0$….(5.2)

If there are two vectors: $A = A_X I + A_Y J + A_Z K, B = B_X I + B_Y J + B_Z K$. The scalar product of A & B is

$$A.B = (A_X I + A_Y J + A_Z K).(B_X I + B_Y J + B_Z K)$$
$$= A_X B_X + A_Y B_Y + A_Z B_Z$$

Example 5.1 If two vectors $P = I + 2mJ + mK$ and $Q = 4I - 2J + mK$ are perpendicular to each other. Then the value of m will be:

Solution: $P.Q = (I + 2mJ + mK).(4I - 2J + mK) = 0$

$$\Rightarrow 4 - 4m + m^2 = 0 \quad \Rightarrow m = 2$$

Example 5.2 Find the angle between force $F = (3I + 4J - 5K)$ unit and displacement $D = (5I + 4J + 3K)$. Also find the projection of F on D.

Solution: We find the angle between two vectors using the dot product formula.

$$cos\,\theta = \frac{F.D}{|F||D|}$$
$$= \frac{(3I+4J-5K).(5I+4J+3K)}{|3I+4J-5K||5I+4J+3K|}$$
$$= \frac{16}{\sqrt{50}\sqrt{50}} \Rightarrow cos\,\theta = \frac{16}{50}$$

$\theta = \cos^{-1}(0.32)$, Projection of F on D

$$\Rightarrow \sqrt{50}\frac{16}{50} = 2.26 \text{ units}.$$

5.2 Work

If an object of mass m undergoes a displacement d in the positive x direction due to a constant force F acting on it (as shown in Figure 5.1). The work done by the force on the object is equal to the product of the component of the force in the direction of the displacement and the magnitude of the displacement.

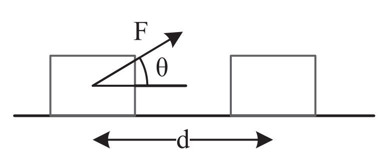


Fig 5.1

$$W = \boldsymbol{F}.\boldsymbol{d} = Fcos\,\theta\, d = Fdcos\,\theta \qquad \text{…(5.3)}$$

Work is a scalar quantity, which is a resultant of the dot product of two vectors namely force and displacement.

There are some facts we can configure from Equation 5.3 .

- There is no work done on a body by a force if displacement and the force are perpendicular to each other (It implies if there is a force acting but displacement happens in a perpendicular direction to the force, in this case there is no work done due to the force)
- There is no work done by a force if there is no displacement due to it.
- Work done can be positive or negative, if θ is between 0° and 90° the work done is positive, if θ is between 90° and 180° The work done is negative.
- In the case of kinetic friction, the friction force acts in the opposite direction to the body's displacement. In such a case the angle θ between force and displacement is 180°.

Example 5.3 A man lifts a suitcase weighing 10 kg from a platform and puts it on his head $4m$ above the platform. Calculate the work done by the man on the suitcase.

Solution: Work = Force × Displacement

$$= mg \times h$$

$$= 10kg \times 9.8m/s^2 \times\ 4\,m = 392\,J\ .$$

A girl sitting just in the row next to Aakash turned back and asked Aakash if he could help her understand the suitcase problem while casually arranging her hair. Aakash could have done it easily, but he felt uncomfortable talking with her. So he suggested she ask the teacher, acting as if he hadn't understood the concept clearly. The girl then asked him pointedly, "Okay, you ask or I ask?" Aakash responded, " you ask."

Upon being asked for a re-explanation, Gita first looked at Aakash and asked whether he understood it. This time, Aakash was in a dilemma. He wondered what the girl would think of him if he said 'yes'. If he said 'no,' it would be embarrassing in front of the whole class, considering he always acted like 'James Bond,' a guy known for his cool and confident style, and struggling to understand a simple solution would undermine that image. However, Aakash had realized a few times that the teacher, Gita, had some attraction to him and he thought she might have asked him because of this. He paused for a bit; but as soon as he thought of responding, the teacher said, "ok, I explain it from the basics."

"Every object of mass 'm' experiences a gravitational force 'mg' downwards. Therefore, the suitcase of mass 10 kg experiences a force of 98 N downwards. To raise it to height h, we need to push the suitcase upward with a force of 98 N for a height h. Therefore the product of the force and height is $98\,N \times 4\,m = 392\,J$. This is equal to the work done by the man because he is lifting the suitcase to height 4 m with a force of 98 N."

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Example 5.4 If a force $F = (2I + 4J)$ acts on a particle, and there is a displacement $d = (3I + 2J)$ due to it. Find the total work done due to the force.

Solution: we know that $W = F.D$

Given that $F = (2I + 4J), D = (3I + 2J)$

$$W = F.D = (2I + 4J).(3I + 2J)$$

$$\Rightarrow 6 + 8 = 14Joule$$

Example 5.5 A force of $F = (5y + 20)j\ N$ acts on a particle. The work done by this force when the particle is moved from $y = 0m$ to $y = 10m$ is ___________ J.

Solution: Given F = $(5y + 20)j$

$$W = \int F dy = \int_0^{10} (5y + 20) dy$$

$$= \left(\frac{5y^2}{2} + 20y\right)_0^{10} = \left(\frac{5}{2}\right) \times 100 + 20 \times 10 = 450J.$$

5.3 Kinetic Energy

Kinetic energy = The kinetic energy (KE) of an object with mass 'm' moving with velocity 'v' is given by the formula $KE = \frac{1}{2}mv^2$. It is a scalar quantity, and its SI unit is the joule (J).

Example 5.6 What will be the kinetic energy of a spherical ball of mass $100gm$ at $t = 4s$, having speed $20m/s$, acceleration 10 m/s^2?

Solution: we know $KE = \frac{1}{2}mv^2$

$$KE_{t4} = \frac{1}{2} m(v_{t4})^2$$

$$V_{t4} = 20 + 10(4) = 60m/s$$

Hence, $K_{t4} = \frac{1}{2} \times 0.1kg \times (60m/s)^2 = 180J$

5.4 Work-Energy Theorem

It states that the change in kinetic energy of an object is equal to the net work done on it.

$$K_f - K_i = W \qquad \text{.....(5.4)}$$

Where K_f and K_i are final and initial kinetic energies of the object.

Example 5.7 A ball of mass 200 gm is dropped from a building, it has a speed of 60 m/s when it hits the ground. (I) Calculate the work done by the gravitational force (II) Height of the building

Solution: (I) change in kinetic energy of the ball is

$$\Delta k = \frac{1}{2}mv^2 - 0,$$

$$= \frac{1}{2} \times .2\ kg \times (60\ m/s)^2$$

$$= 360\ J$$

Let the value of $g = 10m/s^2$

From the work-energy theorem

$\Delta K = w_g$ (work done due to gravitational force)

So $w_g = 360\ J$

(B) $W_g = \Delta k$

$$Mgh = 360\ J$$

$$0.2\ kg \times 9.8\ m/s^2 \times h = 360\ J$$

$$h = \frac{360J}{0.2\ kg \times 9.8 m/s^2} = 184\ m.$$

Example 5.8 As we know that a raindrop falls under the influence of gravity of the downward gravitational force and the opposite resistive force. Assume a drop of mass 10 gm falling from a height 2.00 km. It hits the ground with a speed of 100 m/s. What is the work done by the unknown resistive force?

Solution: The change in kinetic energy of the drop is

$$\Delta K = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2} \times 10^{-2} kg \times (100 m/s)^2$$

$$= 50\ J$$

From the work-energy theorem

$\Delta k = w_g + w_r$ (w_r is the work done by the resistive force on the raindrop)

$W_r = \Delta k - w_g (wg = mgh = 10^{-2} kg \times 9.8\ m/s^2 \times 2000\ m = 196\ J)$

$$W_r = 50J - 196J = -146\ J.$$

$-ve$ sign shows work done by the resistive force is against the motion of the raindrop or opposite to the direction of the motion of the raindrop.

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Upon Aditi's arrival after attending the class, her mother told her that Arnold had come and invited her to his home. Aditi asked her mother, “Is the invitation only for me, or for all the guys?” “I don’t know this! But you should go there” her mother responded. Aditi: “Ok mom.”

Within an hour, almost all of them (the guys) were in attendance at Arnold’s house. Arnold welcomed them with a sweet smile. Offering them tea, he said, “It’s just like a casual call, everything is fine now. However, last night I didn’t sleep properly. A tourist girl named Amila was robbed last evening while she was resting at the beach and was hit by a knife when she tried to follow the thief.” He continued, adding, “She was in a bad state when I found her. My parents and I have been looking after her since then, but now she is better and recovering.”

Currently, Amila was on the balcony on the right side of his house, drinking orange juice served by Arnold’s mother and examining the bandages on her wounds.

Suddenly, David's phone rang. The phone's ringing sound was “ Kiss me hard before you go/ Summertime Sadness………” It was his mother's call, which he rejected. The sound caught Amila's attention towards the dining room, where the group was discussing. Amila waved her hand and said, ‘Hi,’ with a weak voice. She then came over and introduced herself to them.

From the brief introduction, it became known that she majored in Physics from one of the oldest and finest institutions of the country. She was also a professional tennis player who retired at a very young age due to regular injury and some personal reasons. However, she seemed more interested in introducing herself as a Physics graduate rather than a former tennis player.

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As the rest of the group prepared to depart for their homes, Arnold informed them that he wouldn’t be able to attend any session this week. This was because Amila was staying at his home and she needed care, a task his mom had also instructed him to do.

Arnold was secretly listening to their conversation. Amila initially speechless with gratitude for their kindness and help, later stepped forward and offered “ if you guys have any problem academically, I will be more than happy to assist you,” pretending she hadn’t overheard their discussion. “If you teach the modules that are to be taught in our institution, it would be great, because we will be uncomfortable attending a few next academic sessions," Dolly told Amila. "Okay, I'll try," Amila responded. The guys were back in the dining room. Arnold: "There is no need for this, not all of you require to skip the sessions."

However, with a slight blush, Amila began teaching.

5.4 How to calculate the work done by a variable force?

To calculate the work done by a variable force, we cannot simply use the formula $F.d$, because the force is not constant over the entire displacement. Instead, we use integration.

Let a body move from x_i to x_f under a variable force $F(x)$ as shown in Figure 5.2.

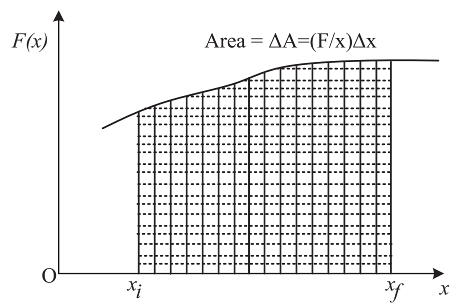


Figure 5.2

To find the total work done, we can first consider the work done over a very small displacement, Δx, at position x. If the displacement Δx is infinitesimally small, we can take the force $F(x)$ as constant during that displacement.

The small amount of work done, Δw, is then: $\Delta w = F(x)\Delta x$

And the total work done $W(total)$, is the sum of all such small amounts of work over the entire displacement from x_i to x_f:

$$W(total) = \sum_{x_i}^{x_f} F(x)\Delta x$$

$$= \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\Delta x$$

$$= \int_{x_i}^{x_f} F(x)\Delta x \; .... (5.5)$$

This total work done is also equal to the area under the force-displacement curve.

Example 5.9 A force $F = 2x + 4$ acts on a body, due to this the body displaces from $x = 2$ to $x = 4$. Find the total work done on the body by the force.

Solution: Given that $F = 2x + 4$, we know that $w = F.d$

Work done when the body displaces dx at $dw = (2x + 4)dx$

$$\int_2^4 (2x + 4)\, dx = \left[\frac{2x^2}{2} + 4x\right]_2^4$$

$= 28\,J.$

5.5 The Work-Energy Theorem for a variable force

We have come across the work-energy theorem of a constant force, now we will read about the work-energy theorem for a variable force.

$$\Rightarrow \frac{dk}{dt} = \frac{d\left(\frac{1}{2}mv^2\right)}{dt}$$

$$\Rightarrow m\left(\frac{dv}{dt}\right)v = Fv$$

$$\Rightarrow F\left(\frac{dx}{dt}\right)$$

Thus, $dk = Fdx$

Integrating from the initial position (x_i) to final position (x_f),

We have $\int_{k_i}^{k_f} dk = \int_{k_i}^{k_f} Fdx$ …..(5.6)

Where, k_i and k_f are the initial and final kinetic energies corresponding to x_i and x_f.

Or $k_f - k_i = \int_{k_i}^{k_f} Fdx$ …..(5.7a)

From Equation (5.5), it follows that

$k_f - k_i = W$….(5.7b)

5.6 Potential Energy

Every object in the world seeks to have a stable state. We can see it in our daily lives. For example, when a block attached to a spring is stretched, it gains some energy due to which when we leave it, it moves speedily and releases this energy. This energy is known as potential energy (potential energy of the spring)

Another example is, when we raise an object lying on the earth's surface to a height h, it again wants to be back at the same position. It gains some energy due to which it intends to go at its previous position. This is again potential energy (gravitational potential energy). It is stored in the object that can be released at a later time when we leave the object. It is often denoted by V. Formally, we can define gravitational potential energy as "The work done by an external force against the gravitational force in raising an object to height h is known as gravitational potential energy."

$V(h) = mgh$ ……(5.8)

If h is taken as a variable, the gravitational force f equals the negative of the derivative of $V(h)$ with respect to h, Thus, $f = -\frac{dV(h)}{dh} = -mg$ …(5.9)

$-ve$ sign indicates the force is in downward direction.

5.7 Conservative and Non Conservative Forces

Conservative Force: It is the force for which the work done on an object depends only on the initial and final position of the object. It implies that the work done in this case is independent of the path taken by the object. For example, gravitational force.

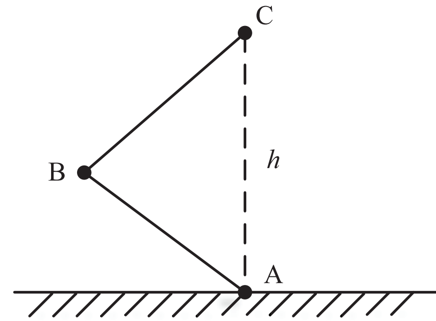


Fig 5.3(a)

Case (i) If an object moves from A to C under the influence of gravitational force F_g by following the path ABC, as shown in Figure 5.3 (a), the total work done by the gravitational force is $F_g.D = F_g h$ (as h is the total displacement in the direction of the force)

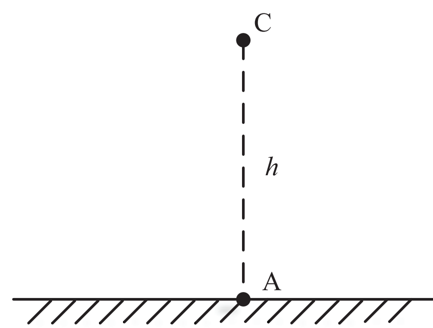


Fig 5.3(b)

Case (ii) If the object moves along the straight path AC, as shown in Fig 5.3 (b), the work done by the gravitational force is again $F_g h$, that is the same.

Conclusion: Since the work done by gravity is the same in both cases, this demonstrates that the work done by a conservative force is independent of the path taken.

Non-Conservative Force: It is the force for which the work done on an object depends on the path taken by the object. This implies that the work done is path-dependent. For example, frictional force. Let an object move from position d to f by following path '$decf$' as shown in Figure 5.3 (c). The total work done by the frictional force is the sum of work done along each segment:

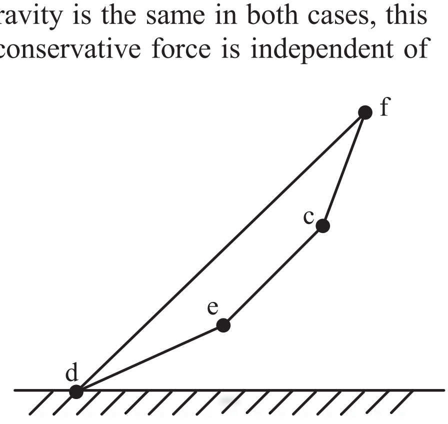


Fig 5.3(c)

$W_{decf} = W_{de} + W_{ec} + W_{cf} = u_k de + u_k ec + u_k cf$. If the object instead follows a direct path from d to f, the work done by the frictional force along is $W_{df} = u_k df$. Since the distance along path '$decf$' is greater than the distance along path 'df', the work done by the friction will be different. This illustrates that the work done by non-conservative forces is path-dependent.

5.8 The Conservation of Mechanical Energy

The conservation of mechanical energy states that the total mechanical energy (the sum of potential and kinetic energy) of a system is constant, provided that internal forces within the system are conservative and no external forces act upon it.

$K_i + V_i = K_f + V_f$…(5.10)

This implies that if we begin monitoring the total energy at $t = 0$, and if the kinetic energy decreases at a later instant, the potential energy must increase by an equal amount, and vice-versa. This ensures the total mechanical energy remains constant at any instant, as long as the conditions for conservation are met.

Example 5.10 A stone dropped from the roof of a bus if the stone hits the ground with a velocity of $10m/s$. Find the height of the bus.

Solution: let assume the mass of the stone be M, and height of the bus h

Then according to conservation of mechanical energy theorem, kinetic energy at bus roof + potential energy at bus roof = kinetic energy at ground + potential energy at ground

$$\Rightarrow 0 + M \times 10m/s^2 \times h = \frac{1}{2} \times M \times (10m/s^2)^2 + 0$$

$$\Rightarrow h = 5m$$

Example 5.11 Find minimum horizontal speed that should be given to the bob of a simple pendulum of length L so that it describes a complete circle?

Solution: let the horizontal speed given at bottom is v_o and it completes a vertical circle

Assume its speed at the highest point be v.

Taking gravitational potential energy zero at the bottom,

The conservation of energy gives $\frac{1}{2}mv_0^2 = \frac{1}{2} mv^2 + 2mgl$

$$mv^2 = mv_0^2 - 4mgl \qquad \text{…(i)}$$

The forces acting on the bob at the highest point are mg due to the gravity and T due to tension in the string. The resultant force towards the centre is $mg + T$.

The ball is moving in a circle, therefore its acceleration towards the centre is v^2/l.

Applying Newton's second law & using eq(i)

$$mg + T = \frac{mv^2}{l} = \frac{1}{l}(mv_0^2 - 4mgl)$$

$$mv_0^2 = 5mgl + Tl$$

Now, for v_o to be minimum, T should be minimum and the minimum value of T could be zero

So, for minimum speed, $mv_0^2 = 5mgl, v_0 = \sqrt{5gl}$

In the system, there is string, ball and earth (gravity).

Example 5.12 The stone attached to a string as shown in Figure 5.4 is able to complete the vertical circular motion. What is the ratio of kinetic energy at B to the kinetic energy at A?

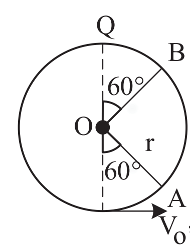
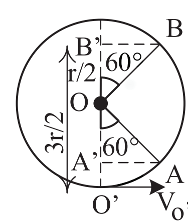


Fig 5.4

Solution: $OA = r$ (given)

$$OA' = r\cos 60° = \frac{r}{2}$$

$$\Rightarrow A'O' = r - \frac{r}{2} = \frac{r}{2} \;\&\; v_{o'} = \sqrt{5gl}$$



Now taking potential energy at bottom o' is zero.

The conservation of energy gives $\frac{1}{2}mv^2 = ke_a + \frac{mgr}{2}$

$$ke_a = \frac{1}{2}m\left(\sqrt{5gr}\right)^2 - \frac{mgr}{2} = 2mgr$$

Again conservation of energy at b gives $ke_b + mgoB' = \frac{1}{2}mv^2$

$$ke_b = \frac{1}{2}mv^2 - \frac{mg3r}{2} = mgr$$

So, $ke_b/ke_a = \frac{mgr}{2mgr} = 1:2$

Example 5.13 Find minimum horizontal speed that should be given to the bob of a simple pendulum of length L so that it describes a complete circle?

Solution: let the horizontal speed given at bottom is v_0 and it completes a vertical circle

Assume its speed at the highest point be v.

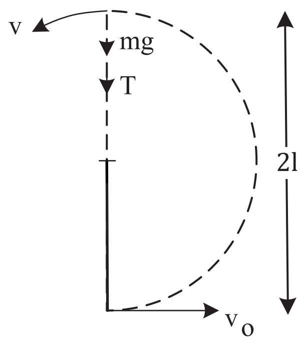
Taking gravitational potential energy zero at the bottom,

The conservation of energy gives $\frac{1}{2}\ mv_0^2 = \frac{1}{2}\ mv^2 + 2mgl$

$mv^2 = mv_0^2 - 4mgl$ ...(i)

The forces acting on the bob at the highest point are mg due to the gravity and T due to tension in the string. The resultant force towards the centre is $mg\ +\ T$.

The ball is moving in a circle, therefore its acceleration towards the centre is v^2/l.



Applying Newton's second law & using equation(i)

$mg\ +\ T = \frac{mv^2}{l} = \frac{1}{l}\ (mv_0^2 - 4mgl)$

$mv_0^2 = 5mgl\ +\ Tl$

Now, for v_o to be minimum, T should be minimum and the minimum value of T could be zero

So, for minimum speed, $mv_0^2 = 5mgl, v_0 = \sqrt{5gl}$.

Example 5.14 A tiny piece ball is kept at the point A of a frictionless path ABC as shown in Figure 5.5. It is pushed slightly towards the right. What will be its speed when it reaches point B?

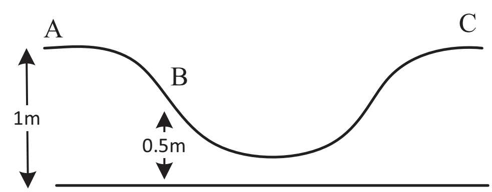


Fig 5.5

Solution: Let us take the gravitational potential energy is 0 at the horizontal surface shown in the figure.

By conservation of energy $U_A + K_A = U_B + K_B$

$Mg(1m) = Mg(0.5m) + \frac{1}{2}\ MV_B^2$

$\Rightarrow \frac{1}{2}V_B^2 = g(1m - 0.5m)$

$\Rightarrow (10m/s^2)\ \times 0.5m$

$\Rightarrow 5m^2/s^2$

$\Rightarrow V_B = \sqrt{10}m/s$

The class was going long, although Amila was enjoying it. It was looking like she had forgotten everything and chosen a new career as 'Physics teacher'. However, David, with a low sound said to Arnold, "could I tell her to pause the class for today, although her way of teaching is wonderful, my mind is already overloaded with concepts today." Arnold replied, 'Yeah, sure, why not.'

But at the same time, Aditi told him to hold on for a bit and let her ask Amila to explain a few solutions of the problems based on 'conservation of mechanical energy' that had been solved recently. When Amila learned of Aditi's concern, she pointed to the first question and its solution, which she had worked out in Arnold's rough notebook and began to explain as follows:

"For such problems, we assume the earth is also part of the system. In this specific problem, the stone and Earth are considered as two particles of a single system. We assume there are no external forces acting on the system, and the internal force of gravity is conservative.

Note that there is contact of hand with the ball as well but we analyze the motion after the hand has released the stone, so no force is exerted by the hand during the period of observation."

Arnold asked Amila, 'Can the rest of the modules be taught the next day?' Dolly replied teasingly, 'Don't say the next day; it will be the next day of the next day. We are already loaded with many concepts in our memory today.' "Ok, I leave you all but I am eager to get feedback on my teaching, can I get it?" Amila said quietly. Almost everyone was satisfied and happy with the style of her teaching. Amila was blooming knowing this.

After the session concluded. The guys were talking silently among themselves, while Amila was engaged in conversation with Arnold's Mother. Meanwhile, Aditi suggested calling a good doctor and a motorist from the city to assess Amila's physical condition and the state of her car, which she had used to travel there to enjoy the coastal village life.

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The guys headed for the city. However, it was hard for them to convince any doctor to come to the village. But they managed to secure the services of a good physician by offering plenty of money.

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The doctor changed Amila's bandages, gave her some medicine, and advised against travelling for at least a month.

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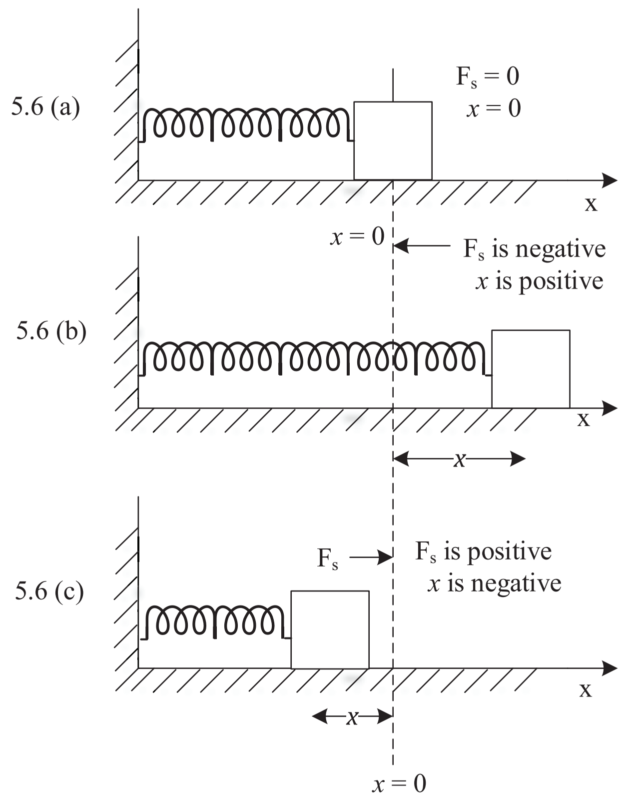
The whole group was getting benefitted largely from Amila's Physics knowledge. It was Friday evening and they again gathered according to their plan to complete some part of their syllabus. Amila was already ready to make them learn. She embarked the class with the topic 'The potential energy of a spring.'

5.9 The potential energy of a spring



To know about the potential energy of a spring, we need to be aware of spring force, which is a variable and conservative force.

To understand it, we assume a block is attached to a wall by a massless spring on a smooth horizontal surface, as shown in Figure 5.6 (a). When the block is compressed or extended from the equilibrium position, there is always a force on the block towards the equilibrium position by the spring which is given as $F_s = -kx$…..(5.11)

In other words, F_s is proportional to x, where x is displacement of the block from the equilibrium position. The constant k is called the spring constant and this force law of spring ($F_s = -kx$) is called Hooke's law. Its unit is Nm^{-1}. In Figure 5.6 (b), x is positive (extension: displacement is positive), the spring force is in the negative direction, pulling the block towards the equilibrium position. In Figure 5.6 (c) x is negative (compression: displacement is negative), the spring force is in the positive direction, pushing the block towards the equilibrium position. Hence in both cases (b) & (c), force direction is towards the equilibrium position .

Suppose, we pull the block outwards as shown in Figure b, and if the extension is $x_m, x > 0$, the work done by the spring force is $w_s = \int_0^{X_m} F_s \,.\, dx = \int_0^{X_m} (-kx)\, dx$

$= -\frac{kx_m^2}{2}$ ....(5.12)

The same is true when the spring is compressed with a displacement $x_c (< 0)$. The spring force does work $-\frac{kx_c^2}{2}$.

Here are some key points to note for solving questions based on this topic.

- If the block is moved from an initial position x_i to a final position x_f, the work done by the spring force w_s is $w_s = \int_{x_i}^{x_f} kxdx = \frac{kx_i^2}{2} - \frac{kx_f^2}{2}$....(5.13)
- The work done by the spring force in a cyclic process is 0 because it depends on initial and final position, and in this case both are the same. Therefore the spring force is conservative.
- The potential energy of the spring is zero, when the block and spring system is in the equilibrium position. It is given as $v(x) = \frac{kx^2}{2}$...(5.14)
- External force always do positive work (F_{ext} will be always against F_{int}, here F_{int} is spring force)
- Suppose the block of mass m is extended to x_m and released. Then its total mechanical energy at any point x is constant, where x lies between $-x_m$ to x_m and is given as $\frac{1}{2} kx_m^2 = \frac{1}{2} kx^2 + \frac{1}{2} mv_x^2$...(5.15)
- One fact can be figured out from the conservation of total mechanical energy. That is, kinetic energy is zero when potential energy is maximum and vice versa.

The speed and kinetic energy will be maximum at the equilibrium position $x = 0$, and Equation (5.15) becomes $\frac{1}{2} mv_m^2 = \frac{1}{2} kx_m^2$.

Therefore $v_m = \sqrt{\frac{k}{m}} x_m$.....(5.16)

Example 5.15 If a spring of mass $10\ kg$, spring constant $4 \times 10^3\ Nm^{-1}$ and is moving with a velocity $5m/s$ at $x = 1m$. What would be its maximum compression?

Solution: Let maximum compression of the spring = x, then total mechanical energy = $\frac{kx^2}{2}$ …(I)

Total mechanical energy at $x = 1m$

$\Rightarrow \frac{1}{2} \times 10kg \times 25m^2/s^2 + \frac{4\times10^3 Nm^{-1}\times1m^2}{2}$ ….(II)

By conservation of Mechanical energy

$\frac{kx^2}{2} = (5\ kg\ \times\ 25\ m^2/s^2) + \frac{4\times10^3 Nm}{2}$

$\Rightarrow x^2 = \frac{2125}{2000} = 1.0625$

$\Rightarrow x = 1.03m$

Example: A mass of block $5\ kg$, spring constant $2 \times 10^3 Nm^{-1}$ and is moving with a velocity of $10m/s$ at $x = 0.5m$. Find maximum extension of the spring.

Solution: Let maximum extension of the spring = x, At x, total mechanical energy = $\frac{kx^2}{2} = \frac{2\times10^3 Nm^{-1}\times x^2}{2}$

Total mechanical energy at $x = 0.5m$

$$\frac{1}{2} \times 2 \times 10^3 Nm^{-1}(0.5m)^2 + \frac{1}{2} \times 5\ kg \times 100\ m^2/s^2$$

By conservation of mechanical energy

$\frac{2\times10^3 Nm^{-1}(x^2)}{2} = \frac{1}{2} \times 5 \times 10^3 Nm^{-1} \times (0.5)^2$

Solving it we get $x = 0.93m$

Example 5.16 If a car of mass $1000kg$ moving with a speed 36 km/h on a smooth road and colliding with a horizontally mounted spring of spring constant $5.25 \times 10^3\ Nm^{-1}$. What is the maximum compression of the spring?

Solution: it will follow hooke's law (spring law)

At maximum compression the kinetic energy of the car is converted entirely into the potential energy of the spring.

The kinetic energy of the moving car is $k = \frac{1}{2}\ mv^2$

$\frac{1}{2} \times 10^3 \times 10 \times 10 = 5 \times 10^4\ J$

At maximum compression x_m, the potential energy v of the spring is equal to the kinetic energy k of the moving car from the principle of conservation of mechanical energy.

$$v = \frac{1}{2}\, kx_m^2 \quad \Rightarrow \; 5 \times 10^4 J = \frac{1}{2} \times 5.25 \times 10^3 Nm^{-1} x_m^2$$

$$\Rightarrow x_m^2 = \frac{5 \times 10^4}{5} \times 10^2 \times 5.25 = 4.36\, m$$

5.10 Power

It is defined as the rate at which work is done. It is calculated by dividing the work done by the total time taken to do that work. $P = \frac{work(w)}{time(t)}$ ...(5.17), its SI unit is watt (w)

Instantaneous Power: It is the amount of power consumed or the rate of work done at a specific moment or time. $P = \frac{dw}{dt}$....(5.18)

The work, dw, done by a force F for a displacement dr is $dw = F.dr$. The instantaneous power at time t is $P = F.\frac{dr}{dt} = F.V$....(5.19) where V is the instantaneous velocity and dt approaches 0.

Power is a scalar quantity, its SI unit is j/s.

Example 5.17 An electric vehicle is carrying a ground roller on a smooth cricket ground. If power generated by the vehicle is $1000\, watt$ & speed is $10\, m/s$. Find the force exerted on the system.

Solution: We know that $P = F.V$

$\Rightarrow 1000\, watt = F \times 10\, m/s$ (P is a scalar product of two vectors F & V)

$$\Rightarrow \frac{1000\, watt}{10\, m/s} = 100\, N$$

5.11 Collisions

(I) when two or more objects come into contact and exert force on each other for a short period of time. This process is called collision.

(II) But here we will read collision of mainly two objects

(III) The laws of momentum and energy conservation are widely used here for solving problems

(IV) Examples of collisions are carrom: when a striker hits a coin, a bat hits a baseball.

There are mainly two types of collisions

(I) Elastic Collision (II) Non-Elastic Collision

In both collisions, the total linear momentum is conserved. When two objects collide, there are impulsive forces on colliding objects for a very short period of time Δt, which is also called collision time. Due to it there is a change in their respective momenta: suppose there are two objects o_1 and o_2, after collision change in momenta of o_1 is Δp_1

$\Delta p_1 = F_{12}\Delta t$, same way there is change of Δp_2 in particle o_2

$\Delta p_2 = F_{21}\Delta t$, Where F_{12} is the force exerted on the first object by the second object and F_{21} is the force exerted on the second object by the first object. $F_{12} = -F_{21}$ (By newton's third law) Therefore $\Delta p_1 + \Delta p_2 = 0$

Elastic Collision: The collision in which initial kinetic energy (before collision) is equal to the final kinetic energy (after collision), is called elastic collision.

Example - when a ball at a billiard table hits another ball, when a ball is thrown on the ground and it bounces back. In these examples, there is no net change in kinetic energy of the system.

Inelastic collision: A collision in which initial kinetic energy is not equal to the final kinetic energy. Some part of the initial kinetic energy is lost in the form of heat, sound etc. If two colliding objects move together after the collision, the collision is called a completely inelastic collision. Otherwise, the collision is of intermediate case where the deformation is partly relieved and some of the initial kinetic energy is lost, is more common and is commonly called an inelastic collision.

- **Collision in one dimension**

First we consider an inelastic collision in one dimension, we assume m_2 is at rest initially $\theta_1 = \theta_2 = 0$ as shown in Figure 5.7 (angle between both objects before collision is equal to the angle between two objects after the collision)

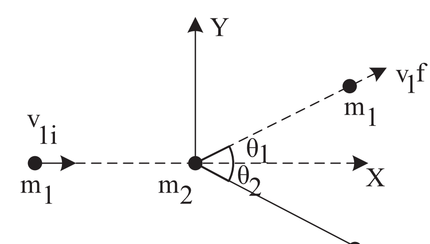


Fig 5.7

$m_1 v_{1i} = (m_1 + m_2)v_f$ (by conservation of momentum)

$v_f = \left(\frac{m_1 v_{1i}}{m_1+m_2}\right)$ ...(I) (v_f is the velocity of the object after collision)

The loss in kinetic energy on collision is

$= \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}\frac{m_1^2}{m_1+m_2} v_{1i}^2$ (using eq I)

$= \frac{1}{2}\, m_1 v_{1i}^2 \left[1 - \frac{m_1}{m_1+m_2}\right]$

$= \frac{1}{2}\left(\frac{(m_1 m_2)}{(m_1+m_2)}\right) v_{1i}^2$, which is a positive quantity as expected.

Elastic Collision

Here again the case of $\theta_1 = \theta_2 = 0$, the momentum and kinetic energy conservation equations are

$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$ ...(i)

$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$ ....(ii)

From eq (i) and (ii), it follows that

$m_1 v_{1i}(v_{2f} - v_{1i}) = m_1 v_{1f}(v_{2f} - v_{1f})$

Or, $v_{2f}(v_{1i} - v_{1f}) = v_{1i}^2 - v_{1f}^2$

$= (v_{1i} - v_{1f})\,(v_{1i} + v_{1f})$

Hence, $v_2 f = (v_{1i} + v_{1f})$ ....(iii)

Substituting this in equation (i), we obtain

$v_{1f} = \frac{(m_1 - m_2)v_{1i}}{m_1+m_2}$ ....(iv)

And $v_{2f} = \frac{2m_1 v_{1i}}{m_1+m_2}$ .....(v)

Thus, the 'unknowns' $\{v_{1f}, v_{2f}\}$ are obtained in terms of the 'knowns' $\{m_1, m_2, v_{1i}\}$.

Special cases of our analysis are interesting.

Case I: If the two masses are equal $v_{1f} = 0, v_{2f} = v_{1i}$

The first mass comes to rest and pushes off the second mass with its initial speed on collision.

Case II: If one mass dominates e.g. $m_2 >> m_1$

$v_{1f} \cong -v_{1i}\;\; v_{2f} \cong 0$

The heavier mass is undisturbed while the lighter mass reverses its velocity.

Collisions in Two Dimensions

We know that momentum is a vector quantity $P = mV$, and is conserved in nature, that is known as law of conservation of momentum. Therefore in a collision, the initial momentum is equal to the final momentum unless and until there is no net external force acting on the system. Here suppose one ball of mass m_1 coming with velocity v_{1i} collides with another ball of mass

m_2 which is at rest. After collision, suppose the m_1 ball makes angle θ_1 from v_{1i} and m_2 makes angle θ_2 from v_{1i} as shown in the figure .

Here initial momentum = $m_1 v_{1i}$

final momentum = $m_1 v_{1f} cos\ \theta_1\ + m_2 v_{2f} cos\ \theta_2$

$m_1 v_{1i} = m_1 v_{1f} cos\ \theta_1\ + m_2 v_{2f} cos\ \theta_2$ ……..(vi)

Since, there is no net momentum in y direction before the collision so after collision the net momentum must be zero in y direction as well according to conservation of momentum. Therefore, $0 = m_1 v_{1f}$ $\sin\theta_1 - m_2 v_{2f} \sin\theta_2$ …..(vii)

If $\theta_1 = \theta_2 = 0$, we get back equation (i) for one dimensional collision. And if the collision is elastic,

$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2}\ m_1 v_{1f}^2 + \frac{1}{2}\ m_2 v_{2f}^2$ ….(viii)

Example 5.18 Consider the collision depicted in Figure 5.7 to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue while the second ball is called the target. The billiard player wants to 'sink' the target ball in a corner pocket, which is at an angle $\theta_2 = 37°$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ_1

Solution: $v_{1i} = v_{1f} + v_{2f}$

Or, $v_{1i}^2 = (v_{1f} + v_{2f}).(v_{1f} + v_{2f})$

$= \ v_{1f}^2 + v_{2f}^2 + 2v_{1f}.v_{2f}$

$= \ v_{1f}^2 + v_{2f}^2 + 2v_{1f}.v_{2f} cos\ (\theta_1 + 37°)$ ....(i)

Since the collision is elastic and $m_1 = m_2$ it follows from conservation of kinetic energy that $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$ ….(ii)

Comparing equation i and ii, we get

$cos\ (\theta_1 + 37°) = 0$

Or, $\theta_1 + 37° = 90°$

Thus $\theta_1 = 53°$.

Exercises

1. Have a look at these cases and tell whether the work done is positive, negative or zero.
   (a) The work done by kinetic friction on an object
   (b) The work done by static friction on an object

(c) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket

2. The negative of the work done by the conservative internal forces on a system equals the change in

   (a) Total energy (b) Potential energy

   (c) Kinetic energy (d) None of these

3. The total work done on a particle is equal to change in its kinetic energy
   (a) always
   (b) only if gravitational force alone acts on it
   (c) only if the forces acting on it are conservative
   (d) none of these

4. When a person lifts a suitcase from the floor and keeps it on a table. The work done by you on the suitcase does not depend on
   (a) The path taken by the suitcase
   (b) The time taken by the person doing so
   (c) The weight of the suitcase
   (d) Your weight

5. An object of mass 4 kg initially at rest moves under the action of an applied horizontal force of 10 N on a table with coefficient of kinetic friction = 0.2. Calculate the (a) work done by the applied force in $10\ s$, (b) work done by friction in $10\ s$, (c) work done by the net force on the body in $4s$

6. A block of mass 2 kg is pulled up on a smooth incline of angle 30° with the horizontal. If the block moves with an acceleration of $1m/s^2$, find the power delivered by the pulling force at a time $4s$ after the motion starts. What is the average power delivered during the $4s$ after the motion starts?

7. A force $F = (10 + .50\ x)$ acts on a particle in the x direction, where F is in Newton and x is in meter. Find the work done by this force during a displacement from $x = 0$ to $x = 2m$.

8. A body dropped from a height H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate the work done by air-friction.

9. A force $(3x^2 + 2x - 5)N$ displaces a body from $x = 2m$ to $x = 4m$. Calculate the work done by this force is ………J.

10. A body of mass $5kg$ is moving with a momentum of $10kgms^{-1}$. Now a force of $2N$ acts on the body in the direction of its motion for $5s$. The increase in the kinetic energy of the body is ………..J

11. A block of mass 25 kg is pulled along a horizontal surface by a force at an angle 45° with the horizontal. The friction coefficient between the block and the surface is 0.25 . The block travels at a uniform velocity.

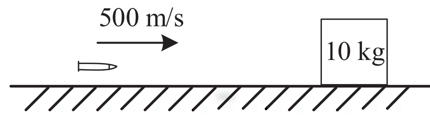
The work done by the applied force during a displacement of $5m$ of the block is :

12. A ball of mass $100g$ is dropped from a height $h = 10cm$ on a platform fixed at the top of a vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $\frac{h}{2}$. The spring constant is ____________ NM^{-1}. (use $g = 10m/s^2$)
13. A 0.5 kg block moving at a speed of $12\,m/s$ compresses a spring through a distance $30cm$ when its speed is halved. The spring constant of the spring will be ________________ Nm^{-1}.
14. A block moving horizontally on a smooth surface with a speed of $40m/s$ splits into two equal parts. If one of the parts moves at $60m/s$ in the same direction, then the fractional change in the kinetic energy will be $x : 4$ where $x =$____________. (Note : apply conservation of momentum and mechanical energy)
15. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angle 45° and 60°with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ....
16. A ball of mass $4kg$, moving with a velocity of $10ms^{-1}$, collides with a spring of length $8m$ and force constant $100Nm^{-1}$. The length of the compressed spring is $x\,m$. The value of x, to the nearest integer, is ______________.
17. A body of mass 4 kg is placed on a plane at a point P having coordinate $(3, 4)m$. Under the action of force $F = (2i + 3j)N$, it moves to a new point Q having coordinates $(6, 10)m$ in 4 sec . The average power and instantaneous power at the end of 4 sec are in the ratio of :
18. A ball having kinetic energy KE, is projected at an angle of 60° from the horizontal. What will be the kinetic energy of ball at the highest point of its flight?
19. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass is

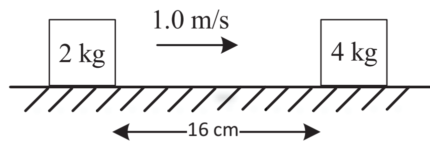
    (a) $\frac{1}{2}kx^2$ (a) $-\frac{1}{2}kx^2$ (c) $\frac{1}{4}kx^2$ (b) $-\frac{1}{4}kx^2$
20. The negative of the work done by the conservative internal forces on a system equals the change in

    (a) total energy (b) kinetic energy
    (c) potential energy (d) none of these
21. A bullet of mass 20g travelling horizontally with a speed of $500m/s$ passes through a wooden block of mass 100kg initially at rest on a level

surface. The bullet emerges with a speed of $100 m/s$ and the block slides $20 cm$ on the surface before coming to rest. Find the friction coefficient between the block and the surface (figure)



22. The friction coefficient between the horizontal surface and each of the blocks shown in figure is 0.20. The collision between the blocks is perfectly elastic. Find the separation between the two blocks when they come to rest. Take $g = 10 m/s^2$.



Answers

1. (a) Negative (b) zero (c) positive	2. b	3. a	4. a,b,d
5. (a)270J (b) -211.68J (c) 9.33J	6. 24 w	7. 21J	8. $-0.28mgH$
9. 58	10. 30	11. 245	12. 120
13. 600	14. 1	15. 2	16. 6
17. 6:13	18. $\frac{KE}{4}$	19. (d)	20. (c)
21. 10 cm	22. 5 cm		

Chapter - 6
System of Particles and Rotational Motion

It was Monday and the group were heading to the school to attend the class. It was known among them through a phone call with a non-teaching staff member, that the chapter 'System of Particles and Rotational Motion' was scheduled to start that day. However, the conversation on the way to the city was dominated by thoughts of Amila, who was still residing at Arnold's house. The group had learned from some sources that she won several knockout matches and had been on the front pages of several newspapers and magazines during her tennis career, even though she always preferred to lead a private life.

*****

Dolly expressly said, "And in addition to this she must have been a unique & hard-working student, that's why she is still able to teach concepts clearly in her own style that is quite understandable. She could be an asset to us especially when any topic is challenging and difficult to grasp either by the formal class or by our own understanding."

*****

When they reached the classroom, many of the students glanced towards them, and some started talking with them happily. It was a nice environment; it seemed that many students had missed them, much like how a team misses its good players in crucial matches. This was especially noteworthy as the guys typically kept to themselves during class.

*****

Gita was looking a little fatigued and sleepy that day, but she had to start the session, and so she did.

6.1 Basics

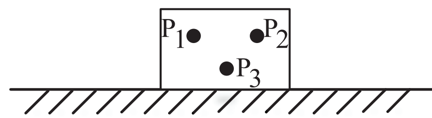
In all of the previous chapters, we assumed all objects as point mass (having no size) but we included the objects having finite size in this category as well.

For example, when a block of mass $4kg$ moves with velocity $10m/s$ in x direction from origin 0. What will be its position after 4 seconds?

Solution: By kinematics equation, we calculate the position of the object is: $x = 40m\ after\ 4s$. But the question is that, is every part of this object is at

$x = 40m$. The answer is obviously no, its back part position is less than $40m$ (i.e. $x < 40m$), front part is more than $40m$ (i.e. $x > 40m$). However we have assumed $40m$ for the whole object: considering the whole mass as a single point mass.

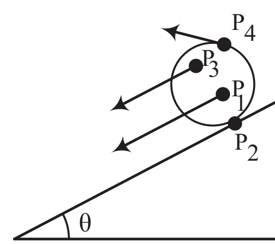
But in this chapter we are going to deal with such small infinitesimal parts within a body as well. Such a body is usually considered as a rigid body. Formally, we can define a rigid body as an extended solid body having a number of small particles. An extended body is considered as a system of a number of particles.



Let's have a look at the types of motions a rigid body follows:

Pure Translational motion: In pure translational motion, all particles of the body have the same velocity and move in a straight line at any instant.

For example, consider a block moving on a frictionless surface. At any instant, particles p_1, p_2 and p_3 will have the same velocity.

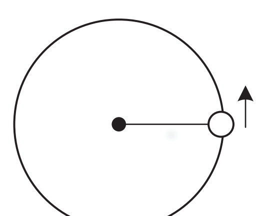


Consider the rolling motion of a wooden cylinder down an inclined plane (as shown in the figure). Here linear velocities at p_4 p_3 p_2 are not equal ($p_4 \neq p_3 \neq p_2$). Here particles move translationally and something else.

This something else is none other than rotational motion, about which we are going to read in detail in this chapter.

Students often get confused between Circular motion and Rotational motion. Here let be cleared with these terms.

The most important difference between these two terms is that a body performing a circular motion rotates around an axis which is outside of the body, while in rotational motion the body rotates around an axis that is inside the body. For example, when a stone attached to a string undergoes a motion. The motion described here is known as circular motion. But the motion described by an electric fan when we on its switch, is rotational motion. When blades of an electric fan start to run, it is an example of rotational motion.

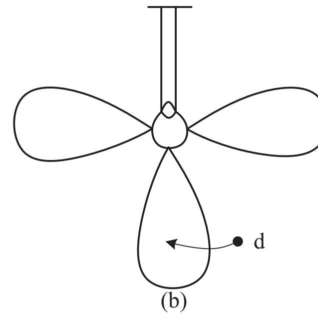


(a)

There are a few facts to note for a rigid body performing rotational motion.

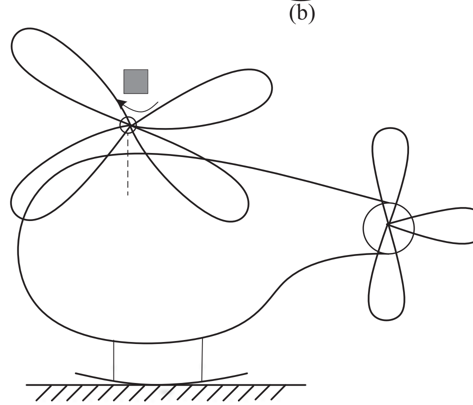
(i) The axis of rotation must pass through a point that lies within the body.

(ii) A body could have rotational motion as well as translational motion at the same time, but in such a case the axis of rotation is not fixed



For example, a circular disc performs a rolling motion on a smooth surface

The fan shown in the figure is performing pure rotational motion because it has a fixed axis of rotation d about which it performs motion. However, in the case of a circular disc the axis of rotation is not fixed because it is performing a translational motion as well, but the axis lies within the body, which is a basic feature of rotational motion.



Hence, a rigid body typically performs three types of motion: (i) pure translational motion (ii) pure rotational motion or (iii) a combination of both, translational and rotational motion.

6.2 Centre of Mass

Now, we need to become familiar with the term 'centre of mass' and its importance. The centre of mass is a crucial concept in the study of the motion of a rigid body.

Let's consider a system of two particles located at positions x_1 and x_2 from an origin o, with masses m_1 and m_2, respectively, as shown in Figure 6.1. The position of the centre of mass (x_{com}) of this system is given by: $x_{com} = \frac{(m_1x_1+m_2x_2)}{m_1+m_2}$ …(6.1)

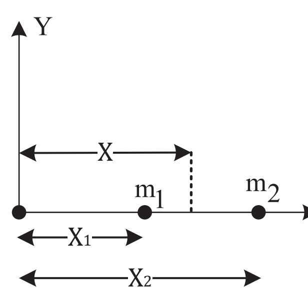


Figure 6.1

Similarly, if we have n particles of masses $m_1\, m_2\, m_3\, \ldots..m_n$ respectively along the x axis. The position of the centre of mass of the system of the particles is given by: $x'_{com} = \frac{(m_1x_1+m_2x_2+m_3x_3+ \ldots.+ m_nx_n)}{m_1+ m_2+m_3 + \ldots..+ m_n} =$

$\frac{\sum_{i=1}^{n} m_1 x_1}{\sum_{i=1}^{n} m_1}$....(6.2) $x_1 x_2 x_3 \ldots\ldots x_n$ are the distances of the particles from the origin.

$\Sigma m_1 = M, M$ is the total mass of the system.

Let $r_1\,(x_1 i + y_1 j + z_1 k)$ and $r_2 (x_2 i + y_2 j + z_2 k)$ be the position vectors and masses are m_1 and m_2 of two particles p_1 and p_2 respectively. Let R be the position vector of the centre of mass of the particles p_1 and p_2, then R is given as: $R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$ ....(6.3), similarly for n particles we find the position vector of the centre of mass $R' = \frac{\sum_{i=1}^{n} m_i r_i}{M}$.....(6.4), M is the total mass of the system of particles.

All the above equations are applicable to a rigid body and a rigid body is just a system of a large number of closely packed particles. The number is so large that it is impossible to carry out the summations over individual particles. Since it is closely packed, we can treat the body as a continuous distribution of mass.

We subdivide the body into n small elements of mass $\Delta m_1, \Delta m_2, \ldots \ldots \Delta m_n$. Let the i[th] element Δm is located at the point $(x_i\ y_i\ z_i)$. The coordinates of the centre of mass are then approximately given by $x = \frac{\sum \Delta m_i x_i}{\sum \Delta m}$, $y = \frac{\sum \Delta m_i y_i}{\sum \Delta m}$, $z = \frac{\sum \Delta m_i z_i}{\sum \Delta m}$ and $\sum \Delta m_i \to \int dm = M$

$\sum \Delta m_i x_i \to \int xdm, \sum \Delta m_i y_i \to \int ydm, \sum \Delta m_i z_i \to \int zdm$

M is the total mass of the body. The coordinates of the Centre of mass are $x = \frac{1}{M}\int xdm\,, y = \frac{1}{M}\int ydm$ and $z = \frac{1}{M}\int zdm$ ……. (6.5) The vector expression $R = \frac{1}{M}\int rdm$ …..(6.6) is equivalent to these three scalar expressions.

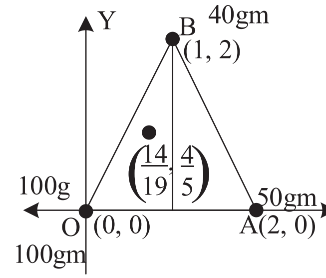
When we need to calculate the centre of mass of homogeneous bodies of regular shapes like circular discs, spheres, rods. (Homogeneous body $-a$ body with uniformly distributed mass at every point).

By using symmetry consideration, we can say easily that the centre of mass of homogeneous bodies lie at their geometric centres.

Let us consider a thin rod, we choose origin o, taking the origin to be at the geometric centre of the rod and x −axis to be along the length of the rod, then for every dm at x there would be dm at $-x$. So the net contribution of every such pair to the integral $\int xdm$ is zero in this case

This symmetry rule applies for other homogeneous bodies as well like rings, discs, spheres or even thick rods of circular or cross section area A. For all such bodies, for every element dm at a point (x, y, z) there is always an element of the same mass at the point $(-x, -y, -z)$. As a result, the integrals in Eq. (6.5) all are zero.

Example 6.1 If each side of an equilateral triangle is $2m$ long and three particles are placed at the vertices of the triangle of masses $100gm$ $50gm$ and $40gm$ respectively. Find the centre of mass of the three particles.



Solution: First we choose x and y axes as shown in the figure. The coordinates of points O, A & B forming the equilateral triangle are $(0, 0), (2, 0), (1, 2)$.

The masses are $100gm, 50gm$, and $40gm$ respectively. Then

$$X = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$$

$$= \frac{.1(0)+.05(2)+.04(1)}{.1+.05+.04}$$

$$= \frac{14}{19}$$

$$Y = \frac{.1(0)+.05(0)+.04(2)}{(.01+.05+.04)}$$

$$= \frac{4}{5}\ m\ .\ \text{Thus COM is}\ \frac{14}{19}, \frac{4}{5}$$

6.3 Motion of centre of Mass

$$\boldsymbol{P = MV}$$

Where $MV = m_1v_1 + m_2v_2 + m_3v_3 + \ldots\ldots.m_nv_n$....(6.7)

Also $\frac{MdR}{dt} = \frac{m_1dr_1}{dt} + \frac{m_2dr_2}{dt} + \frac{m_3dr_3}{dt} + \ldots.\frac{m_ndr_n}{dt}$...... (6.8)

$(V = \frac{dR}{dt}, \frac{dr_1}{dt} = v_1, \frac{dr_2}{dt} = v_2)$

Differentiating equation (6.7) with respect to time, we obtain

$$\frac{MdV}{dt} = \frac{mdv_1}{dt} + \frac{mdv_2}{dt} + \ldots\ldots. + \frac{m_ndv_n}{dt}$$

$$MA = m_1a_1 + m_2a_2 + \ldots\ldots m_na_n$$

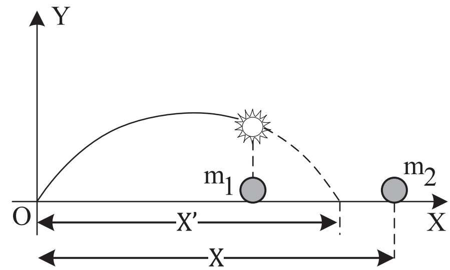
($A = \frac{dV}{dt}$ is the acceleration of the centre of mass of the system of particles)

And, $MA = F_{ext}$ ....(6.9)

F_{ext} represents the sum of all external forces acting on the system of the particles.

Equation (6.9) states that the centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point (COM)

If a ball follows a parabolic path under the influence of gravity, and the ball explodes into two parts at $x = x_1$. However, the COM of the ball will continue to follow the same path as shown in the figure.



$$COM = \frac{m_1x_1+m_2x_2}{m_1+m_2}$$

Here COM will not change due to explosion because explosion takes place due to internal force.

Example 6.2 If a ball of mass $1kg$ is following a parabolic path under the gravitational force and the ball is to fall on the earth surface at a distance of $8m$ from the origin from where the ball is thrown. But the ball explodes into two parts each of mass $.5kg$, the first ball falls at $x = 5m$. Find the distance at which the second ball falls.

Solution: Given that COM $= 8m$,

So, $8m = \frac{(.5kg \times 5m + .5kgx_2))}{1kg}$

$\Rightarrow x_2 = 11m.$

Dolly appeared to be struggling with the example number 6.2. She was about to ask the teacher, but Aditi stopped her saying, 'I can make you understand this example with basics.' 'Okay, you do one thing: you make me understand it when we're on the bus,' Dolly responded.

Only 5 minutes were left until the bell rang.

The teacher was talking about an author, who was scheduled to visit the school the next day to campaign for his debut novel. Although the guys had nothing to do with it, as tomorrow would be Tuesday and there was no Physics class on that day.

*****

Both Dolly and Aditi sat in the bus cabin. Aditi took a notebook from her bag and explained, 'The net external force (F_{ext}) is only due to the force of gravity, and this force exists before explosion as well as after explosion. In such cases of motion, COM is considered as an important term. The COM must fall at the spot where it is to fall before the explosion. Therefore we locate COM to find the position of the other particle.'

Note: Before the explosion the system consists of a single particle of mass $1kg$ and after the explosion the same system has two particles each of mass $0.5\ kg$.

*****

Arnold had a cheque of INR 2 Lakhs in his name given by Amila this morning and was told to cash it and return the money to her. Arnold had absolutely no idea why she needed such a large amount. The vehicle stopped at a bank, and at the village, the cash was given to Amila . However, Amila gave INR 1.5 Lakhs back to Arnold, saying, 'This is for you and your group.'

Arnold was hesitating to keep it, but finally kept it at her insistence. Dolly was informed first, and then the other members of the group, and they decided to keep it for their common expenses.

*****

The next session starts with the module 'Linear momentum of a system of particles.'

6.4 Linear momentum of a system of particles

$P = mV$ (P is the linear momentum of a system of particles)

Also $F = \frac{dP}{dt}$ (by Newton's second law)

If there are n particles in the system

Then $P = P_1 + P_2 + \dots\dots P_n$

$= m_1 v_1 + m_2 v_2 + \dots\dots\dots m_n v_n$

Comparing the above equation with (6.7), we get P = MV

$\frac{dP}{dt} = \frac{Mdv}{dt} = MA$ ......(6.10)

Comparing equation 6.9 and 6.10 we get

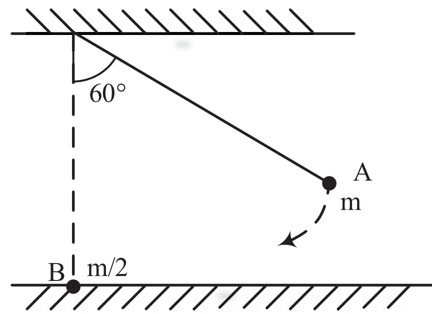
$\frac{dP}{dt} = F_{ext}$.........(6.11)

Equation (6.11) is the definition of Newton's second law of motion for a system of particles.

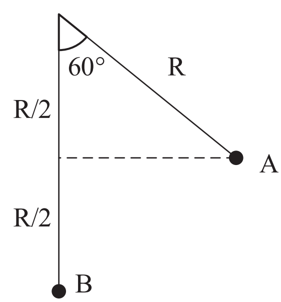
$\frac{dP}{dt} = O$ or $P = \text{Constant}$ ...(6.12) Equation 6.12 is law of conservation of momentum, when F_{ext} is zero the total linear momentum of the system is constant (conserved).

Vector Equation (6.12) is equivalent to three scalar equations, $P_x = C_1, P_y = C_2$ and $P_z = C_3$ (6.13) Here P_x, P_y and P_z are the components of the total linear momentum vector P along the x, y and z axes respectively; C_1, C_2 and C_3 are constants.

Example 6.3 As shown below, bob A of a pendulum having a massless string of length $'R'$ is released from 60° to the vertical. It hits another bob B of half the mass that is at rest on a frictionless table in the center. Assuming elastic collision, the magnitude of the velocity of bob A after the collision will be (take g as acceleration due to gravity.)



Solution: Velocity of A just before hitting: $u = \sqrt{\frac{2gR}{2}} = \sqrt{gR}$ (by the kinematic equation of motion $v^2 - u^2 = 2as$)



Just after collision, let velocity of A and B are v_1 and v_2 respectively

$\therefore mu = mv_1 + \frac{mv_2}{2}$ (by conservation of momentum)

$2v_1 + v_2 = 2u$ …….(i)

$e = 1$ (for elastic collision)

$\Rightarrow \frac{v_2 - v_1}{u} = 1$

$\Rightarrow v_2 - v_1 = u$ ……(ii)

Subtracting eq (ii) from eq (i)

$\Rightarrow 3v_1 = u$

$\Rightarrow v_1 = \frac{u}{3} = \frac{1}{3}\sqrt{gR}$

The solution of example number 6.3 was difficult to understand not only to the group, but to almost the entire class, with the exception of Aakash. The second row humbly requested the teacher to detail it comprehensively. Gita, the teacher, asked, “Has anyone understood it, and could explain here? Anyone? No one? Dolly? Aakash?” But as she called the name Aakash, everyone started shouting “Aakash! Aakash! Aakash!”.

The teacher screamed, “What the hell is this? I can’t compromise with the decorum of this class.”

Aditi, speaking in a moderate tone towards the back row, exclaimed, ‘What nasty stuff this is!’ He is like a son to Gita. A low sound came from the same row, ‘who are we to fix any type of relationship?’ Aditi retorted, ‘This is terrible and nothing less!’ It looked like she was intensely jealous of Gita and possessive of Aakash.

Aakash walked to the board even amidst the ongoing commotion and explained the solution to the whole class, using the same diagram that was drawn on the large blackboard at the front of the classroom, this way.

“The ball is having 0 initial velocity at the released point (as the ball is at rest). Once the ball is released it starts performing circular motion due to the tension of the string which develops due to the force of gravity. And, ball b is located vertically at the table in the centre which must be in the path of ball a, therefore collision takes place. We can apply the kinematic equation of motion and conservation of momentum easily in this case to find the physical quantity.”

*****

6.4 Vector product of two vectors

In the previous chapter, we defined the scalar product of two vectors (Force and Displacement). In this chapter, we will define the vector product (or cross product) of two vectors, an operation, involving two vector quantities, that results in a new vector quantity.

Let a and b are two vectors. Their vector product denoted as $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$, is defined as follows:

The magnitude of the resultant vector c is given by: $|c| = ab\sin\theta$

where a and b are the magnitudes of vectors a and b, and θ is the angle between them.

The direction of the resultant vector c is perpendicular to the plane containing a and b. Its specific orientation is determined by the 'Right-Hand Thumb Rule.'

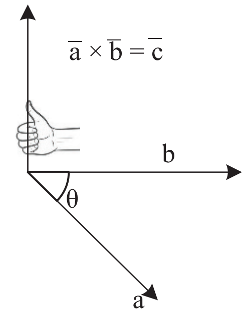


Fig 6.2 (a)

Right Hand Thumb Rule: Open up your right hand palm and curl the fingers pointing from a to b and if $a \times b = c$, your stretched thumb points the direction of c that is in $+ve\ y$ direction, as shown in Figure 6.2(a).

It should be remembered that we always encounter two angles θ and $360° - \theta$ between any two vectors (suppose a and b) while applying the rule. The rotation should be taken through the smaller angle ($< 180°$) between a and b, that is θ here. It is also called cross product.

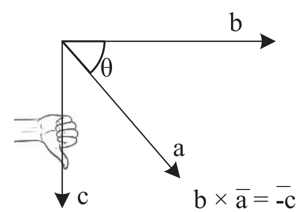


Fig 6.2 (b)

What will be the vector product of $b \times a$, here you need to open your right hand palm and all the fingers pointing from b to a, and your stretched thumb points in opposite direction of c (as shown in Figure 6.2(b)) and still it is perpendicular to the plane containing b and a (but in negative y direction). Therefore $\boldsymbol{b \times a = -c}$. However, the magnitude of both $a \times b$ and $b \times a$ is the same that is $ab \sin\theta$.

We have some points to note here

(i) $i \times i = 0, j \times j = 0, k \times k = 0$
(ii) $i \times j = k$
(iii) $j \times k = i$
(iv) $k \times i = j$
(v) $j \times i = -k$
(vi) $k \times j = -i$
(vii) $i \times k = -j$

If $a = a_x i + a_y j + a_z k \;\; b = b_x i + b_y j + b_z k$

$$\text{Then } a \times b = \begin{matrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{matrix}$$

$$= (a_y b_z - a_z b_y)i - (a_x b_z - a_z b_x)j + (a_x b_y - a_y b_x)k$$

Example 6.4 Find the vector product of the vectors $a = 3i + 4j - 5k$ and $b = -2i + 3j - 3k$

$$\textbf{Solution: } a \times b = \begin{array}{ccc} i & j & k \\ 3 & 4 & -5 \\ -2 & 3 & -3 \end{array}$$

$$= 3i - 19j + 17k$$

6.5 Angular velocity and its relation with linear velocity

When a rigid body performs rotational motion about an axis, every particle of the body moves in a circle such that its centre lies on the axis and the circle itself is in a plane perpendicular to the axis of rotation.

There is also linear velocity of every particle undergoing rotational motion at every instant during the motion (see Figure 6.3). Its direction at point p is along the tangent at p to the circle, at p' it is again along the tangent at p' to the circle and so on. The angular velocity of the particle at p is $\frac{\Delta\theta}{\Delta t}$ for $\Delta t \to 0$ (Δt tends to be 0), and is denoted by ω. We know $v = \omega \times r$ (from circular motion), where r is the radius of the circle. $v = \omega \times r$, applies for all particles of the body, so $v_i = \omega \times r_i$ (where v_i is the linear velocity of i_{th} particle and r_i is the radius of i_{th} particle). The index i runs from 1 to n, where n is the total number of particles of the body.

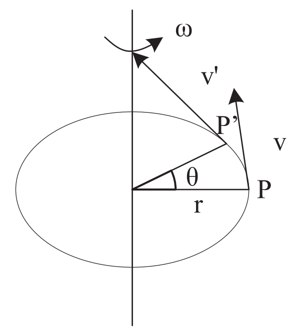


Fig 6.3

Therefore, $v = \omega \times r$ is the relation held by a particle between its angular velocity and linear velocity.

Angular acceleration $a = \frac{d\omega}{dt}$ …(6.13)

'It could be noted here for any particle of a rigid body undergoing rotational motion about a fixed axis, suppose y, has same angular velocity ω . However, their radius and linear velocity could vary particle to particle.'

6.6 Torque

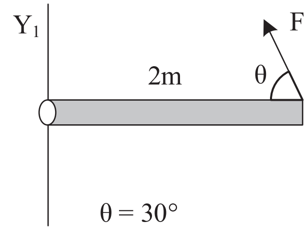
We have seen a rigid body performing both rotational and translational motion at the same instant in case the axis of rotation is not fixed.

But if the axis of rotation is fixed, the body undergoes only rotational motion. However particles in the body still have linear velocities (given as $v = \omega \times r$) at any instant. Same way there is a term known as Torque (τ), which is defined as $\tau = r \times F$ ……(6.14)

$|\tau| = rFsin\,\theta$ …….(6.15)

Torque is also known as moment of force. It is a vector product of r and F. The magnitude of torque is $rFsin\,\theta$ and direction is given according to the right-hand thumb rule.

If a force of $4N$ is applied to a rod stick of length $2m$ at its end as shown in the figure. Then find the torque on it.



Solution: the torque here produced will be about y_1 axis

And we know that $\tau = r \times$ F

$= 2m \times 4N \times \sin 30°$

$= 2m \times 4N \times \frac{1}{2} = 4N.m$

And direction will be upward that is along y_1 axis (a/c to right hand thumb rule)'

6.7 Angular momentum of a particle

Assume a particle of mass m and linear momentum P at a position r relative to the origin o. Then the angular momentum L of the particle having origin 0 is $L = r \times P$, $|L| = rpsin\,\theta$.

$\frac{dL}{dt} = \frac{d(r\times P)}{dt}$

$\Rightarrow \frac{d(r\times P)}{dt} = \left(\frac{dr}{dt}\right) \times P + r \times \frac{dp}{dt}$ (A/c to the product rule of differentiation)

$= v \times P + r \times \frac{dp}{dt} \left(\frac{dr}{dt} = v\right)$

$= 0 + r \times \frac{dp}{dt}$ (vector product of two parallel vectors is always zero)

$= r \times F \left(\frac{dp}{dt} = F\right)$

$= \tau$, Hence $\frac{dL}{dt} = \tau$…..(6.16)

Torque and angular momentum for a system of particles

If there are a number of particles in the system and need to find angular momentum about a point. Then, we have to add vectorially the angular momenta of individual particles. Thus for a system of n particles

$L = L_1 + L_2 + \ldots\ldots + L_n = \sum_{i=1}^{n} L_i$

Angular momentum for any i_{th} particle $L_i = r_i \times P_i$

r_i is the position vector of i_{th} particle with respect to given origin o and P_i ($m_i v_i$ is the linear momentum of the particle) so we may write the total angular momentum of a system of particles as $L = \sum L_i = \sum_i r_i \times P_i$

Similarly, we have $\frac{dL}{dt} = \sum \frac{dL_i}{dt} = \sum \tau_i = \tau_{ext}$

*****

In the village, Amila was almost completely recovered now, able to walk and drive, although she still had some bandages wrapped around the wrist of her left hand.

Amila was going down the stairs, saying to Arnold, 'I am going for a small drive just to check the state of my car.'

Arnold responded, "Okay, wait, I'm also coming." Both Amila and Arnold exchanged a sweet look to each other at the spot the car was parked. This was the first time they both felt attracted to each other. On the drive, Arnold felt voiceless; even a normal word was getting tough to utter, probably because his feeling was transitioning from a well-wisher to something new.

That night after the drive, Amila had many wonderful conversations with Arnold's parents. She talked about her time at University and Tennis career. She had many great memories. She also mentioned that in a single year she failed in two subjects but managed to finally pass with a flying percentage.

*****

The next formal session started with the module 'Conservation of angular momentum'

6.8 Conservation of angular momentum

We will define conservation of angular momentum in the same way we have defined linear momentum.

Conservation of angular momentum states that the total angular momentum of a system is conserved (i.e. constant) if the net external torque on the system is zero. If $\tau_{ext} = 0$

$$\text{Or, } \frac{dL}{dt} = 0 \;\; (\tau_{ext} = \frac{dL}{dt})$$

It implies L = constant…….(6.17)

Equation 6.17 is equivalent to three scalar equations

$L_x = K_1, L_y = K_2$ & $L_z = K_3$ …….(6.18)

Here K_1, K_2 and K_3 are constants; L_x L_y and L_z are the components of total angular momentum vector L along the x y and z axes respectively.

Example 6.5 A circular ring of mass 2 kg and radius 1 m is rotating along an axis passing through its centre and perpendicular to its plane with 2 rad/sec. If an object of mass 100 gm comes and sticks at the end parts of the disc and starts moving together with the disc. Find the new angular velocity of the disc.

Solution: By conservation of angular momentum

$L_1 = L_2 \Rightarrow 1m \times 2kg \times 2m/s = 1m \times 2.1kg \times v,$ (Note: using the relationship $v = \omega \times r$)

$\Rightarrow v, = 1.9m/s$ Therefore $\omega, = 1.9ms^{-1}/1m = 1.90\ rad/s.$

Example 6.6 Find the torque of a force $4i + 6j + 2k$ about the origin of a particle whose position vector is $i - 2j + 4k$.

Solution: Given $r = i - 2j + 4k$ and $F = 4i + 6j + 2k$

We will use the determinant rule to find the torque $\tau = r \times F$

$$\tau = \begin{matrix} i & j & k \\ 1 & -2 & 4 \\ 4 & 6 & 2 \end{matrix} = -28i + 14j + 14k$$

6.9 Equilibrium of a rigid body

The term comes when we extend the knowledge of motion of a rigid body. We know force can change translational as well as rotational state of a rigid body.

A rigid body is said to be in mechanical equilibrium if it is in translational equilibrium as well as rotational equilibrium (i) $F_1 + F_2 + \ldots\ldots F_n = \sum_i^n F_i = 0$…..(6.19) (for translational equilibrium)

(ii) $\tau_1 + \tau_2 + \ldots\ldots \tau_n = \sum_i^n \tau = 0$ …..(6.20) (for rotational equilibrium)

A body satisfying both equations is said to be in mechanical equilibrium.

Example 6.7 A $3m$ long ladder weighing $20kg$ leans on a frictionless wall. Its feet rest on the floor $1m$ from the wall as shown in Figure 6.4. Find the reaction forces of the wall and the floor.

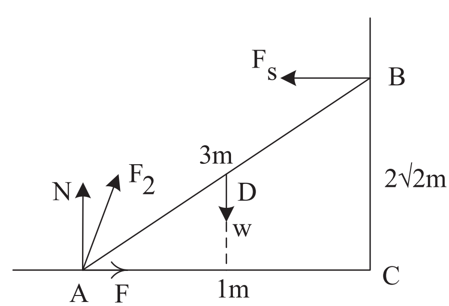


Fig 6.4

Solution: The ladder AB is $3m$ long, its foot A is at distance $AC = 1m$ from the wall. From Pythagoras theorem, $BC = 22m$. The forces on the ladder are its weight W acting at its centre of gravity D, reaction forces F_1 and F_2 of the wall and the floor respectively. Force F_1 is perpendicular to the wall, since the wall is frictionless. Force F_2 is resolved into two components, the normal reaction N and the force of friction F. Note that F prevents the ladder from sliding away from the wall and is therefore directed toward the wall.

For translational equilibrium, taking the forces in the vertical direction, $N - W = 0$….. (i)

Taking the forces in the horizontal direction, $F - F_1 = 0$ ...(ii) For rotational equilibrium, taking the moments of the forces about $A, 2\sqrt{2}F_1 - \left(\frac{1}{2}\right) W = 0$ ……. (iii)

Now $W = 20\ g = 20 \times 9.8 = 196.0\ N$ From equation (i) $N = 196\ N$

From (iii) $F_1 = \mathrm{w}/4\sqrt{2} = 196/4\sqrt{2} = 34.6$ N

From (ii) $F = F_1 = 34.6\ N$, $F_2 = \sqrt{F^2 + N^2} = 199N$

The force F_2 makes an angle α with the horizontal.

$\tan\alpha = \frac{N}{F} = 4\sqrt{2}, \ \alpha = \tan^{-1}(4\sqrt{2}) = 80°.$

6.9 Moment of Inertia

Moment of inertia is analogous to mass in linear motion. Just as $F = ma$ in linear motion, $\tau = I\alpha$, where I is the moment of inertia and α is angular acceleration. The moment of inertia I is always defined about a specific axis of rotation. For a single point mass, its moment of inertia is given by $I = mr^2$. However, in any rigid body, the mass is distributed, meaning different particles are located at varying distances from the axis of rotation. Therefore for a system of particles, the moment of inertia is calculated as the sum: $I = \sum_{i=1}^{n} m_i r_i^2$, where m_i is the mass of the i^{th} particle and r_i is its perpendicular distance from the axis of rotation.

6.10 Radius of gyration

Radius of gyration of a body is defined as the perpendicular distance between the axes of rotation and a point at which the whole mass of the body is supposed to be concentrated (i.e. COM).

$I = mk^2$ (I is the moment of inertia,m is the mass of the body and k is the radius of gyration)

$$K = \sqrt{\frac{I}{m}}$$

6.11 Parallel axis theorem

It states that the moment of inertia of a rigid body about an axis parallel to its center of mass is equal to the sum of the moment of inertia about its center of mass plus the product of the body's mass and the square of distance between the two parallel axes (Figure 6.5).

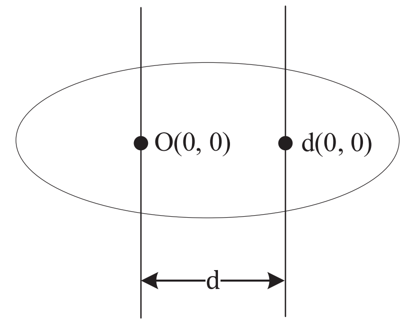


Fig 6.5

$$I = I_o + md^2$$

6.12 Perpendicular axes theorem

This theorem is applicable to planar bodies only. It states for any planar object, the moment of inertia (I) about an axis Z perpendicular to the plane is equal to the sum of the moments of inertia about two mutually perpendicular axes (X and Y axes) lying in the plane and passing through the same point as the Z axis (Figure 6.6).

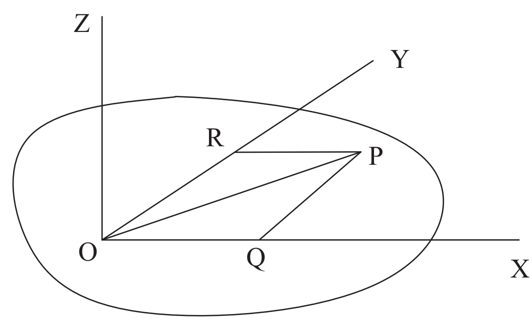


Fig 6.6

$$I_z = I_x + I_y$$

6.13 Moment of Inertia of Continuous Mass Distribution

(i) Uniform rod about a perpendicular bisector= $\frac{Ml^2}{12}$
M is mass of the rod, l is length of the rod

(ii) A rectangular plate about a line parallel to an edge and passing through the centre = $\frac{Ml^2}{12}$

(iii) A circular ring about its axis (the line perpendicular to the plane of the ring and passing through its centre) = MR^2

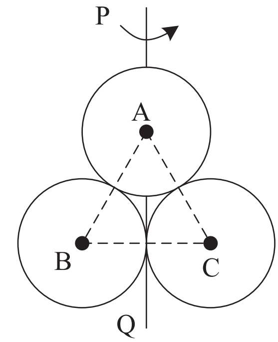
(iv) A uniform circular plate about its axis = $\frac{MR^2}{2}$

(v) A hollow cylinder about its axis = MR^2

(vi) A uniform solid cylinder about its axis = $\frac{MR^2}{2}$

(vii) a uniform hollow sphere about a diameter $= \frac{2}{3}MR^2$

(viii) a uniform solid sphere about a diameter $= \frac{2}{5}MR^2$



Example 6.8 A, B and C are disc, solid sphere and spherical shell respectively with same radii and masses. These masses are placed as shown in the figure. The moment of inertia of the given system about the PQ axis is $\left(\frac{x}{15}\right)I$, where I is the moment of inertia of the disc about its diameter. The value of x is ____________.

Solution: All bodies have the same mass and the same radius.

A → Disc

B → Solid Sphere

C → Spherical shell

And, moment of inertia of the disc about the axis $PQ = \frac{MR^2}{4}$

Moment of the inertia of the solid sphere about the axis $PQ = \frac{2}{5} MR^2 + MR^2$

Moment of the inertia of the spherical shell about the axis $PQ = \frac{2}{3} MR^2 + MR^2$

The moment of inertia of the given system about PQ axis is $= \frac{MR^2}{4} + \left(\frac{2}{5} MR^2 + MR^2\right) + \left(\frac{2}{3} MR^2 + MR^2\right)$

Therefore $I_{PQ} = \frac{MR^2}{4} + \left(\frac{2}{5} MR^2 + MR^2\right) + \left(\frac{2}{3} MR^2 + MR^2\right)$

$$I_{PQ} = \frac{15\ MR^2 + 24\ MR^2 + 60MR^2 + 40\ MR^2 + 60MR^2}{60} = 199\frac{MR^2}{60}$$

$$I_{PQ} = \left(\frac{199}{15}\right)\left(\frac{MR^2}{4}\right) = \left(\frac{199}{15}\right) I. \text{ Therefore } x = 199.$$

Note: Angular Momentum $L = I\omega$, this is the formula of angular momentum in terms of moment of inertia about a given axis of an object

6.14 Equations of Rotational Motion (about a fixed axis)

The equations of rotational motion are analogous to the equations of linear motion. However, they describe the relationship between angular displacement, angular velocity, angular acceleration, and time. These equations are $\omega = \omega_0 + at$ ......(i)

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\,\mathrm{a}t^2 \qquad ....(ii)$$

$$\omega^2 = \omega_0^2 + 2\mathrm{a}(\theta - \theta_0) \qquad ....(iii)$$

Equation (i) relates final angular velocity ω to initial angular velocity ω_0 angular acceleration It essentially states that the change in angular velocity is directly proportional to the angular acceleration and the time it acts for.

Equation (ii) This equation relates final angular displacement θ to initial angular displacement θ_0, initial angular velocity ω_0, angular acceleration α, and time t. It describes how the angular displacement changes with time, considering both the initial angular velocity and the effect of constant angular acceleration.

Equation (iii) This equation relates final angular velocity ω to initial angular velocity ω_0, angular acceleration α and the change in angular displacement $(\theta - \theta_0)$.

Example 6.9 A wheel of perimeter $240cm$ rolls on a level road at a speed of $12km/h$. How many revolutions does the wheel make per second?

Solution: Perimeter of the wheel $(p) = 240\ cm = 2.4\ m$

Speed of the wheel $(v) = 12\ km/hr = \left(\frac{10}{3}\right) m/s$

Therefore number of revolution, the wheel does make per second = $\frac{speed\ of\ the\ wheel\ (v)}{perimeter\ of\ the\ wheel\ (p)}$

$$\Rightarrow \frac{\left(\frac{10}{3}\right) m/s}{2.4m} = 1.38\ rev/s$$

6.15 Comparisons of Linear Motion and Rotational Motion

Linear Motion	Rotational Motion about a Fixed Axis
1. Displacement x	Angular displacement θ
2. Velocity $v = \frac{dx}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
3. Acceleration $a = \frac{dv}{dt}$	Angular acceleration $\alpha = \frac{d\omega}{dt}$
4. Mass M	Moment of inertia I
5. Force $F = Ma$	Torque $\tau = I\ \alpha$

6. Work $dW = Fds$	Work $W = \tau\, d\theta$
7. Kinetic energy $K = \frac{Mv^2}{2}$	Kinetic energy $K = \frac{I\omega^2}{2}$
8. Power $P = Fv$	Power $P = \tau\omega$
9. Linear momentum $p = Mv$	Angular momentum $L = I\omega$

Example 6.10 A thin solid disk of $1kg$ is rotating along its diameter axis at the speed of $1800rpm$. By applying an external torque of $25\pi\ Nm$ for $40s$, the speed increases to $2100rpm$. The diameter of the disk is ___________ m.

Solution: Given that initial angular velocity, $w_i = 1800rpm = 1800 \times \frac{2\pi}{60} = 60\pi\ rad/sec$

Final angular velocity, $w_f = 2100rpm = 2100 \times \frac{2\pi}{60} = 70\pi\ rad/sec$

By applying the equation of rotational motion $w_f = w_i + \alpha t$

$70\pi = 60\pi + \alpha(40)$

$\Rightarrow\ \alpha = \frac{70\pi - 60\pi}{40} = \frac{\pi}{4}\ rad/s^2$

The torque $\tau = I\alpha$ (I = moment of inertia)

For a thin solid disk rotating along its diameter, the moment of inertia I is $mR^2/4$. Thus: $25\pi = \frac{1 \times R^2}{4} \times \frac{\pi}{4}$

$\Rightarrow\ 25\pi = \frac{\pi R^2}{16}\ \Rightarrow\ R^2 = 25\pi \times \frac{16}{\pi} = 400$

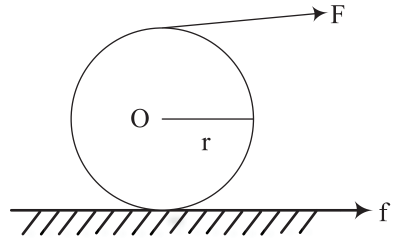
$\Rightarrow\ R = 20m$. The diameter of the disk is: $Diameter = 2R = 40m$.

Rolling without slipping: In this case, an object is rotating and translating forward at the same time and the point of contact between the object and the surface it is rolling on, is instantaneously at rest relative to the surface. (This condition implies the surface itself is not slipping)

Rolling and slipping: In this condition, an object is rotating and translating at the same time but slipping on the surface on which it is rotating. It implies that the point of contact of the object and the surface is not stationary.

Example 6.11 A force F acts tangentially at the highest point of a sphere of mass m kept on a rough horizontal plane. If the sphere rolls without slipping, find the acceleration of the centre of the sphere?

Solution: When the force F rotates the sphere, the point of contact has a tendency to slip towards the right. Let r be the radius of the sphere and α be the linear acceleration of the centre of the sphere. The angular acceleration about the centre of the sphere is $\alpha = \frac{a}{r}$, as there is no slipping.



For the linear motion of the centre $F + f = ma$ ……(i)

And for the rotational motion about the centre,

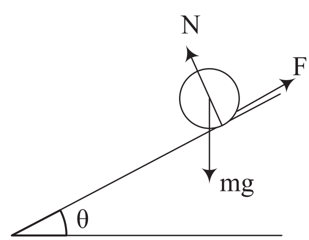
$Fr - fr = I\alpha = \left(\frac{2}{5}\, mr^2\right)\left(\frac{a}{r}\right)$

Or, $F - f = \frac{2}{5}\, ma$ …….(ii)

From equation (i) and (ii) $2F = \frac{7}{5}ma$ or $a = \frac{10F}{7m}$.

Example 6.12 A sphere of mass m rolls without slipping on an inclined plane of inclination θ. Find the linear acceleration of the sphere and the force of friction acting on it. What should be the minimum coefficient of static friction to support pure rolling?

Solution: Suppose the radius of the sphere is r. The forces acting on the sphere are as shown in the figure. They (a) weight mg, (b) normal force N and (c) friction f.



Let the linear acceleration of the sphere down the plane be a. Then the equation for the linear motion of the centre of mass is $mg\ sin\ sin\ \theta\ - f = ma$ …(i)

As the sphere rolls without slipping, its angular acceleration about the centre is $\frac{a}{r}$. The equation of rotational motion about the centre of mass is,

$fr = \left(\frac{2}{5}\, mr^2\right)\left(\frac{a}{r}\right)\ or, f = \frac{2}{5}\, ma$ ……(ii)

From (i) and (ii) $a = \left(\frac{5}{7}\right)\ gsin\,\theta$ and $f = \left(\frac{2}{7}\right)mgsin\,\theta$

The normal force is equal to $mg\cos\theta$ as there is no acceleration perpendicular to the incline. The maximum friction that can act is, therefore, $\mu mg\ cos\ cos\ \theta$ (μ is the coefficient of static friction). Thus for pure rolling

$\mu mgcos\,\theta\ > \left(\frac{2}{7}\right)mgsin\,\theta$

$\mu > \left(\frac{2}{7}\right) tan\ \theta$.

Example 6.13 A cylinder is released from rest from the top of an incline of inclination θ and length l. If the cylinder rolls without slipping, what will be its speed when it reaches the bottom?

Solution: Let the mass of the cylinder be m and its radius is r.

Suppose the linear speed of the cylinder when it reaches the bottom is v. as the cylinder rolls without slipping, its angular speed about its axis is $\omega = \frac{v}{r}$. The kinetic energy at the bottom will be

$$K = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

$$= \frac{1}{2}\left(\frac{1}{2} mr^2\right)\omega^2 + \frac{1}{2} mv^2 = \frac{1}{4} mv^2 + \frac{1}{2} mv^2 = \frac{3}{4} mv^2$$

This should be equal to the loss of potential energy $mglsin\ \theta$ Therefore, $\frac{3}{4}\ mv^2 = mglsin\ \theta$

$$V = \sqrt{\left(\frac{4}{3}\right) gl \sin\theta}$$

Exercises

1. The torque of the weight of any body about any vertical axis is zero. Is it always correct?
2. Can an object be in pure translational as well as in pure rotation?
3. If A is a unit vector along the axis of rotation of a purely rotating body and B is a unit vector along the velocity of a particle P of the body away from the axis. The value of $A.B$ is
   (a) -1 (b) 1 (c) 0 (d) none of these
4. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1Å $= 10^{-10}m$). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.
5. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.
6. A solid cylinder of mass $20kg$ rotates about its axis with angular speed 100 $rads^{-1}$. The radius of the cylinder is $0.25m$. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

7. A rope of negligible mass is wound round a hollow cylinder of mass $4kg$ and radius $20cm$. What is the angular acceleration of the cylinder if the rope is pulled with a force of $40N$? What is the linear acceleration of the rope? Assume that there is no slipping.
8. A thin solid disk of $1kg$ is rotating along its diameter axis at the speed of $1800rpm$. By applying an external torque of $25\pi Nm$ for $40s$, the speed increases to $2100rpm$. The diameter of the disk is ________ m.
9. A horizontal circular platform of negligible friction with a radius of $2m$ and a mass of $120kg$ is rotating at an angular velocity of $0.50\ rev/s$. A child of mass $22kg$ runs tangent to the rim at a speed of $4.0m/s$ and jumps onto the edge of the platform. What is the angular velocity of the platform just after the child jumps on?
10. A solid sphere with uniform density and radius R is rotating initially with constant angular velocity ω_1 about its diameter. After some time during the rotation it starts losing mass at a uniform rate, with no change in its shape. The angular velocity of the sphere when its radius become $\frac{R}{2}$ is $x\omega_1$. The value of x is (Note: There is no external torque on the system, therefore this is application of conservation of momentum case)
11. Four particles each of mass $1kg$ are placed at four corners of a square of side $2m$. Moment of inertia of the system about an axis perpendicular to its plane and passing through one of its vertex is _____ kgm^2.
12. A cord of negligible mass is wound round the rim of a fly wheel of mass $20\ kg$ and radius $20\ cm$. A steady pull of $25\ N$ is applied on the cord as shown in Fig. 6.31. The flywheel is mounted on a horizontal axle with frictionless bearings. (a) Compute the angular acceleration of the wheel. (b) Find the work done by the pull, when 2m of the cord is unwound. (c) Find also the kinetic energy of the wheel at this point. Assume that the wheel starts from rest.
13. Three balls of masses $2kg, 4kg$ and $6kg$ respectively are arranged at the centre of the edges of an equilateral triangle of side $2m$. The moment of inertia of the system about an axis through the centroid and perpendicular to the plane of the triangle, will be ……… Kgm^2.
14. A wheel of radius $0.2m$ rotates freely about its center when a string that is wrapped over its rim is pulled by force of $10N$ as shown in figure. The established torque produces an angular acceleration of $2rad/s^2$. Moment of inertia of the wheel is___________ kgm^2. (Acceleration due to gravity $= 10\ m/s^2$)
15. A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $\frac{x}{5}$. The value of x is ………..
16. A solid sphere and a hollow cylinder roll up without slipping on the same inclined plane with the same initial speed v. The sphere and the

cylinder reaches upto maximum heights h_1 and h_2 respectively, above the initial level. The ratio $h_1 : h_2$ is $\frac{n}{10}$. The value of n is

17. A circular ring and a solid sphere having the same radius roll down on an inclined plane from rest without slipping. The ratio of their velocities when reached at the bottom of the plane is $\sqrt{\frac{x}{5}}$ where $x =$
18. A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $x5$. The value of x is _________.
19. Consider a Disc of mass $5kg$, radius $2m$, rotating with angular velocity of $10rad/s$ about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is ________ J.
20. A force of $49N$ acts tangentially at the highest point of a sphere (solid) of mass $20kg$, kept on a rough horizontal plane. If the sphere rolls without slipping, then the acceleration of the center of the sphere is……………

Answers

1. No	2. Yes	3. C	4. Located on the line joining H and cl nuclei at a distance of 1.24 Å from the H end
5. 5145N	6. 3125J, 62.5 kgm^2/s	7. 50 rad/s^2, 10 m/s^2	8. 40
9. 2.8 rad/s or .446 rev/s	10. 32	11. 16	12. (a) 12.5 rad s^{-2} (b) 50 J (c) 50 J
13. 4	14. 1	15. 2	16. 7
17. 3.5	18. 2	19. 250	20. 3.5 m/s^2

Chapter - 7
Gravitation

It was Friday morning, and Amila was finally leaving the village. The whole group had gathered at Arnold's house and asked her to come again in her free time. Arnold, meanwhile, seemed quiet. He watched Amila adjust her jeans, his mind replaying their shared moments-the drive, all the beautiful conversation, the unexpected jolt of attraction. He wanted to say something meaningful, something that captured the shift he felt, but the words caught in his throat.

As Amila boarded the car, she turned to face the group, a grateful smile on her face. Her soft but clear voice spoke above the engine's rumble, "Thank you all for everything. This village… and all of you…." Her eyes met Arnold's briefly, and a spark passed between them. However, both of them restrained their feelings for the moment. After some discussions, the guys also headed to the city for the formal class.

The class had already started and they had missed a little part, though it was not particularly useful. Now, however, Kepler's laws were being taught.

7.1 Kepler's Laws

Kepler gives three laws for gravitation, which are as follows:

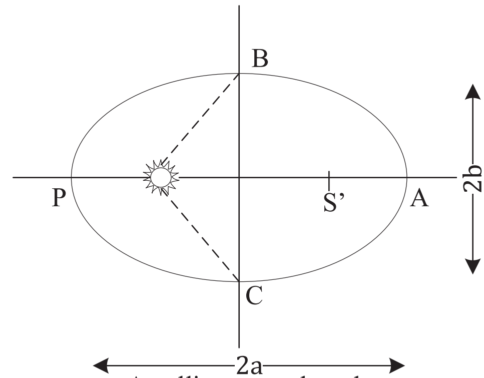
(I) **Law of orbits-** All planets move around the sun in elliptical orbits with the sun located at one of the foci of the ellipse.

(II) **Law of Areas-** It states, a line joining a planet and the sun sweeps equal areas in equal intervals of time.

Conservation of angular momentum (which is valid for any central force) can play a crucial role in letting us understand 'The law of areas.'

Arnold exclaimed, "What the hell is the central force?" Dolly teased, "Amila knows it better." "I don't like such jokes," Arnold shouted. Now, the whole class's attention was on their row. The teacher knew this sound must be from that row. She asked "what happened David?" David was speechless as he had no idea what led Arnold to make such a loud sound. Aditi tried to handle the situation and said to the teacher kindly, "Actually Anold has a concern regarding central force and the second law's application."

"It doesn't mean anyone does such nasty stuff during lecture! the teacher scolded. "Okay, now pay attention closely."



An ellipse traced out by a planet ground the sun

Central force is the force on the planet that is along the vector connecting the sun and the planet. It is towards the sun, and the law of conservation of angular momentum is valid for any central force so it is true here.

Let the Sun be at the origin and let the position and momentum of the planet be denoted by r and p respectively. Then the area swept out by the planet of mass m in the time interval Δt is ΔA given by $\Delta A = \frac{1}{2}\,(r \times v\Delta t)$.... (7.1) (See Figure 7.1)

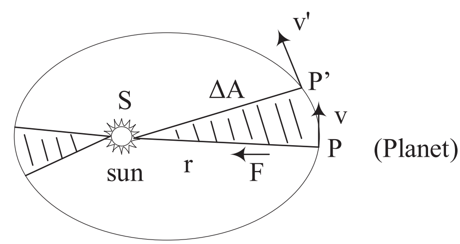


Fig 7.1

Hence, $\frac{\Delta A}{\Delta t} = \frac{1}{2}\frac{(r \times p)}{m}\left(since\ v = \frac{p}{m}\right)$

$= \frac{L}{(2\,m)}$.......... (7.2) , where v is the velocity, p is the linear momentum, and $L(r \times p)$ is the angular momentum

For a central force, which is directed along r, L is a constant as the planet goes around. Hence, $\frac{\Delta A}{\Delta t}$ is a constant according to equation 7.1. This is the law of areas

(III) **Law of Periods-** It states, the square of the time period of revolution of a planet is proportional to the cube of the ellipse traced out by the planet.

Example 7.1 A Saturn year is 29.5 times the earth year. How far is Saturn from the sun if the earth is $1.50 \times 10^8 km$ away from the sun?

Solution: Distance of the earth from the sun,

$r_e = 1.5 \times 10^8 km = 1.5 \times 10^{11} m$

Time period of the earth $= T_e$

Time period of saturn, $T_s = 29.5T_e$

Distance of saturn from the sun $= r_s$

From Kepler's third law of planetary motion, we have,

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{1/2}$$

For saturn and sun, we can write,

$$\frac{r_s^3}{r_e^3} = \frac{T_s^2}{T_e^2}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} \left(\frac{29.5T_S}{T_e}\right)^{2/3}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} (29.5)^{2/3}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} \times 9.55$$

$$\Rightarrow r_s = 14.32 \times 10^{11} m.$$

7.2 Universal law of Gravitation

It states that every particle attracts every other particle in the universe with a force that is directly proportional to the product of their masses and inversely proportional to the square of distance between their centres of masses.

$F = \frac{GM_1M_2}{r^2}$ (F is the force on a point mass M_1 due to another point mass M_2) where M_1 and M_2 are masses of the particles p_1 and p_2 respectively and r is the distance between them.

In vector form $F = \frac{GM_1M_2}{r^2}(-\hat{r}) = -\frac{GM_1M_2}{r^2}(\hat{r})$ ($\hat{r}$ is the unit vector from M_1 to M_2)

G is the universal gravitational constant, $r = r_2 - r_1$, and note down $F_{12} = -F_{21}$.

One point to note in this topic is that the gravitational force on a point mass due to an extended object is calculated vectorially by adding forces due to each point mass of the extended object. It is done easily by calculus.

The results of two such special cases are as

(i) The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.

(ii) The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.

Example 7.2 Three equal masses of $m\ kg$ each are fixed at the vertices of an equilateral triangle ABC. (a) What is the force acting on a mass $2m$ placed at the centroid G of the triangle? (b) What is the force if the mass at the vertex A is doubled? Take $AG = BG = CG = 1m$ (see Figure 7.2)

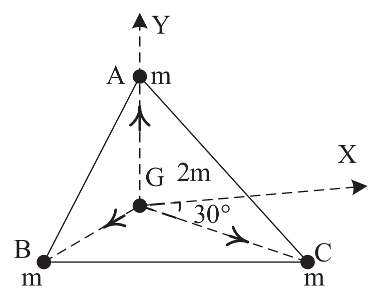


Fig 7.2

Solution: (a) The angle between GC and the positive x −axis is 30° and so is the angle between GB and the negative x −axis. The individual forces in vector notation are

$$F_{GA} = \frac{Gm(2m)j}{1}$$

$$F_{GB} = \frac{Gm(2m)(-i\ cos30° - j\ sin30°)}{1}$$

$$F_{GC} = \frac{Gm(2m)(+i\ cos30° - jsin30°)}{1}$$

The resultant gravitational force F_R on $(2m)$ is

$$F_R = F_{GA} + F_{GB} + F_{GC}$$

$$FR = 2Gm^2j + 2Gm^2(-icos30° - jsin30°) + 2Gm^2(icos30° - jsin30°) = 0$$

0

7.3 Acceleration due to gravity of the earth

We can assume earth as a sphere consists of a large number of concentric spherical shells.

(I) For a point mass m outside the earth the gravitational force due to the earth is such that all the shells' masses are concentrated at the centre of the sphere. Suppose the point mass m located at the surface of earth. Then the point mass m experiences a force $\frac{GM_em}{R_E^2}$, so $ma = \frac{GM_em}{R_E} \Rightarrow a = \frac{GM_e}{R_E^2}$, a is also known as gravitational field, G is gravitational constant and is equal to

$6.67 \times 10^{-11} Nm^2/kg^2$ and M_E is mass of the earth, R_E is the radius of the earth. Hence, we can apply the universal law of gravitation $\left(F = \frac{Gm_1m_2}{r^2}\right)$ by assuming all the masses are at the centre of the sphere.

(II) For any point mass m inside the earth, the gravitational force is 0 on the point mass. Therefore acceleration due to gravity of the earth is 0 in this case.

Both points (I) and (II) are conclusions of adding forces vectorially due to each point mass dm located at the earth.

The first one is on the point located outside the earth, the second one is inside the earth.

7.4 Acceleration variation with altitude and depth due to gravity, that is above and below the surface of Earth

Acceleration above the surface of the earth

If a point mass m is considered at a height h above the surface of the earth as shown in Figure 7.3(a). Then the force on the point mass m due to Earth is $F_m = \frac{GM_E m}{(R_E+h)^2}$ ……(7.3) (where M_E is mass of the earth and R_E is radius of the earth)

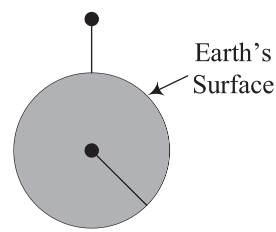


Fig 7.3 (a)

And by Newton's second law, the acceleration experienced by the point mass m is $\frac{F_m}{m} \cong \frac{GM_E}{(R_E+h)^2}$ …..(7.4)

For $h \lll R_E$, expanding the R.H.S of equation 7.4

We have $g(h) = \frac{GM_E}{R_E^2\left(1+\frac{h}{R_E}\right)^2} = g\left(1 + \frac{h}{R_E}\right)^{-2}$

For $\frac{h}{R_E} \lll 1$, (by binomial expression)

$g(h) = g\,(1 - \frac{2h}{R_E})$........(7.5)

Example 7.3 A body weighs $48\ N$ on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to one third the radius of the earth ?

Solution: Weight of the body, $W = 48\ N$

Acceleration due to gravity at height h from the earth's surface for $h = \frac{R_e}{3}$, is given by :

$$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$$

$$g_h = \frac{g}{\left(1+\frac{1}{3}\right)^2} = \frac{g}{\frac{16}{9}} = \frac{9g}{16}$$

Therefore $w = m \times \frac{9g}{16} = mg \times \frac{9}{16}$

$\Rightarrow 27N$

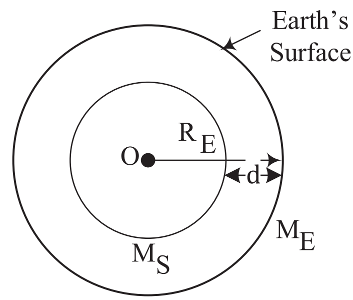


Fig 7.3 (b)

Acceleration below the surface of the earth

Consider a point mass m at depth of d below the surface of Earth. Then $g(d) = g\left(1 - \frac{d}{R_E}\right)$ .... (7.6) (Fig 7.3(b) and M_s is the mass of the smaller sphere)

7.5 Gravitational Potential Energy

We are already familiar with the term potential energy. Now, we are going to learn about Gravitational Potential Energy.

The amount of work done by an external agent against gravitational force in bringing an object of mass m from infinity to a specific distance R from the center of the earth of mass m is called gravitational potential energy of the object at position R. Force due to the external agent $= \frac{GMm\hat{r}}{r^2}$ (here external agent always acts in the direction opposite to the direction of gravitational force). Work done due to this force against a small displacement dr in the direction of gravitational force $d(w) = \frac{GMm(-dr)\hat{r}}{r^2}$. Therefore, work done in displacing an object from position infinity (∞) to R, $w = \int_{\infty}^{R} \frac{GMm}{r^2}(-dr)\hat{r}$.... (7.7)

Integrating Equation (7.7), we get $w = -\frac{GMm}{R}$ ...(7.8) which is potential energy of the object at position R, and is denoted as U_R.

Therefore, $(U_R) = -\frac{GMm}{R}$ ...(7.8)

7.6 Gravitational Potential

The amount of work done against gravitational force per unit mass in bringing an object from infinity to a distance R from the centre of the earth is called gravitational potential at position R and is denoted as V_R. $V_R = -\frac{GM}{R}$.....(7.9)

7.7 Escape Velocity

Escape velocity or Escape speed is the minimum speed an object must have to completely escape the gravitational field of a primary body.

Let the primary body we assume is 'Earth', and we throw an object with speed v_i, so that the object reaches infinity where there is no effect of gravitational force. The effect of gravitational force is given as: $\frac{GMm}{(R_E+h)^2}$, where M is mass of the earth, m is mass of the object, R_E is radius of the earth and h is the distance of the object from the surface of the earth.

Conservation of energy helps us find the escape speed of an object.

Suppose the object did reach infinity and its speed there was 0.

The energy of an object is the sum of potential and kinetic energy. $E\,(\infty) = P.E + K.E = 0$ ….(7.10)

If the object was thrown initially with speed v_i, from a point at a distance $(h + R_E)$ from the centre of Earth, its energy initially was $E(h + R_E) = \frac{1}{2}\,mv_i^2 - \frac{GM_Em}{(h+R_E)}$ ...(7.11). According to the principle of conservation of Energy, Equation (7.10) and (7.11) must be equal

$$0 = \frac{1}{2}\,mv_i^2 - \frac{GM_Em}{(h+R_E)}$$

$$\Rightarrow \frac{1}{2}mv_i^2 = \frac{GM_Em}{(h+R_E)}$$

$$\Rightarrow v_i^2 = \frac{2GM_E}{(h+R_E)}$$

$\Rightarrow v_i$ (escape speed) $= \sqrt{\frac{2g(R_E+h)^2}{(h+R_E)}}$......(7.12) using the relation $g = \frac{GM_E}{(R_E+h)^2}$

If the object is thrown from the surface of the earth, $h = 0$, we get $v_i = \sqrt{2gR_E}$

Example 7.4 The escape speed of a projectile on the earth's surface is 25 km/hr. A body is projected out with twice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Solution: $V_{esc} = \frac{25km}{hr}, V_p = 2V_{esc}$

Let mass of the projectile is m and velocity of the projectile far away from the earth (infinity) is V_f

Projectile's total energy on the earth $= \frac{1}{2}\, mV_p^2 - \frac{1}{2}\, mV_{esc}^2$

The projectile's gravitational energy far away from the earth (infinity) is 0.

Total energy of the projectile far away from the earth $= \frac{1}{2}\, mV_f^2$

From the law of conservation of energy, we have

$$\frac{1}{2}\, mV_p^2 - \frac{1}{2}\, mV_{esc}^2 = \frac{1}{2}\, mV_f^2$$

$$V_f^2 = V_p^2 - V_{esc}^2 \ = 4V_{esc}^2 - V_{esc}^2$$

$$\Rightarrow V_f = \sqrt{3}\, V_{esc} \ \Rightarrow V_f = \sqrt{3} \times 25km/hr = 43.3km/hr$$

The class concluded here. All the guys were on their way to the village.

*****

The rainy season had already begun, and laltens– portable devices using kerosene oil for lighting,--were widely used throughout the village during the season. Electricity was scarce due to heavy rainfall, sometimes lasting continuously for 2 or 3 days, or even more. The Annual Exam was also getting closer although most of them seemed to be as usual as they had been since the start of the year; cool, assured, unstressed and hopeful. However, Dolly was somewhat a little concerned regarding electricity. She was in a deep conversation that day with some guys of the group about the impact a shortage of electricity could have, especially on the group study that they were planning for the next chapter.

They approached a wedding decorator to hire a generator for electricity, but it was not finalized due to some problems. Later they decided to buy a diesel generator rather than renting it. The generator was set up in Aditi's second house and a few pieces of furniture were also arranged: two large sofas, a medium sized wooden table, a TV, and a landline phone.

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It had been three days since the setup, but not a single topic had been discussed there. Instead, they were just watching football matches, making different food items, duplicating their teacher's voice in a funny way. It looked like they had made the house a new permanent hangout spot. As the entertainment filled Aditi's second house, Dolly couldn't help but wonder if their fun would soon outweigh the importance of the exam preparation, especially given their weak approach to the Gravitation chapter. However, considering the guys approach Dolly said, "The next session probably would be the last day of the gravitation chapter and we need to be attentive."

*****

In the very next session, the teacher walked to the front of the makeshift classroom, a gentle smile gracing her lips. She tapped a marker lightly on the whiteboard, drawing the group's attention from their lingering conversations. "Alright everyone," she began, her voice clear and calm, "let's dive into some concepts"

7.8 Motion of Planets and satellites

Planets

Planets move around the sun due to the gravitational attraction of the Sun. The path of these planets are elliptical with the Sun at one focus. However, due to the small differences in major and minor axes. The orbits can be treated as circular. Now, we are going to derive some formulas for planetary motion in terms of the radius of the orbit assuming it to be circular.

Let the mass of the Sun be M and the planet we are considering be m.

Orbital Speed (velocity)

Let the radius of the orbit be r and the speed of the planet in the orbit is v. By Newton's Second Law, the force on the planet equals its mass times the acceleration. Therefore,

$$\frac{GMm}{a^2} = m\left(\frac{v^2}{r}\right)$$

$$v = \sqrt{\frac{GM}{r}} \quad \ldots\ldots (7.13)$$

Time period

The time taken by a planet in completing one revolution is its Time period T. In one revolution it travels a linear distance of $2\pi r$ at speed v.

Thus, $T = \frac{2\pi r}{v}$

$$T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} \text{ (by putting } v = \sqrt{\frac{GM}{r}}$$

$$T = \frac{2\pi (r)^{3/2}}{\sqrt{GM}} \quad \ldots\ldots (7.14)$$

Motion of a satellite

When we discuss satellites, we will primarily focus on Earth satellites, satellites which revolve around the Earth. Their motion is similar to the motion of planets around the Sun, and therefore, Kepler's laws of planetary motion are applicable to them. The orbits of these satellites around the Earth are either circular or elliptical. The Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days.

We will consider a satellite in a circular orbit of a distance $(R_E + h)$ from the centre of the earth, where R_E = radius of the earth. If m is the mass of the satellite and V be its speed, the centripetal force required for this orbit is

$F_{centripetal} = \frac{mV^2}{(R_E+h)}$ ..(7.15). It is directed towards the centre. This centripetal force is provided by the gravitational force of the earth, which is

$F_{gravitational} = \frac{GmM_E}{(R_E+h)^2}$ .....(7.16) (M_E is the mass of the earth.)

Orbital speed (Velocity)

Equating R.H.S of Equation (7.15) and (7.16) and cancelling out m, we get $V^2 = \frac{GM_E}{(R_E+h)}$ or $V = \sqrt{\frac{GM_E}{(R_E+h)}}$ .....(7.17)

Thus, V decreases as h increases. From equation (7.17), the speed V for $h = 0$ is $V = \sqrt{\frac{GM_E}{R_E}} = \sqrt{gR_E}$ (7.18), we have used the relation $g = \frac{GM_E}{R_E^2}$.

Time period

In every orbit, the satellite traverses a distance $2\pi(R_E + h)$ with speed V.

Therefore, its time period (T),

$T = \frac{2\pi(R_E+h)}{V} = \frac{2\pi(R_E+h)^{\frac{3}{2}}}{\sqrt{GM_E}}$..... (7.19),

on substituting the value of V from Equation (7.18). Squaring both sides of Equation (7.19), we get $T^2 = k\,(R_E + h)^3$ ....(7.20) (where $k = 4\,\pi^2/GM_E$), which is Kepler's law of periods, as applied to motion of satellites around the Earth.

For a satellite very close to the surface of the Earth, h can be neglected in comparison to R_E in Equation (7.20).

Hence, for such satellites, T is T_0,

where $T_0 = 2\pi\sqrt{\frac{R_E}{g}}$ ....(7.21)

If we substitute the numerical values $g \simeq 9.8ms^{-2}$ and $R_E = 6400km.$, we get $T_0 = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}}$, Which is approximately 85 minutes.

Example 7.5 A rocket is fired from the earth towards the Sun. At what distance from the earth's centre is the gravitational force on the rocket zero?

Mass of the Sun = $2 \times 10^{30} kg$, mass of the Earth = $6 \times 10^{24} kg$. Neglect the effect of other planets (orbital radius = $1.5 \times 10^{11} m$).

Solution: Mass of the Sun = $2 \times 10^{30} kg$, mass of the Earth = $6 \times 10^{24} kg$, orbital radius $r = 1.5 \times 10^{11} m$.

Let mass of the rocket = m

Let x be the distance from the earth center where the gravitational force working on the rocket becomes zero.

From Newton's law of gravitation, we can equalize gravitational forces acting on the rocket under the effect of the sun and the earth: $\frac{GmM_S}{(r-x)^2} = \frac{GmM_E}{x^2}$

$$\Rightarrow \left(\frac{r-x}{x}\right)^2 = \frac{M_S}{M_E}$$

$$\Rightarrow \frac{r-x}{x} = \frac{2\times10^{30} kg}{60\times10^{24}} = 577.35$$

$$\Rightarrow 1.5 \times 10^{11} - x = 577.35x$$

$$\Rightarrow x = \frac{1.5\times10^{11}}{578.35} = 2.6 \times 10^{8}\ m.$$

ENERGY OF AN ORBITING SATELLITE

The kinetic energy of a satellite in a circular orbit with speed v is $\frac{1}{2} mv^2 = \frac{1}{2}\frac{mGM_E}{(R_e+h)} = \frac{GM_E m}{2(R_e+h)}$…(7.22) (where m is the mass of the satellite and M_E the is the mass of Earth)

The gravitational potential energy at distance $(R_e + h)$ from the centre of the earth is $-\frac{GMm}{R_e+h}$ ….(7.23)

The $K.E.$ is positive whereas the $P.E.$ is negative. However, in magnitude the $K.E.$ is half the $P.E.$ (we can say that from Equations 7.22 and 7.23) so that the total E is negative.

The total energy of a circularly orbiting satellite is thus negative, with the potential energy being negative but twice the magnitude of the positive kinetic energy.

When the orbit of a satellite becomes elliptic, both the $K.E.$ and $P.E.$ vary from point to point.

The total energy which remains constant is negative as in the circular orbit case. This is what we expect, since as we have discussed before if the total energy is positive or zero, the object escapes to infinity. Satellites are always at a finite distance from the earth and hence their energies cannot be positive or zero.

Example 7.6 A satellite of mass $1000\ kg$ is launched to revolve around the earth in an orbit at a height of $270\ km$ from the earth's surface. Kinetic energy of the satellite in this orbit is____________ $\times 10^{10} J$. (Mass of earth $= 6 \times 10^{24}\ kg$, Radius of earth $= 6.4 \times 10^{6} m$, Gravitational constant $= 6.67 \times 10^{-11} Nm^2 kg^{-2}$)

Solution: The kinetic energy (KE) of a satellite revolving around the Earth can be expressed with the following formula:

$$K.E. = \frac{1}{2}\ mv^2 = \frac{1}{2}\frac{mGM_e}{r} = \frac{GM_e m}{2r}$$

For this satellite, the radius r is the sum of the Earth's radius R_E and the height h above the Earth's surface:

$$r = R_E + h$$

Given: Mass of the satellite $(m) = 1000kg$, Mass of the earth $(M_e) = 6 \times 10^{24} kg$.

Radius of the earth $(R_E) = 6.4 \times 10^6 m$

Height of the orbit above the earth's surface

$(h) = 270\ km = 2.7 \times 10^5 m$

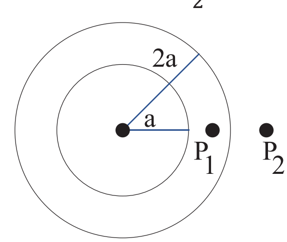
Gravitational constant $(G) = 6.67 \times 10^{-11} Nm^2 kg^{-2}$.

Substituting these values into the equation:

$$K.E. = \frac{(6.67 \times 10^{-11} Nm^2 kg^{-2} \times 6 \times 10^{24} kg \times 1000 kg)}{2(6.4 \times 10^6 + 2.7 \times 10^5)}$$

$$= 3 \times 10^{10} J$$

Example 7.7 A uniform solid sphere of mass M and radius a is surrounded symmetrically by a uniform thin spherical shell of equal mass and radius $2a$. Find the gravitational field at a distance (a) $\frac{3}{2}a$ from the centre, (b) $\frac{5}{2}a$ from the centre.



Solution: Figure shows the situation. The point P_1 is at a distance $\frac{3}{2}a$ from the centre and P_2 is at a distance $\frac{5}{2}a$ from the centre. As P_1 is inside the cavity of the thin spherical shell, the field here due to the shell is zero. The field due to the solid sphere is $E = \frac{GM}{\left(\frac{3}{2}a\right)^2} = \frac{4GM}{9a^2}$

This is also the resultant field. The direction is towards the centre. The point P_2 is outside the sphere as well as the shell. Both may be replaced by single particles of the same mass at the centre. The field due to each of them is $E' = \frac{GM}{\left(\frac{5}{2}a\right)^2} = \frac{4GM}{25a^2}$

The resultant field is $E, = 2\ E' = \frac{8GM}{25a^2}$ towards the centre.

Example 7.8 A satellite is to revolve round the earth in a circle of radius 800km. With what speed should this satellite be projected into orbit? What will be the time period? Take g at the surface = $9.8ms^{-2}$ and radius of the earth = 6400 km.

Solution: Suppose, the speed of the satellite is v. The acceleration of the satellite is $\frac{v^2}{r}$, where r is the radius of the orbit. The force on the satellite is $\frac{GMm}{r^2}$ with usual symbols. Using Newton's second law, $\frac{GMm}{r^2} = m\frac{v^2}{r}$

Or, $v^2 = \frac{GM}{r} = \frac{gR^2}{r} = \frac{(9.8ms^{-2})(6400km)^2}{(8000\ km)}$

Giving $v = 7.08kms^{-1}$

The time period is $\frac{2\pi r}{v} = \frac{2\pi(8000km)}{(7.08kms^{-1})} \approx 118minutes.$

Example 7.9 A satellite of $\boldsymbol{10^3 kg}$ mass is revolving in circular orbit of radius $\boldsymbol{2R}$. If $\boldsymbol{\frac{10^4 R}{6} J}$ energy is supplied to the satellite, it would revolve in a new circular orbit of radius _____

(use $\boldsymbol{g = 10m/s^2}$, R = radius of earth)

Solution: To find the new radius of the orbit after the energy $\boldsymbol{\frac{10^4 R}{6} J}$ is supplied. We will use the concept of total mechanical energy $(\boldsymbol{E})$ in an orbit which is given by

$$\boldsymbol{E = -\frac{GMm}{2r}}$$

Where $\boldsymbol{G}$ is the gravitational constant

$\boldsymbol{M}$is the mass of the earth

$\boldsymbol{r}$is the radius of the orbit

$$\boldsymbol{E_i = -\frac{GMm}{2\times(2R)}} \qquad \text{(given } \boldsymbol{r = 2R}\text{)}$$

$$\boldsymbol{= -\frac{GMm}{4R}}$$

Now, the energy supplied to the satellite is given as $\frac{\mathbf{10^4 R}}{\mathbf{6}}\boldsymbol{J}$. The new total energy $\boldsymbol{E_f}$ will be the sum of the initial energy plus the supplied energy.

$\boldsymbol{E_f} = \boldsymbol{E_i} +$ Energy supplied

$$\boldsymbol{E_f} = -\frac{\boldsymbol{GMm}}{\boldsymbol{4R}} + \frac{\mathbf{10^4}\boldsymbol{R}}{\mathbf{6}}$$

And $\boldsymbol{E_f} = -\frac{\boldsymbol{GMm}}{\boldsymbol{2r_f}}$

Therefore

$$-\frac{\boldsymbol{GMm}}{\boldsymbol{2r_f}} = -\frac{\boldsymbol{GMm}}{\boldsymbol{4R}} + \frac{\mathbf{10^4}\boldsymbol{R}}{\mathbf{6}}$$

$$\Rightarrow \frac{\boldsymbol{GMm}}{\boldsymbol{2r_f}} = \frac{\boldsymbol{GMm}}{\boldsymbol{4R}} - \frac{\mathbf{10^4}\boldsymbol{R}}{\mathbf{6}}$$

(Since $\boldsymbol{GMm} = \boldsymbol{gR^2m}$ using $\frac{\boldsymbol{GMm}}{\boldsymbol{R^2}} = \boldsymbol{mg}$)

$$\frac{\boldsymbol{gR'm}}{\boldsymbol{2r_f}} = \frac{\boldsymbol{gR'm}}{\boldsymbol{4R}} - \frac{\mathbf{10^4}\boldsymbol{R}}{\mathbf{6}}$$

$$\frac{\mathbf{1}}{\boldsymbol{2r_f}} = \frac{\mathbf{1}}{\boldsymbol{4R}} - \frac{\mathbf{10'}\boldsymbol{R}}{\boldsymbol{6gR^2m}}$$

(using $\boldsymbol{g} = \mathbf{10}\boldsymbol{m/s^2}$ mass of the satellite $\boldsymbol{m} = \mathbf{10^3}\boldsymbol{kg}$)

$$\frac{\mathbf{1}}{\boldsymbol{2r_f}} = \frac{\mathbf{1}}{\boldsymbol{4R}} - \frac{\mathbf{10'}\boldsymbol{R}}{\mathbf{6\times10\times}\boldsymbol{R^2}\mathbf{\times10^3}}$$

$$\Rightarrow \frac{\mathbf{1}}{\boldsymbol{2r_f}} = \frac{\mathbf{1}}{\boldsymbol{4R}} - \frac{\mathbf{1}}{\boldsymbol{6R}}$$

Therefore $\boldsymbol{2r_f} = \mathbf{12}\boldsymbol{R}$

Thus $\boldsymbol{r_f} = \boldsymbol{6R}$

Hence, the new radius of the circular orbit is $\boldsymbol{6R}$**.**

Exercises

1. A body weighs $32N$ on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to one third the radius of the earth?
2. Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed $400N$ on the surface?

3. A rocket is fired vertically with a speed of $5kms^{-1}$from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth $= 6.0 \times 10^{24}kg$; mean radius of the earth $= 6.4 \times 10^{6}m;\ G = 6.67 \times 10^{-11}Nm^{2}kg^{-2}$.
4. The escape speed of a projectile on the earth's surface is $20kms^{-1}$. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.
5. A satellite of mass $1000kg$ is launched to revolve around the earth in an orbit at a height of $270\ km$ from the earth's surface. Kinetic energy of the satellite in this orbit is____________$\times 10^{10}J$.
6. Two planets A and B are orbiting a common star in circular orbits of radii R_A and R_B respectively, with $R_B = 2R_A$. Planet B is $4\sqrt{2}$ times more massive than planet A. The ratio $\left(\frac{L_B}{L_A}\right)$ of angular momentum L_B of planet B to that of planet $A(L_A)$ is closest to integer ……..
7. If the earth suddenly shrinks to $\frac{1}{64}th$ of its original volume with its mass remaining the same, the period of rotation of earth becomes $\frac{24}{x}\ h$. The value of x is: (Hint : apply conservation of momentum)
8. If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at a depth $\alpha h (h << Re)$from the earth surface. The value of α will be __________. (use $R_E = 6400\ KM$)
9. A satellite of mass $1000\ kg$ is launched to revolve around the earth in an orbit at a height of $270\ km$ from the earth's surface. Kinetic energy of the satellite in this orbit is_____________ $\times 10^{10}J$ (Mass of earth $= 6 \times 10^{24}\ kg$, Radius of earth $= 6.4 \times 10^{6}m$, Gravitational constant $= 6.67 \times 10^{-11}Nm^{2}kg^{-2}$)
10. Acceleration due to gravity on the surface of earth is $'g'$. If the diameter of earth is reduced to one third of its original value and mass remains unchanged, then the acceleration due to gravity on the surface of the earth is ________ g.
11. If the radius of earth is reduced to three-fourth of its present value without change in its mass then the value of duration of the day of earth will be _________ hours 30 minutes.
12. If the earth suddenly shrinks to $\left(\frac{1}{64}\right)$th of its original volume with its mass remaining the same, the period of rotation of earth becomes $\left(\frac{24}{x}\right)h$. The value of x is ___________.
13. Two heavy spheres each of mass $100kg$ and radius $0.10m$ are placed $1.0m$ apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres ?

Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

14. If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at a depth $\alpha h(h << R_e)$ from the earth surface. The value of α will be __________. (use $R_e = 6400KM$)
15. Two satellites $S1$ and $S2$ are revolving in circular orbits around a planet with radius $R1 = 3200km$ and $R2 = 800km$ respectively. The ratio of speed of satellite $S1$ to the speed of satellite $S2$ in their respective orbits would be $\frac{1}{x}$ where $x =$____________.
16. The radius in kilometers to which the present radius of earth ($R = 6400\ km$) to be compressed so that the escape velocity is increased 10 times is ____________.
17. A ball is dropped from the top of a $100m$ high tower on a planet. In the last $12s$ before hitting the ground, it covers a distance of $19m$. Acceleration due to gravity (in ms^2) near the surface on that planet is ______.

Answers

1. 18	2. 200	3. $1.6 \times 10^6 m$	4. 56.57 km/h
5. 3	6. 8	7. 16	8. 2
9. 3	10. 9	11. 13	12. 16
13. 0, $-2.7 \times 10^{-8} J/kg$	14. 2	15. 2	16. 64
17. 2			

Chapter - 8
Mechanical Properties of Solids and Liquids

Gita stepped into the classroom to start the first session of the chapter 'Mechanical Properties of Solids and Liquids.' She turned to the blackboard, picked up a marker, and with a quiet confidence that settled the bustling room, began. "Good morning everyone. Let's start."

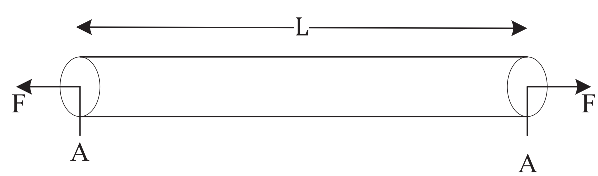
8.1 Elasticity:

The property by which an object tends to attain its original shape and size once a deforming force is removed. This property is known as elasticity.

Deforming Force- An external force applied to an object that causes it to change its shape, size, or both.

8.2 Stress and Strain

Stress: When a deforming force acts on a body, a restoring force is developed within the body, equal in magnitude and opposite in direction to the deforming force. The restoring force per unit area is known as stress. If a deforming force F is applied to a body with a cross- sectional area A, as shown in the figure. The magnitude of the stress is given as: $Stress = F/A$ ....(8.1) Its SI unit is N/m^2 or pascal.



Strain: Strain is a measurement of the deformation of a material under stress. For example, if the length of the cylinder increases by ΔL due to stress, then strain is given as $Strain = \frac{\Delta L}{L}$....(8.2), where L is the original length of the cylinder.

When a force is applied to a solid body, it experiences both stress (the internal force per unit area) and strain (the resulting deformation). There are different types of stress each of which produces a corresponding strain within the body.

(i) **Tensile stress**: Tensile stress is the force per unit area that is developed when a body is stretched, resulting in an increase in its length.

(ii) **Compressive stress:** It is the force per unit area that is developed when a body is compressed, resulting in decrease in its length.

Both tensile stress and compressive stress are also known as longitudinal stress, because they act along the length of the body.

Longitudinal strain: In both cases, tensile stress and compressive stress, there is a change in length of the body ΔL. The change in length ΔL to the original length L of the body is known as longitudinal strain.

(iii) **Tangential (shearing) stress:** It is the stress (restoring force) that occurs when a force acts parallel to the cross sectional area of an object as shown in Figure 8.1.

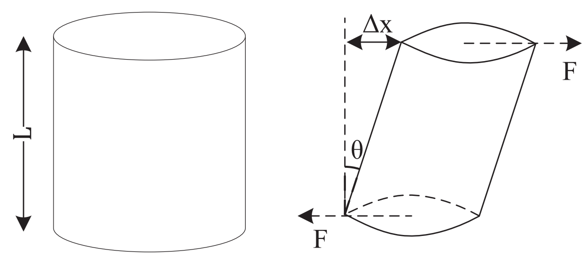


Fig 8.1

Shearing strain: It is a type of strain that occurs when an object experiences change in its shape rather than a change in length or volume, due to tangential forces acting on it. Shearing strain $= \frac{\Delta x}{L} = tan\ \theta$ …..(8.3)

(iv) **Hydraulic stress:** When a body is submerged in fluid under high pressure and is compressed uniformly on all sides, the body develops internal restoring forces that are equal and opposite to the forces applied by the fluid. This internal force developed per unit area is called hydraulic stress. In magnitude, hydraulic stress is equal to the hydraulic pressure, which is the applied force per unit area.

Volume strain: The strain produced by a hydraulic pressure is called volume strain and is defined as the ratio of change in volume Δv to the original volume v.

$Volume\ strain = \frac{\Delta v}{v}$…….(8.4)

8.3 Hooke's Law:

Hooke's law states that the strain in a solid is proportional to the applied stress within the elastic limit of that solid. Hence, stress ∝ strain

$stress = k \times strain$....(8.5)

Where k is the proportionality constant and is known as modulus of elasticity.

Example 8.1 The length of a light string is $1.4m$ when the tension on it is $5N$. If the tension increases to $7N$, the length of the string is $1.56m$. The original length of the string is__________ m.

Solution: To find the original length of the string, we use the relationship between tension, elasticity constant (K), and the change in length of the string that is Hooke's law.

The equation for tension: $\frac{T}{(l-l_0)} = K$, where l is the length of the string under tension and l_0 is the original length.

When the tension is $5N$, the equation becomes: $\frac{5}{(1.4-l_0)} = K$ .....(i)

When the tension increases to 7 N, the equation is: $\frac{7}{(1.56-l_0)} = K$......(ii)

By setting up a ratio from the two equations, we have: $\frac{5}{(1.4-l_0)} = \frac{7}{(1.56-l_0)}$ ....(iii)

Solving this proportion gives us the original length l_0: $l_0 = 1m$. Thus, the original length of the string is 1 meter.

8.4 Stress-Strain Relationship:

To understand the relationship between stress and strain, we plot a typical graph for a metal between its stress and the strain it produces as shown in Figure 8.2. A stress-strain graph could vary from material to material, but here we are plotting a typical or general graph for a material between its stress and strain. The graph is also known as the stress-strain curve. In the graph, we can see that in the region between O to A, the curve is linear. In this region,

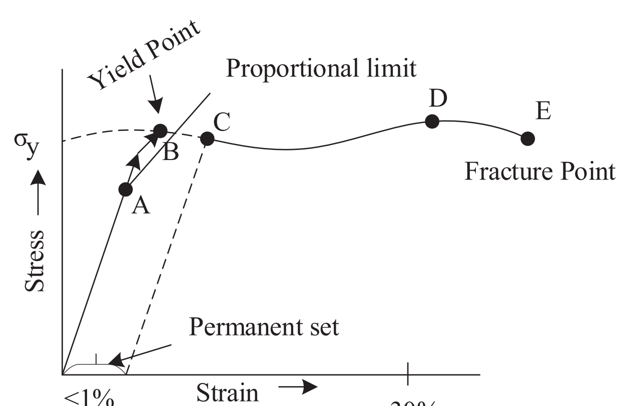


Fig 8.2

Hooke's law is obeyed. The body regains its original dimensions when the applied force is removed. In this region, the solid behaves as an elastic body.

In the region from A to B, stress and strain are not proportional. Nevertheless, the body still returns to its original dimension when the load is removed. The point B in the curve is known as yield point (also known as elastic limit) and the corresponding stress is known as yield strength (σy) of the material. If the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress. The portion of the curve between B and D shows this. When the load is removed, say at some point C between B and D, the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not zero. The material is said to have a permanent set. The deformation is said to be plastic deformation. The point D on the graph is the ultimate tensile strength (σu) of the material. Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at point E. If the ultimate strength and fracture points D and E are close, the material is said to be brittle. If they are far apart, the material is said to be ductile.

8.5 Modulus of elasticity:

The ratio of stress and strain is called modulus of elasticity. It is an important characteristic of a material.

8.6 Young's Modulus:

It is a property of material that tells us how easily the material could be stretched or deformed (meaning how stiff the material is), and is defined as the ratio of tensile stress $\left(\sigma = \frac{F}{A}\right)$ to tensile strain $\left(\varepsilon = \frac{dl}{L}\right)$. $Y = \frac{\sigma}{\varepsilon}$

$$Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)}$$

$= \frac{(F \times L)}{(A \times \Delta L)}$......(8.6) The SI unit of Young's modulus is Pascal(pa) or N/m^2.

Example 8.2 A steel rod has a radius of $20mm$ and a length of $1.0m$. A $200kN$ force stretches it along its length. Calculate (a) stress (b) elongation (c) strain on the rod. Given that young's modulus of steel is $2.0 \times 10^{11} Nm^{-2}$.

Solution: $Stress = \frac{F}{A} = \frac{F}{\pi r^2}$

$$= \frac{200 \times 10^3 N}{3.14 \times (0.020m)^2}$$

$$= 1.59 \times 10^8 Nm^{-1}$$

The elongation,

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right)$$

$$= \frac{(1.59 \times 10^{8}\ Nm^{-1})(1m)}{2.0 \times 10^{11} Nm^{-2}}$$

$$= 0.796 \times 10^{-3}\ m$$

$$\text{Strain} = \frac{\Delta L}{L}$$

$$= \frac{0.796 \times 10^{-3} m}{1m}$$

$$= 7.957 \times 10^{-4}$$

Example 8.3 A copper wire of length 2.2m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net elongation is found to be 0.70 mm. Obtain the load applied.

Solution: The copper and steel wires are under a tensile stress because they have the same tension (equal to the load W) and the same area of cross-section A.

From Eq. (8.7) we have stress = strain × Young's modulus. Therefore $\frac{W}{A} = Y_c \times \left(\frac{\Delta L_c}{L_c}\right) = Y_s \times \left(\frac{\Delta L_s}{L_s}\right)$ where the subscripts c and s refer to copper and stainless steel respectively.

Or, $\frac{\Delta L_c}{\Delta L_s} = \left(\frac{Y_s}{Y_c}\right) \times \left(\frac{L_c}{L_s}\right)$

Given $L_c = 2.12\ m, L_s = 1.6\ m,$

$Y_c = 1.1 \times 10^{11}\ N.m^{-2}$, and $Y_s = 2.0 \times 10^{11}\ N.m^{-2}$.

$$\frac{\Delta L_c}{\Delta L_s} = \left(\frac{2.0 \times 10^{11}}{1.1 \times 10^{11}}\right) \times \left(\frac{2.2}{1.6}\right) = 2.5$$

The total elongation is given to be

$$\Delta L_c + \Delta L_s = 7.0 \times 10^{-4} m$$

Solving the above equations, $\Delta L_c = 5.0 \times 10^{-4} m,$ and $\Delta L_s = 2.0 \times 10^{-4}\ m$.

$$\text{Therefore } W = \frac{A \times Y_c \times \Delta L_c}{L_c} = \pi\ (1.5 \times 10^{-3})^2 \times \left[\frac{5.0 \times 10^{-4} \times 1.1 \times 10^{11}}{2.2}\right]$$

$$= 1.8 \times 10^{2}\ N$$

Example 8.4 A $14.5 kg$ mass, fastened to the end of a steel wire of unstretched length $1.0 m$, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

Solution: In the question it is given that mass m is $14.5kg$

Length of the steel wire is $l = 1m$

Angular velocity is $\omega = 2rev/s$

Cross sectional area of the wire is $a = .065cm^2 = 6.5 \times 10^{-6}\, m^2$

Considering the elongation of the wire when the mass is at the lowest point of its path to be Δl.

The total force on the mass when the mass is placed at the position of the vertical circle is given by

$F = mg + ml\omega^2$

$\Rightarrow F = 14.5 \times 9.8 + 14.5 \times 1 \times 2^2 = 200.1\, N$

Young's modulus is given by:

$Y = \frac{stress}{strain}$

$\Rightarrow\ Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta l}{l}\right)}$

$\Rightarrow\ \Delta l = \frac{Fl}{AY}$

We know that Young's modulus for steel is $2 \times 10^{11} Pa$

Therefore,

$\Delta l = \frac{(220.1 \times 1)}{(6.5 \times 10^{-6})} \times (2 \times 10^{11}) = 1.87 \times 10^{-3} m.$

Example Two persons pull a wire towards themselves. Each person exerts a force of $200N$ on the wire. Young's modulus of the material of wire is $1 \times 10^{11} Nm^{-2}$. Original length of the wire is $2m$ and the area of cross section is $2cm^2$. The wire will extend in length by ________ cm.

Solution: We know young's modulus $(y) = \frac{Stress}{Strain}$

Where stress (σ) is given by: $\sigma = \frac{F}{A}$

And strain (E) is: $E = \frac{\Delta l}{L}$

And, we are given $F = 200N$

(Because each person pulls the wire with this force in just opposite direction (i.e. themselves) but in this case the total force in the wire should be considered as tension by one side, so it will remain $200N$).

$A = 2cm^2 = 2 \times 10^{-4}m^2,$

$L = 2m$

$Y = 1 \times 10^{11}Nm^{-2}$

The stress (σ):

$\sigma = \frac{200N}{2\times10^{-4}m^2} = 10^6Nm^{-2}$

Using young's modulus formula:

$y = \frac{Stress}{strain} \Rightarrow Strain = \frac{stress}{y}$

Thus, the strain (E) is:

$E = \frac{10^6Nm^{-2}}{1\times10^{11}Nm^{-2}} = 10^{-5}$

Now

E = $\Delta l/$L

$\Delta l =$ E×L

$\Rightarrow 10^{-5} \times 2m = 2 \times 10^{-5}m$

In micrometer

$= 20\mu m$

Therefore, the wire will extend in length by $20\mu m$.

David was unable to understand many steps of the solution of examples 2 and 3. He expressed his concern to Arnold, but Arnold replied, "There is nothing much challenging in these problems, it is just about remembering definitions and formulas, once important formulas and definitions are in your mind, it won't be tough." However he made him understand that the tension here is completely due to the load attached to the wire (of copper and steel) and it is along the length of the wire. "I got it!" David exclaimed, a broad grin spreading across his face, a sudden change of realization echoing in his mind.

*****

Most of them, after attending the institution, stopped at Aditi's second house. The house was found dirty and wet. " Damn it, check if there is water on the electrical appliances," Kimya screamed, adding "It's our mistake; all of the windowpanes are open." Within a few hours, the water was removed from the house, and then the generator was started, and the TV and lights were checked. “Thank God, it is working,” Aditi said, relaxing. Evening had already passed, and it was getting late into the night. Some were looking through class notebooks while Kimaya and Dolly were talking about the savings that were left for the group's joint expenses. They departed for their homes at around 2 AM.

*****

It was during the next day afternoon the institution’s gateman made a brief call to Aditi (Aditi and the gateman were well-known to each other as Aditi used to give money as tip for sometimes to him), informing her of massive political protests happening in the city. He further relayed the principal’s advice to students using vehicles for their transportation to exercise caution as reports of such property damage had increased across the city. The urgency of this message felt harsh to Aditi, prompting her to suggest to the others that they should finish the remaining modules of the chapter through group study in the village compound.

*****

It was Dolly who prepared notes on the modules to deliver among the classmates and the next day at the established spot (Aditi’s second house). She started sharing her notes

8.7 Bulk Modulus:

It is defined as the ratio of hydraulic stress to the corresponding hydraulic strain. $B = -\frac{p}{\left(\frac{\Delta v}{v}\right)}$ ....(8.7). Here p is the hydraulic pressure applied to the object. The negative sign ensures that the bulk modulus is a positive value, since an increase in pressure (positive p) causes a decrease in volume (Δv is negative). The SI unit for bulk modulus is N/m^2 or pascal, which is the same as the unit of pressure.

Hydraulic stress is the internal restoring force per unit area that develops in response to the applied pressure.

The value of bulk modulus is always positive and its SI unit is Nm^{-2} or pascal (same as of pressure).

The reciprocal of bulk modulus is known as compressibility and denoted by k. It is termed as the fractional change in volume per unit increase in pressure.

$$k = -\left(\frac{1}{\Delta p}\right) \times \left(\frac{\Delta v}{v}\right) = \left(\frac{1}{B}\right) \text{.....(8.8)}$$

Example A sample of a liquid is kept at 1 atm. It is compressed to 5atm which leads to a change of volume of $0.8cm^3$. If the bulk modulus of the liquid is $2GPa$, the initial volume of the liquid was _______ litre.

(Take 1atm = $10^5 pa$)

Solution: Given, initial pressure of liquid

$(P_i) = 1atm$

Final pressure of liquid $\left(P_f\right) = 5atm$

Change in pressure $(dp) = P_f - P_i = 4\ atm$

$$= 4 \times 10^5 pa$$

Change in volume $(dr) = -0.8cm^3$

Bulk modulus $(B) = 2 \times 10^{-9} pa$

Now, $B = -\frac{dp}{\left(\frac{dv}{v}\right)} \Rightarrow v = -B\left(\frac{dv}{dP}\right)$

$\Rightarrow v = -2 \times 10^{-9} \times \left(\frac{0.8\times10^{-6}}{4\times10^5}\right)$

$= 4 \times 10^{-3} m^3 = 4litre$

8.8 Modulus of rigidity:

It is also called shear modulus. It is the ratio of shearing stress to the corresponding shearing strain, and is denoted by G.

$$G = \frac{shearing\ stress\ (\sigma_S)}{shearing\ strain}$$

$$G = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta x}{L}\right)}$$

$$= \frac{(F\times L)}{(A\times\Delta x)} \quad ..... (8.9)$$

Similarly, from Eq. (8.9) $G = \frac{\left(\frac{F}{A}\right)}{\theta} = \frac{F}{(A\times\theta)}$ ...(8.10)

The shearing stress σ_s can also be expressed as

$\sigma_s = G \times \theta \ldots$ (8.11)

The SI unit of shear modulus is Nm^{-2} or Pa.

8.9 POISSON'S RATIO

The strain perpendicular to the applied force is called lateral strain. It is found that within the elastic limit, lateral strain is directly proportional to the longitudinal strain. The ratio of the lateral strain to the longitudinal strain in a stretched wire is called Poisson's ratio.

If the original diameter of the wire is d and the contraction of the diameter under stress is Δd, the lateral strain is $\frac{\Delta d}{d}$. If the original length of the wire is L and the elongation under stress is ΔL, the longitudinal strain is $\Delta L/L$. Poisson's ratio is then $\frac{\left(\frac{\Delta d}{d}\right)}{\left(\frac{\Delta L}{L}\right)}$ or $\left(\frac{\Delta d}{\Delta L}\right) \times \left(\frac{L}{d}\right)$ .....(8.12)

Poisson's ratio is a ratio of two strains, the lateral strain to the longitudinal strain; it is a pure number and has no dimensions or units. Its value depends only on the nature of the material.

8.10 Elastic Potential Energy in a Stretched Wire

When a wire is put under a tensile stress, work is done against the interatomic forces. This work is stored in the wire in the form of elastic potential energy. When a wire of original length L and area of cross-section A is subjected to a deforming force F along the length of the wire, let the length of the wire be elongated by l. Then from Equation (8.8), we have $F = YA \times \left(\frac{l}{L}\right)$. Here Y is the Young's modulus of the material of the wire. Now for a further elongation of infinitesimal small length dl, work done dW is Fdl or $\frac{YAldl}{L}$. Therefore, the amount of work done (W) in increasing the length of the wire from L to $L + l$, that is from $l = 0$ to $l = l$ is

$$W = \int_0^l YAl\frac{dl}{L} = YA \times \frac{l^2}{2l}$$

$$= \frac{1}{2} \times Y \times \left(\frac{l}{L}\right)^2 \times AL$$

$$= \frac{1}{2} \times Young's\ modulus \times (strain)^2 \times volume\ of\ the\ wire$$

$$= \frac{1}{2} \times stress \times strain \times volume\ of\ the\ wire$$

This work is stored in the wire in the form of elastic potential energy (U). Therefore the elastic potential energy per density of the wire (u) is

$$u = \frac{1}{2}\,\sigma\,\varepsilon.$$

Example 8.5 A steel wire of length $2m$ and Young's modulus $2.0 \times 10^{11} Nm^{-2}$ is stretched by a force. If Poisson ratio and transverse strain for the wire are 0.2 and 10^{-3} respectively, then the elastic potential energy density of the wire is ___________ $\times 10^5$. (In SI units).

Solution: To find the elastic potential energy density of the steel wire, we need to use the given information and formulae for strain and energy density. Given:

The length of the wire, $l = 2m$

Young's modulus $Y = 2.0 \times 10^{11} N/m^2$

Poisson's ratio, $\mu = 0.2$

Transverse strain, $\frac{\Delta r}{r} = 10^{-3}$

The formula for Poisson's ratio is: $\mu = -\frac{\left(\frac{\Delta r}{r}\right)}{\left(\frac{\Delta l}{l}\right)}$

From this, we solve for the longitudinal strain $\frac{\Delta l}{l}$:

$\frac{\Delta l}{l} = \left(\frac{1}{\mu}\right) \times \left(\frac{\Delta r}{r}\right)$

Substitute the given values:

$\frac{\Delta l}{l} = \left(\frac{10}{2}\right) \times 10^{-3} = 5 \times 10^{-3}$

The elastic potential energy density (u) is given by: $u = \left(\frac{1}{2}\right) Y \varepsilon_l^2$, where $\varepsilon_l = \frac{\Delta l}{l}$. Plug in the values: $u = \frac{1}{2} \times 2 \times 10^{11} \times (5 \times 10^{-3})^2$ .Simplify further: $u = \left(\frac{1}{2}\right) \times 2 \times 10^{11} \times 25 \times 10^{-6}$ (in SI units). $u = 25 \times 10^5$ (in SI units) Thus, the elastic potential energy density of the wire is 25×10^5 SI units.

*****

After an hour break, the sharing process continued in the next casual session for the 'prepared notes'.

8.10 Pressure

It is the perpendicular force applied to the surface of an object per unit area over which the force is distributed. $P = \frac{F}{A}$. For example, when an object is submerged in a fluid at rest, the fluid exerts a force on its surface as shown in Figure 8.3. This force is always normal to the object's surface. Pressure is a scalar quantity, its SI unit is Pascal or Nm^{-2}.

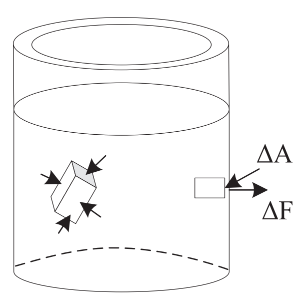


Fig 8.3

Another important quantity in this module is density represented as ρ and it is equal to $\rho = \frac{m}{v}$. Its SI unit is kgm^{-3}.

Other common units of pressure are $1\ atm = 1.01 \times 10^5\ Pa, 1\ bar = 10^5\ Pa, 1\ torr = 133\ Pa = 0.133\ kPa, 1\ mm\ of\ Hg = 1\ torr = 133\ Pa$

Hg = 1torr = 133Pa

Pressure due to a fluid column: It is pressure exerted by a fluid at rest, caused by the weight of the fluid above and is determined as $p_{due\ to\ fluid\ column} = \rho g h, p_{fluid} = p_o + \rho g h$ ….(8.13)

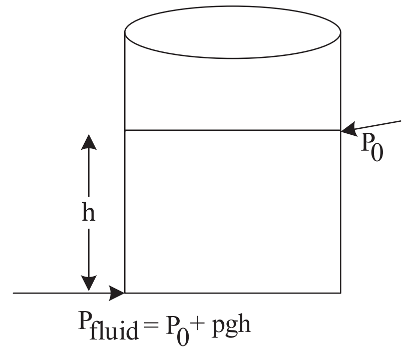


Fig 8.4

Where,

P_o denotes the pressure at the point of reference, P_{fluid} denotes the pressure at a specific point in a fluid, ρ denotes the density of the fluid. g denotes the acceleration due to gravity (taking into consideration for earth $g = 9.8\ m/s^2$) and h denotes the height from the point of reference (Figure 8.4).

Example 8.6 What is the pressure on an object lying below $1000m$ below the surface of an ocean?

Solution: $h = 1000m$ and $\rho = 1000\ kgm^{-3}$. Take $g = 10ms^{-2}$

$$P = P_a + \rho g h = 1.01 \times 10^5 Pa + 1000\ kgm^{-3} \times 10ms^{-2} \times 1000m$$

$$= 1.01 \times 10^5 Pa + 100 \times 100000\ Pa$$

$$\cong 1\ atm + 100\ atm = 101\ atm.$$

8.11 Pascal's Law:

It states that the pressure in a fluid at rest is the same at all points at the same height. Suppose a small cubicle size material is put in fluid. The material is so small such that we assume all the points are at the same height (Figure 8.5)

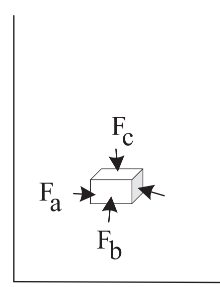


Fig 8.5

Its application-Hydraulic Lift: We can see Figure 8.6, it is a simple diagram of a hydraulic lift. This is the principle of the working of hydraulic lift. It works based on the principle of equal pressure transmission throughout a fluid (Pascal's Law).

Hydraulic Brakes: When you press the brake pedal, you apply pressure to a small piston in the brake system. This pressure is transmitted through the brake fluid to pistons in the wheel brakes, which then push the brake pads against the rotors to slow down or stop the vehicle.

Hydraulic presses: These machines use Pascal's law to exert a large force over a small area, which is used for tasks like compressing materials or pressing objects into molds.

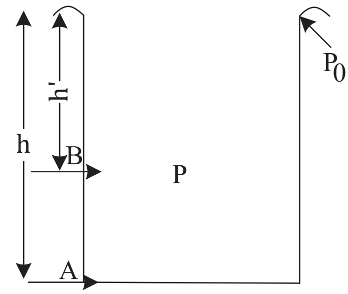


Fig 8.6

Example 8.7 A hydraulic press containing water has two arms with diameters 1.4cm and 28cm respectively. A force of $40N$ is applied on the surface of water in the thinner arm. The force required to be applied on the surface of water in the thicker arm to maintain equilibrium of water is _________ N.

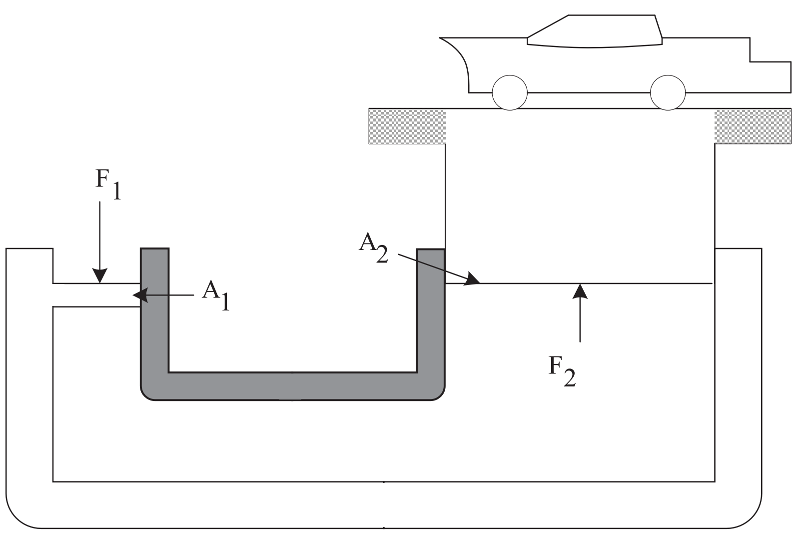
Solution: $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ (by pascal's law)

$$\Rightarrow F_2 = 40 \times \frac{A_2}{A_1}$$

$$\Rightarrow F_2 = 40 \times \left(\frac{28}{1.4}\right)^2$$

$$\Rightarrow F_2 = 40 \times 400$$

$$\Rightarrow F_2 = 16000\ N.$$



All her prepared notes ended after two consecutive casual sessions in the village. One week had already passed since they attended the previous formal school session.

Now, the situation in the city was normal, but the guys enjoyed their own casual study sessions for the chapter.

And, they were in the mood to complete the whole chapter in the same way. Aakash took the responsibility for the remaining parts, preparing notes and delivering them in the next session, scheduled after three days. No one intended to go home even though the session had concluded.

Aditi randomly said, “You know, if we have business of our own, we would go for more weekend trips.” Kimaya gave a hard look and said, "What business! Every business requires a lot of capital, which we don’t.” Aditi reacted gently, “ See the costliest rent we have is for the bus, collectively around 18,000 INR per month, totalling more than 2 Lakhs per year. And if we target to do our college in the same city or nearby, that means three years added to our post-HSC education. That would cost us around a total of 10 Lakhs INR. If we collect it at one time, can we get a second hand bus with it?”

“No way, " Dolly answered. "Even if we convince our parents for the plan and they pay all the transportation fees in advance, we won’t be able to buy a bus,”

The ongoing conversation was suddenly interrupted by a knock at the main door. Their discussion was happening on the first floor, while the house had two floors in total. Aakash opened the main door, located downstairs. It was Arnold’s Mom, with an umbrella in one hand and a tiffin on the other .

Aakash said “oh, it’s you. Welcome, Aunty, to my adda!” Arnold’s mom replied, “Thank You, where is Arnold? It’s very late (It was already 11 PM)” “Actually, we are working on our syllabus,” Aakash simply responded, thereafter showing a little interest in continuing the conversation. Nevertheless, he invited her upstairs, but she refused saying “No, it’s ok, take this launch box, there are some aloo parathas in it and you do one thing; give me the telephone number of this house.” She went back home after taking the number.

*****

As planned, Aakash prepared and began delivering notes in the next casual session.

8.12 Effect of gravity on fluid pressure

“The pressure exerted by a fluid at equilibrium due to the force of gravity increases with depth” as we go deeper in the liquid we experience more pressure. See in Figure 8.6 if rho (ρ) is the density of the liquid (fluid), P_o is the pressure at o, and we want to know the pressure at point A, then it is $P_o + \rho g h$ and at point B it is $P_o + \rho g h$’. We can observe that pressure is increasing with depth of the liquid (as $h > h'$, $P_A > P_B$).

Example 8.7 Mercury is filled in a tube of radius $5cm$ up to a height of $50cm$. The force exerted by mercury on the bottom of the tube is __________ N.

Solution: Density of mercury $= 1.36 \times 10^4\ kgm^{-3}, g = 9.8\ ms^{-2}, \pi = \frac{22}{7}, r = .05\ m, h = .5\ m$

$$A = \pi \times (0.05\ m)^2$$

$$A = 0.007854\ m^2$$

The pressure exerted by the liquid column $= \rho gh = 1.36 \times 10^4 \times 9.8 \times .5$

$$= 66640\ Pa$$

$$F = (\rho gh)A = (66640) \times (22/7) \times 0.007854 = 523.23\ N$$

8.13 Viscosity

The resistance that a fluid provides to its motion is called viscosity. It is like internal friction which acts when there is relative motion between different layers of the fluid. The internal forces between fluid layers create shearing stress and a corresponding strain rate.

The coefficient of viscosity (η), which is pronounced as 'eta', is defined as the ratio of the shearing stress to the strain rate for the fluid.

$$\eta = \frac{Shearing\ Stress}{Strain\ Rate} = \frac{\frac{F}{A}}{\frac{v}{l}} = \frac{Fl}{vA} \text{ ....(8.14)}$$

The SI unit of viscosity is poiseuille (PI), its other units are $N.sm^{-2}$ or $Pa.s$ The dimensions are $[ML^{-1}T^{-1}]$

8.14 Terminal Velocity and Stoke's law

Terminal Velocity: It is the highest speed an object attains while falling through a fluid (like air or water) under the influence of Gravity.

How an object attains Terminal Velocity: When an object starts to fall, it accelerates due to gravity, resulting in an increase of its speed, at the same time there is a drag force on the object acting against the gravitational force. And, when the drag force is equal to the gravitational force there is no net force and hence no further increase in its speed and the object continues to move with the constant speed, and the (constant) speed is called Terminal Velocity.

Stoke's Law

It states "a force exists on a sphere when the sphere moves through a viscous fluid and the force is proportional to the velocity of the object, viscosity η of the fluid and radius a of the sphere and is opposite to the direction of the motion of the sphere.”

$F = 6\pi\eta av$……..(8.15)

This law is an interesting example of retarding force, which is proportional to velocity. We can study its consequences on an object falling through a viscous medium. We consider a raindrop in air. It accelerates initially due to gravity. As the velocity increases, the retarding force also increases. Finally, when viscous force plus buoyant force becomes equal to the force due to gravity, the net force becomes zero and so does the acceleration. The sphere (raindrop) then descends with a constant velocity. Thus, in equilibrium, this terminal velocity v_t is given by $6\pi\eta av_t = \left(\frac{4\pi}{3}\right) a^3(\rho - \sigma)g$ where ρ and σ are mass densities of the sphere and the fluid, respectively.

We obtain $v_t = \frac{2a^2(\rho-\sigma)g}{(9\eta)}$ ..... (8.16) So the terminal velocity v_t depends on the square of the radius of the sphere and inversely on the viscosity of the medium.

8.15 Streamline And Turbulent Flow

Streamline Flow: A type of fluid flow where the velocity of the fluid at a specific point remains constant over time. However, the velocity may change as the fluid moves from one point to another. Streamline flow is also known as steady flow.

From this definition, we can infer that any fluid particle passing through a particular point will have the same velocity in streamline flow.

In steady flow, each particle flows a smooth path and the paths of the particles don’t cross each other as shown in Figure 8.7. The path taken by a fluid particle under a steady flow is streamline. And we know two streamlines can’t cross each other.

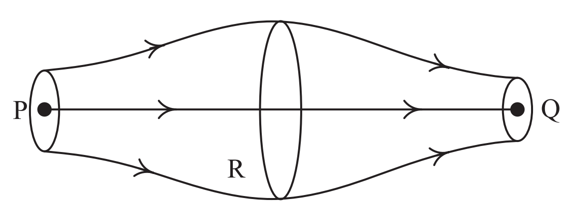


Fig 8.7

Regarding streamline we can also say it is a curve whose tangent at any point is in the direction of the fluid velocity at the point.

From the streamline concept, the number of particles crossing the surfaces at P, R and Q is the same.

If area of cross-sections at these points are A_P, A_R and A_Q and speeds of fluid particles are v_P, v_R and v_Q, then mass of fluid Δm_p crossing at A_P in a small interval of time Δt is $\rho_P A_P v_P \Delta t$.

Similarly mass of fluid Δm_R flowing or crossing at A_R in a small interval of time Δt is $\rho_R A_R v_R \Delta t$ and mass of fluid Δm_Q is $\rho_Q A_Q v_Q \Delta t$ crossing at A_Q.

The mass of liquid flowing out equals the mass flowing in, holds in all cases. Therefore, $\rho_P A_P v_P \Delta t = \rho_R A_R v_R \Delta t \rho = \rho_Q A_Q v_Q \Delta t$ ….. (8.17)

And for flow of incompressible fluids $\rho_P = \rho_R = \rho_Q$

Equation (8.17) reduces to

$A_P v_P = A_R v_R = A_Q v_Q$…. (8.18). The equation (8.18) is called the equation of continuity and it is a statement of conservation of mass in flow of incompressible fluids.

$Av = constant$ (8.19)

Av gives the volume flux or flow rate and remains constant throughout the pipe of flow. Thus, at narrower portions where the streamlines are closely spaced, velocity increases and vice versa.

From (Figure 8.7) it is clear that $A_R > A_Q \; or \; v_R < v_Q$, the fluid is accelerated while passing from R to Q. This is associated with a change in pressure in fluid flow in horizontal pipes.

Example 8.8 The cylindrical tube of a spray pump has a cross-section of $6.0cm^2$ one end of which has 30 fine holes each of 1.0 mm diameter. If the liquid flow inside the tube is $1.5m\,min^{-1}$, what is the speed of ejection of the liquid through the holes?

Solution: Given that Area of the cross section of the spray pump $A_1 = 6 \times 10^{-4}\, m^2$, Number of holes, $n = 30$

Diameter of each hole, $d = 10^{-3}m$. The radius of each hole $r = .5 \times 10^{-3}m$, Area of cross section of each hole, $a = \pi r^2 = \pi\,(.5 \times 10^{-3})^2$

The total area of 30 holes, $A_2 = na = n\pi\,(.5 \times 10^{-3})^2$

$= 44.3 \times 10^{-6}\, m^2$, the speed of flow of liquid inside the tube $= v_1 = 1.5m\,min^{-1} = .025m/s$.

Let the liquid ejected through the holes is at speed v_2

Then according to the continuity equation, we have $v_2 = \frac{A_1 v_1}{A_2} = \frac{(6\times10^{-4}m^2\times.025m/s)}{44.3\times10^{-6}\,m^2} = .338\, m/s$.

Steady flow is achieved at low flow speeds. The steady flow loses its steadiness beyond a limiting value of speed (called critical velocity) and becomes turbulent.

Critical Velocity: It is the speed beyond which a fluid's flow becomes turbulent. It depends on fluid viscosity, density, and the pipe's radius. It is denoted by v_c.

Formula to calculate critical velocity $v_c, v_c = \frac{(K\eta)}{(\rho r)}$

K is a dimensionless quantity

η is the coefficient of viscosity

ρ is the density of the fluid

r is the radius of the pipe.

8.16 Bernoulli's Principle and its applications

First let us assume a fluid flowing in a pipe of varying cross sectional area. And the pipe is placed in such a way that different parts have different heights as shown in Figure 8.8. And the flow is steady.

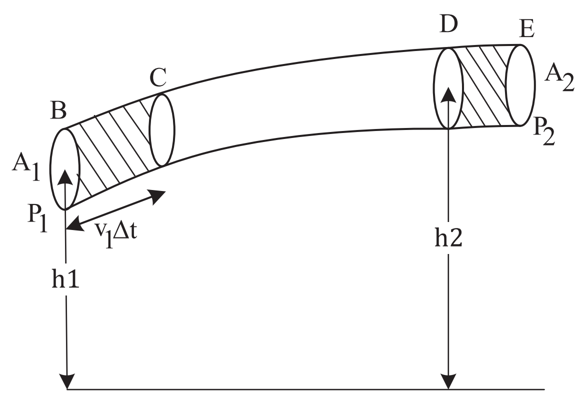


Fig 8.8

Bernoulli's Principle can be stated as follows "As we move along a streamline the sum of pressure (P), the kinetic energy per unit volume $\left(\frac{\rho v^2}{2}\right)$ and the potential energy per unit volume (ρgh) is constant."

The expression may be written as follows

$P + \frac{1}{2}\rho v^2 + \rho gh = constant$ ….(8.20)

Note down, we have also assumed there is not any energy loss due to friction, however it is not possible in practical but we have to be supposed.

When a fluid is at rest, i.e., its velocity is zero everywhere, Bernoulli's equation becomes

$P_1 + \rho gh_1 = P_2 + \rho gh_2$

$\Rightarrow (p_1 - p_2) = pg(h_2 - h_1)$ .....(8.21)

For low viscosity incompressible fluids, Bernoulli's principle works.

It has a large number of practical applications in daily life.

Airplane Wings: The curved shape of an airplane wing causes air to flow faster over the top surface, creating lower pressure above the wing compared to the higher pressure below, resulting in lift.

Carburetor: In a carburetor, a narrowed section (venturi) causes air to flow faster, reducing pressure and drawing in fuel.

Dynamic lift on aeroplanes: The curved shape of an airplane wing causes air to flow faster over the top surface compared to the bottom surface. This faster airflow results in lower pressure above the wing, while the slower airflow under the wing creates higher pressure. The pressure difference generates an upward force (lift) that allows the plane to fly.

Venturi Meters: A venturi meter is a device used to measure the flow rate of fluids in pipes. The device uses a narrowing section/throat to increase fluid velocity, which reduces pressure. The pressure difference between the wider section and the throat is used to calculate the flow rate

Blowing of roofs during cyclones: During a cyclone, the wind speed increases significantly around a house. The higher wind speed above the roof creates lower pressure compared to the higher pressure underneath. The pressure difference can cause the roof to lift and blow away.

8.17 Surface Tension and Surface Energy

Surface Tension: It is the force per unit length acting on the surface of a liquid. As all the molecules in a mass of liquid are subject to equal attractive force from all sides. But the molecules present on the surface only experience the attraction from inside the liquid. To put it simply, the particles on the surface of a liquid experience a pulling force from the rest of the liquid (this force per unit length is surface tension), thereby causing a thin elastic-like film to appear on the surface. Thus, we can say that surface tension is caused by attractive forces that pull liquid molecules together. Its SI unit is N/m.

Surface Energy: It is the amount of work (energy) required to increase the surface area of a liquid by a unit area. Its SI unit is J/m^2.

We know that extra energy is associated with the surface of liquids, if we want to increase the surface (spreading of surface) keeping other things like volume fixed requires a horizontal liquid film ending in bar free to slide over parallel guides.

Figure 8.9(a) shows a film in equilibrium, Figure 8.9(b) shows the film stretched an extra distance.

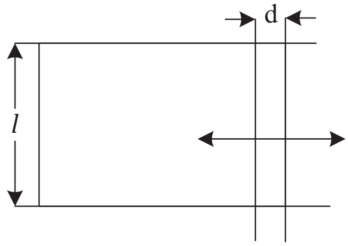


Fig 8.9 (b)

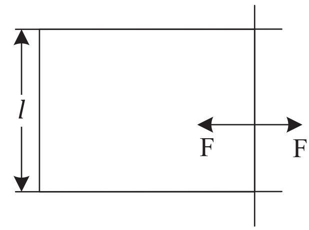


Fig 8.9 (a)

Suppose that we move the bar by a small distance d as shown in the figure. Since the area of the surface increases, the system now has more energy, this means that some work has been done against an internal force. Let this internal force be F, the work done by the applied force is $F.d = Fd$. From conservation of energy, this is stored as additional energy in the film. If the surface energy of the film is S per unit area, the extra area is $2dl$. A film has two sides and the liquid in between, so there are two surfaces and the extra energy is $S(2dl) = Fd$ .... (8.22)

Or, $S = \frac{Fd}{2dl} = \frac{F}{2l}$ .... (8.23)

This quantity S is the magnitude of surface tension. It is equal to the surface energy per unit area of the liquid interface and is also equal to the force per unit length exerted by the fluid on the movable bar.

Example 8.9 A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of $2 \times 10^{-2}N$ (which includes the small weight of the slider). The length of the slider is $40cm$. What is the surface tension of the film?

Solution: The weight that the soap film supports, $W = 2 \times 10^{-2}N$, length of the slider $l = .4m$, a soap film has two free surfaces.

Therefore, the total length $= 2l = .8m$

Hence, the surface tension will be obtained as $S = \frac{Force}{2l} = \frac{2\times10^{-2}}{.8m} = 0.025\ N/m$.

8.18 Angle of Contact

First we draw two figures as shown below, Figure 8.10(a) and Figure 8.10(b).

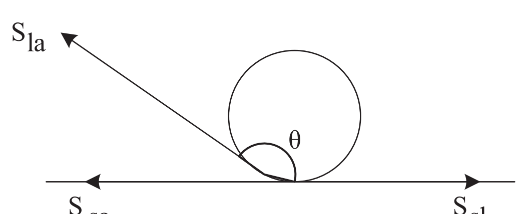


Fig 8.10 (a)

The angle formed between tangent to the liquid surface and solid surface at the point of contact within the liquid is known as angle of contact.

Angle of contact is denoted by θ. Its value determines whether a liquid will form droplets on the surface of a solid where it is lying or will just spread on it.

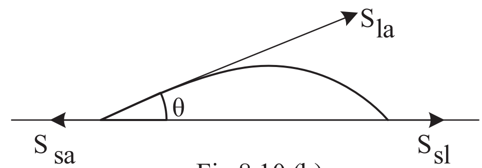


Fig 8.10 (b)

We consider the three interfacial tensions at all the three interfaces, liquid-air, solid-air and solid-liquid denoted by S_{la}, S_{sa} and S_{sl}, respectively as given in Figure 8.10. At the line of contact, the surface forces between the three media must be in equilibrium. From Figure 8.10 (b) the following relation is easily derived.

$S_{la} cos\,\theta\ + S_{sl} = S_{sa}$ …… (8.24)

The angle of contact is an obtuse angle, if $S_{sl} > S_{la}$ as in the case of water-leaf interface while it is an acute angle if $S_{sl} < S_{la}$ as in the case of a water-plastic interface.

When θ is an obtuse angle then molecules of liquids are attracted strongly to themselves and weakly to those of solid, it costs a lot of energy to create a liquid-solid surface, and liquid then does not wet the solid. This is what happens with water on a waxy or oily surface, and with mercury on any surface. On the other hand, if the molecules of the liquid are strongly attracted to those of the solid, this will reduce S_{sl} and therefore, $cos\,\theta$ may increase or θ may decrease. In this case θ is an acute angle. This is what happens for water on glass or on plastic and for kerosene oil on virtually anything (it just spreads).

Soaps, detergents and dying substances are wetting agents. When they are added the angle of contact becomes small so that these may penetrate well and become effective. Water proofing agents on the other hand are added to create a large angle of contact between the water and fibres.

8.19 Application of Surface Tension: Drops and Bubbles

Drops and bubbles form due to the effect of surface tension. There is a cohesive (attractive) force between liquid molecules, causing liquids to minimize their surface area, resulting in the spherical shapes of drops and bubbles. (A liquid-air interface has energy, so for a given volume, the surface with minimum energy is the one with the least area. The sphere has this property.)

There is one important point to note: the pressure inside a spherical drop or bubble is greater than the pressure outside. This is because surface tension acts to minimize the surface area, thus creating an excess pressure inside the curved surface.

And suppose a spherical drop of radius r is in equilibrium and if its radius increases by Δr. Then the extra surface energy is

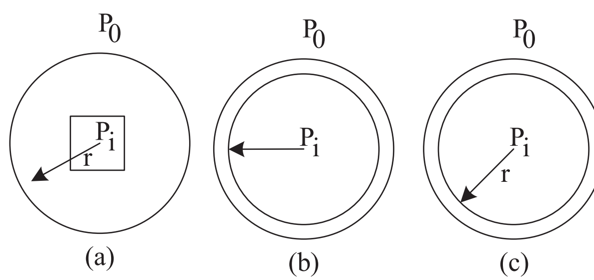
$[4\pi(r+\Delta r)^2 - 4\pi r^2]\, S_{la} = 8\pi r \Delta r S_{la}$ …… (8.25)

If the drop is in equilibrium this energy cost is balanced by the energy gain due to expansion under the pressure difference $(P_i - P_o)$ between the inside of the bubble and the outside.

The work done is $W = (P_i - P_o) 4\pi r^2 \Delta r$ ………. (8.26)

so that $(P_i - P_o) = \left(\frac{2S_{la}}{r}\right)$ ……. (8.27)

In general, for a liquid-gas interface, the convex side has a higher pressure than the concave side. For example, an air bubble in a liquid would have higher pressure inside it. See Figure 8.11(b).



Drop, Cavity and bubble of radius r.
Fig 8.11

A bubble Figure 8.11(c) differs from a drop and a cavity; in this it has two interfaces. Applying the above argument we have for a bubble $(P_i - P_o) = \left(\frac{4S_{la}}{r}\right)$ …… (8.28)

This is probably why you have to blow hard, but not too hard, to form a soap bubble. A little extra air pressure is needed inside.

8.20 Capillary Rise

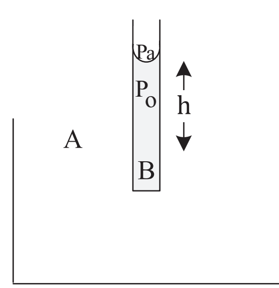


Fig 8.12 (a)

One consequence of the pressure difference across a curved liquid-air interface is the well known effect that water rises up in a narrow tube in spite of gravity. The word capilla means hair in Latin; if the tube were hair thin, the rise would be very large. To see this, consider a vertical capillary tube of circular cross section of radius a inserted into an open vessel of water (Figure 8.12). The contact angle between water and glass is acute. Thus the surface of water in the capillary is concave. This means that there is a pressure difference between the two sides of the top surface.

This is given by $(P_i - P_o) = \left(\frac{2S}{r}\right) = \frac{2S}{(a\sec\theta)} = \left(\frac{2S}{a}\right)\cos\theta$ ......(8.29)
Thus the pressure of the water inside the tube, just at the meniscus (air-water interface) is less than the atmospheric pressure.

Consider the two points A and B as shown in Figure 8.12(a). They must be at the same pressure, namely $P_o + \rho gh = P_i = P_A$ .... (8.30) where ρ is the density of water and h is called the capillary rise (8.12).

Using Equation (8.28) and (8.29) we have $\rho gh = (P_i - P_o) = \frac{2S\cos\theta}{a}$ .... (8.30)

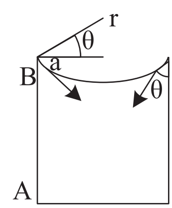


Fig 8.12 (b) Magnified picture near the surface

The discussion here, and Equation (8.26) and (8.27) make it clear that the capillary rise is due to surface tension. It is larger, for a smaller a. Typically it is of the order of a few cm for fine capillaries.

Example 8.10 A capillary tube of radius $0.1mm$ is partly dipped in water (surface tension 70 dyn/cm and glass water contact angle = 0°) with 30° inclined with vertical. The length of water rise in the capillary is ______ cm. (take $g = 9.8m$)

Solution: $h = \frac{2T\cos\theta}{\rho gr} = \frac{2\times70\times1}{1\times980\times10^{-2}}$ (we converted m & mm in cm)

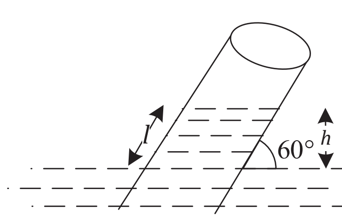
$$h = \frac{100}{7}cm$$

$$\sin 60° = \frac{h}{l}$$

$$l = \frac{h\times 2}{\sqrt{3}}$$

$$l = \frac{100}{7}\times\frac{2}{\sqrt{3}} = \frac{200}{7\times\sqrt{3}}$$

$$= 16.49cm$$



Example 8.11 A liquid column of height $0.04\boldsymbol{cm}$ balances excess pressure of soap bubble of a certain radius. If density of liquid is $\mathbf{8\times10^3}\boldsymbol{kgm^{-3}}$ and surface tension of soap solution is $\mathbf{0.28}\boldsymbol{Nm^{-1}}$. Then diameter of the soap bubble is __________$\boldsymbol{cm}$ (if $\boldsymbol{g} = \mathbf{10}\boldsymbol{ms^{-2}}$)

Solution: To find the diameter of the soap bubble, we will use the formula $\Delta\boldsymbol{P} = \frac{\boldsymbol{4T}}{\boldsymbol{r}}$

Where ΔP is the excess pressure.

T is the surface tension of the soap solution.

r is the radius of the soap bubble.

The column balances this excess pressure, and the pressure exerted by the liquid column is given by

$$P = \rho g h$$

Thus, we have $\rho g h = \frac{4T}{r}$ ...... (i)

(So putting the given values in equation (i))

We have

$8 \times 10^3 \times 10 \times 0.04 \times 10^{-2} = \frac{4\times0.28}{r}$

So, $0.32 = \frac{1.12}{r}$

$r = 3.5cm$

Therefore diameter $= 2r = 7cm$

Example 8.12 An air bubble of radius $1.0mm$ is observed at a depth $20cm$ below the free surface of a liquid having surface tension is $0.095J/m^2$ and density $10^3kg/m^3$. The difference between pressure inside the bubble and atmospheric pressure is __________ N/m^2 (Take $g = 10m/s^2$)

Solution: The internal pressure of the bubble is

$$P_{inside} = P_{atm} + \rho g h + \frac{2T}{r}$$

Therefore, the difference between the bubbles internal pressure and the atmosphere pressure is

$$\Delta P = P_{inside} - P_{atm} = \rho g h + \frac{2T}{r}$$

$\rho g h = (10^3 kg/m^3).(10m/s^2).(0.20m)$

$= 2000N/m^2$

and $\frac{2T}{R} = \frac{2\times0.095J/m^2}{1.0\times10^{-3}m} = 2 \times .95$

$= 190N/m^2$

Therefore $\Delta P_{\text{total}} = 2000N/m^2 + 190N/m^2 = 2190N/m^2$

8.21 Buoyancy

It is the upward force exerted by a fluid on an object when it is submerged in it (the fluid). This force is known as the buoyant forces. The magnitude of buoyant force is equal to the weight of the fluid displaced by it $Fb = \rho v g$

Where Fb is the buoyant force

$\rho(rho)$ is the density of the fluid

v is the volume of the displaced fluid (which is equal to the volume of the submerged portion of the object)

g is the acceleration due to gravity

Exercises

1. The length of a light string is $1.4m$ when the tension on it is $5N$. If the tension increases to $7N$, the length of the string is $1.56m$. The original length of the string is __________m.
2. A $20kg$ mass, fastened to the end of a copper wire of unstretched length $1.0m$, is whirled in a vertical circle with an angular velocity of $2rev/s$ at the bottom of the circle. The cross-sectional area of the wire is $0.065cm^2$. Calculate the elongation of the wire when the mass is at the lowest point of its path. (Young modulus of copper $= 1.17 \times 10^{11}$ pascals)
3. A copper wire of length $2.2m$ and a steel wire of length $1.6m$, both of diameter $3.0mm$, are connected end to end. When stretched by a load, the net elongation is found to be $0.70mm$. Obtain the load applied.
4. The average depth of a big river is about $2000m$. Calculate the fractional compression, $\frac{\Delta V}{V}$, of water at the bottom of the ocean, given that the bulk modulus of water is $2 \times 10^{10}\ Nm^{-2}$. (Take $g = 10ms^{-2}$)
5. A load of 4.0kg is suspended from a ceiling through a steel wire of length of the wire increases by 0.031mm as equilibrium is achieved. Find Young modulus of steel. (Take $g = 3.1\pi ms^{-2}$
6. The volume contraction of a solid copper cube of edge length $10cm$, when subjected to a hydraulic pressure of $7 \times 10^6\ Pa$, would be ....mm^3. (given that bulk modulus of copper $= 1.4 \times 10^{11} Nm^{-2}$)
7. Two persons pull a wire towards themselves. Each person exerts a force of $200N$ on the wire. Young's modulus of the material of wire is $1 \times 10^{11} Nm^{-2}$. Original length of the wire is $2m$ and the area of the cross section is $2cm^2$. The wire will extend in length by _________ μm.
8. A hydraulic lift uses Pascal's Principle to multiply force. If a force of $100N$ is applied to a small piston with an area of $0.01m^2$, and this pressure is transmitted to a larger piston with an area of $0.5m^2$ .What is

the maximum load (force) that can be lifted by the larger piston? ____ N.

9. A liquid column of height $0.04cm$ balances excess pressure of a soap bubble of a certain radius. If density of liquid is $8 \times 10^3\ kgm^{-3}$ and surface tension of soap solution is $0.28\ Nm^{-1}$, then diameter of the soap bubble is __________ cm (if $g = 10ms^{-2}$)
10. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of energy of big drop is $\frac{10}{x}$.The value of x is …..…….
11. A soap bubble is blown to a diameter of $7cm$. 36960 erg of work is done in blowing it further. If the surface tension of soap solution is $40\ dyne/\ cm$ then the new radius is ________ cm. Take $\left(\pi = \frac{22}{7}\right)$
12. The elastic potential energy stored in a steel wire of length $20m$ stretched through $2cm$ is $80J$. The cross sectional area of the wire is __________ mm^2. (Given $Y = 2 \times 10^{11}\ Nm^{-2}$)
13. Two water drops each of radius $'r'$ coalesce to form a bigger drop. If $'T'$ is the surface tension, the surface energy released in this process is :
    (a) $4\pi r^2 T[2 - 2^{2/3}]$ (b) $4\pi r^2 T\left[2 - 2^{1/3}\right]$
    (c) $4\pi r^2 T[1 + \sqrt{2}]$ (d) $4\pi r^2 T[\sqrt{2} - 1]$
14. The fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the depth of $2.5km$ below the sea level is __________%. (Given, the Bulk modulus of water $= 2 \times 10^9\ Nm^{-2}$, density of water $= 10^3 kgm^{-3}$, acceleration due to gravity $g = 10ms^{-2}$)
15. A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter $1.0cm$. What is the pressure exerted by the heel on the horizontal floor?
16. A hydraulic automobile lift is designed to lift cars with a maximum mass of $3000kg$. The area of cross-section of the piston carrying the load is $425cm^2$. What maximum pressure would the smaller piston have to bear?
17. What is the excess pressure inside a bubble of soap solution of radius $5.00mm$, given that the surface tension of soap solution at the temperature $(20°C)$ is $2.50 \times 10^{-2} Nm^{-1}$? If an air bubble of the same dimension were formed at a depth of $40.0cm$ inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble?
18. A $400g$ solid cube having on edge of length $10cm$ floats in water. How much volume of the cube is outside the water? (Given- density of water $= 100kgm^{-3}$)

Answers

1. 1m	2. 4.41mm	3. 177N	4. 1.001
5. $2.0 \times 10^{11} Nm^{-2}$, $1.3 \times 10^{-4} m$	6.50	7. 20	8. 5000
9. 7cm	10. 1	11. 7	12. 40
13. a	14. 1.25	15. $6.24 \times 10^{6} pa$	16. $6.92 \times 10^{5} pa$

17. Excess pressure inside the soap bubble $= 20.0\ Pa$; excess pressure inside the air bubble in soap solution $= 10.0\ Pa$. Outside pressure for air bubble $= 1.01 \times 10^{5} + 0.4 \times 10^{3} \times 9.8 \times 1.2 = 1.06 \times 10^{5} Pa$. The excess pressure is so small that up to three significant figures, total pressure inside the air bubble is $1.06 \times 10^{5} Pa$.

18. 600 cm^{3}

Chapter - 9
Thermal Properties of Matter

The next morning of the last casual study session, Dolly woke up a little later than usual. That morning, she first called Geeta, and asked about the module currently being taught in the class. She was informed generally, that the introduction and general terms had been delivered of the chapter 9, ‘Thermal Properties of Matter,’ and that day, topics like heat capacity, calorimetry and changes of state like topics would be covered. Gita also advised her against skipping too many lectures consecutively, no matter the reason.

She told the others about the call, and after some dramatic deliberation, they decided to attend today’s formal class. They also emphasized the importance of attending school at least one day every week to monitor the syllabus of the formal class.

*****

Arnold's landline phone rang just as he was wearing his Nike sport type shoes to head off to the hangout place and thereafter for the school. It was Amila who, after asking about her and the guys' well-being, told that she wanted to come to the village again that day. Arnold wasn't able to tell that he would not be available for at least a few hours; instead he said, “Okay, I will be waiting.”

Soon after, Amila arrived at the village, and Arnold took him to their established spot and other guys also gathered there. She also learned about some of their concerns, especially regarding finances. Amila offered to help them financially for the bus purchase, and said, “Through this, we have a wonderful opportunity to embark on a beautiful business journey.” Amila had always wanted to be a part of their gang as she found their group attractive, charming and fascinating.

First they seemed to be speechless. Later Aditi said “Okay, first we need to talk to our parents and collect as much money as possible. Then for the rest of the capital, we could rely on you.” “Sure, I’m always ready," Amila responded.

However, after the concluding statement, they realized they were too late. Therefore, they cancelled the plan of attending class that day. And, it was decided that some general terms and the modules that were to be taught in the formal class would be completed casually with Amila's help.

Afterwards, they would depart for their homes to discuss the idea to their parents and collect funds.

They just sat on the big sofa, and Amila started teaching with general terms like temperature, heat and thermal expansion.

9.1 Temperature

Temperature is the measurement of hotness or coldness of an object. It is a relative measure of the average kinetic energy of the particles within a substance or an object. This means both hotness & coldness are described comparatively. For example: A normal human body temperature is generally considered to be within the range of 97.5°F to 99.6°F (36.4°C to 37.2°C). While the average is often cited as 98.6°F (37°C). A temperature above 100.4°F (38°C) is generally considered a fever (hot).

It is measured using an instrument called Thermometre. The two familiar temperature scales are the Fahrenheit temperature scale and the Celsius temperature scale.

A relationship for converting between the two scales are $\frac{t_F-32}{180} = \frac{t_c}{100}$ …..(9.1)

Heat: It is a form of energy that is transferred between two objects or systems when there is temperature difference between these two. It flows from a hotter object to a colder one.

The SI unit of heat energy (transferred) is expressed in joule (J) while SI unit of temperature is Kelvin (K), and degree Celsius (℃) is a commonly used unit of temperature.

9.2 Ideal-Gas Equation and Absolute Temperature

For this chapter, we will know only the equation that an ideal gas follows that is $PV = uRT$…….(9.2) (Its detail we will read in next chapter "Thermodynamics and kinetic theory of Gases")

where μ is the number of moles in the sample of gas, P and V are the pressure and volume of the gas respectively. R is called the universal gas constant: $R = 8.31 J\, mol^{-1} K^{-1}$ and T is temperature in kelvin $(T = t + 273)$, t is the temperature in $°C$.

Absolute temperature

Absolute temperature, also called thermodynamic temperature, is the temperature of an object on a scale where zero is taken as absolute zero. Absolute temperature scales are Kelvin and Rankine. However we commonly consider absolute temperature scales as Kelvin.

Absolute zero is the temperature at which a system is in the state of lowest possible (minimum) energy. As molecules approach this temperature, their movements continue to slow down. The kinetic energy of the molecules becomes negligible. However, the molecules will never stop moving nor have no energy.

It is the lowest temperature a gas thermometer can measure. No electronic devices work at this temperature. No living thing can survive in this temperature.

9.3 Thermal Expansion:

The tendency of a substance to change its size "length, area, or volume" due to change in its temperature is called thermal expansion.

Most substances expand on heating and contract on cooling.

The expansion in length of a material is called linear expansion. The expansion in area is called area expansion and the expansion in its volume is called volume expansion.

If a rod undergoes a change in length Δl due to small change in temperature ΔT, then $\frac{\Delta l}{l}$ is directly proportional to ΔT.

$\frac{\Delta l}{l} = \alpha_1 \Delta T$ ……(9.3) (l is the original length of the material)

Where α_1 is known as the coefficient of linear expansion of the rod. It is characteristic of the material.

Example 9.1 A worker fixing an iron ring on the rim of the wooden wheel of a horse cart. The diameter of the rim and the iron ring are $4.243m$ and $4.231m$, respectively at $36°C$. To what temperature should the ring be heated so as to fit the rim of the wheel? ($\alpha = 1.2 \times 10^{-5} k^{-1}$)

Solution: $\frac{\Delta l}{l} = \alpha \Delta T$

$\Rightarrow \frac{l_{T2} - l_{T1}}{l_{T1}} = \alpha\,(T_2 - T_1)$

$\Rightarrow \frac{4.243 - 4.231}{4.231} = 1.2 \times 10^{-5} (T_2 - 36)$

$\Rightarrow\ 236.35 = T_2 - 36$

$\Rightarrow T_2 = 272.35°C.$

The same way we define the coefficient of volume expansion $a_v = \left(\frac{\Delta v}{v}\right) \frac{1}{\Delta T}$ …..(9.4). Here a_v is also a characteristic of the material but not strictly constant. It is observed that a_v becomes constant at a high temperature.

Figure 9.1: coefficient of volume expansion of copper as a function of temperature

Note: For normal temperature, gases expand more than solids and liquids.

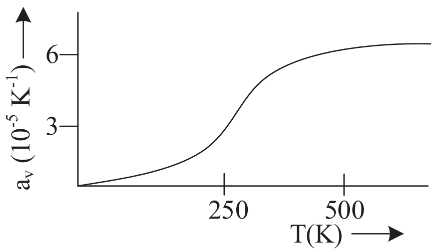


Fig 9.1

9.4 Coefficient of volume expansion:

For gases, the coefficient of volume expansion is dependent on the temperature. In the case of liquid, it is relatively independent of the temperature.

And for an ideal gas at constant pressure, coefficient of volume expansion can be found from the ideal gas equation: $PV = \mu RT$

At constant pressure $P\Delta V = \mu R\Delta T$

$\frac{\Delta V}{V} = \frac{\Delta T}{T}$

i.e., $\alpha_v = \frac{1}{T}$ for ideal gas ……(9.5)

There is a simple relation between the coefficient of volume expansion (α_v) and coefficient of linear expansion (α_l). Suppose a cube of length l that expands equally in all directions, when its temperature increases by ΔT. We have $\Delta l = \alpha_l l\Delta T$

So, $\Delta B = (l + \Delta l)^3 - l^3 \simeq 3l^3\Delta l$ …..(9.6) In Equation (9.6), terms in $(\Delta l)^2$ and $(\Delta l)^3$ have been neglected since Δl is small compared to l.

So $\Delta V = \frac{3V\Delta l}{l} = 3V\alpha_1\,\Delta T$ ….(9.7) which gives $a_v = 3\alpha_1$ ……..(9.8)

9.5 Specific Heat Capacity

First we need to know Heat capacity, Heat capacity of a substance is the amount of heat required to change the temperature of a specific amount of that substance by one degree. $S = \frac{\Delta Q}{\Delta T}$……(9.9)

S is the heat capacity, ΔQ is the amount of heat supplied and ΔT is the temperature changed from T to $T + \Delta T$.

Specific Heat Capacity (s): It is defined as the amount of heat per unit mass absorbed or given off by the substance to change its temperature by one unit.

$s = \frac{\boldsymbol{S}}{m} = \frac{\Delta Q}{m\Delta T}$ .....(9.10) SI unit of specific heat capacity is $Jkg^{-1}k^{-1}$, (m is the mass of the substance)

9.6 Calorimetry

Calorimetry: Calorimetry is the process of measuring heat changes that occur during physical or chemical processes. The device used to perform these measurements is called a calorimeter.

Calorimeter: It is a type of device in which heat measurement can be done. The device is used to measure the heat absorbed or released during physical or chemical processes. The device is isolated from its surroundings, which means heat exchange can't take place between the system (the substance undergoing a change) and its surroundings.

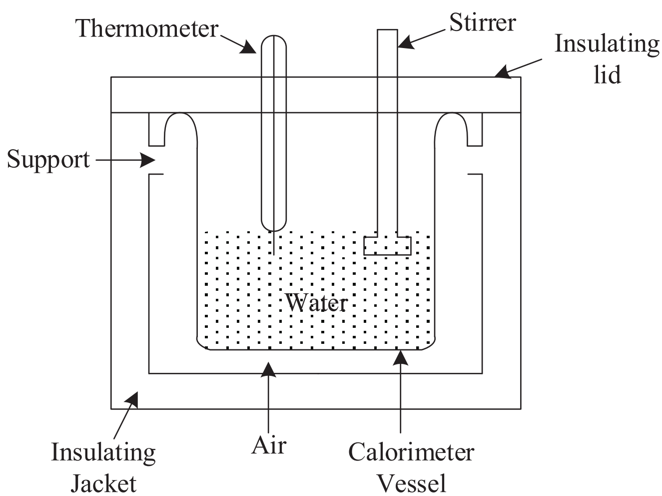


Fig 9.2

Structure of Calorimeter: It consists of a metallic vessel and stirrer of the same material, like copper or aluminium. The vessel is kept inside a wooden jacket, which contains heat insulating material, like glass wool etc. The outer jacket acts as a heat shield and reduces the heat loss from the inner vessel. There is an opening in the outer jacket through which a mercury thermometer can be inserted into the calorimeter (Figure 9.2). The example below provides a method by which the specific heat capacity of a given solid can be determined by using the principle, heat gained is equal to the heat lost.

Example 9.2 A sphere of $0.05kg$ copper sphere and $80°C$ is placed for sufficient time in a vessel of water, of mass $0.4kg$ water at $20°C$. The temperature of water rises and attains a steady state at $40°C$. Calculate the specific heat capacity of aluminium.

Solution: Given mass of the copper sphere $(m_1) = .05\ kg$

Initial temperature of copper sphere $80°C$. Final temperature $= 40°C$. Change in temperature $\Delta T = (80°C - 40°C) = 40°C$

Let the specific heat capacity of copper be s_{cu}.

The amount of heat lost by the copper sphere $= m_1 s_{cu} \Delta T = .05\ kg \times s_{cu} \times 40°C$

Mass of water $(m_2) = .4\ kg$

Change in temperature $\Delta T = 40°C - 20°C = 20°C$

Specific heat capacity of water $= 4.18 \times 10^3\ Jkg^{-1}K^{-1}$

The amount of heat gained by the water $= .4\ kg \times 4.18 \times 10^3\ Jkg^{-1}K^{-1} \times 20°C$

In steady state, the amount of heat lost by the copper sphere is equal to the amount of heat gained by water

$.05\ kg \times s_{cu} \times 40°C = .4\ kg \times 4.18 \times 10^3 Jkg^{-1}K^{-1} \times 20°C$

$\Rightarrow S_{cu} = 16.72 \times 10^3 Jkg^{-1}K^{-1}$

9.7 Change of State

We all know a matter could exist in three states : solid liquid and gas. A conversion of a matter from one of these states to another is called a change of state. The change typically occurs when an exchange of heat takes place between the matter and the surroundings.

The change of state from solid to liquid is called melting or fusion, and the change of state from liquid to solid is called freezing. It is important to note that the temperature remains constant during both these processes. The constancy of temperature implies that both the solid and liquid states of the substance remain in thermal equilibrium during the phase change.

Melting Point: The temperature at which the solid and the liquid states of the substance are in thermal equilibrium to each other is known as melting point. It is considered as a property of the substance. It also depends on the pressure.

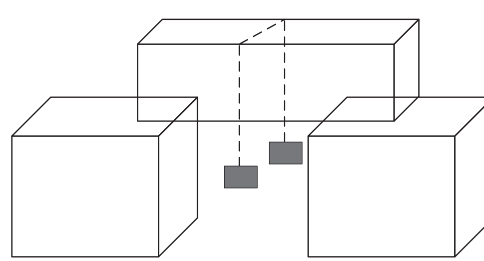


Fig 9.3

The phenomenon in which ice melts to water below 0°C due to applying of pressure and it refreezes back to ice once the pressure is removed is called regelation. For example, take a metallic wire and fix two blocks, say $5kg$ each, at its ends. Put the wire over the slab as shown in Figure 9.3. You will observe that the wire passes through the ice slab. This happens due to the fact that just below the wire, ice melts at lower temperature due to increase in pressure. When the wire has passed, water above the wire freezes again. The whole process is none other than the regelation.

Vaporisation: The process during which a liquid changes into a gas is called vaporisation. In the process of vaporisation, the temperature is constant till the entire amount of the liquid is converted into vapour, implying both the liquid and vapour states of the substance coexist in thermal equilibrium during the change of state from liquid to vapour. The temperature at which the liquid and the vapor states of the substance coexist: exist at the same time is called its boiling point. There is one fact; boiling point increases with increase in pressure.

Sublimation: Sublimation is the transition of a substance directly from the solid to the gas state without passing through the liquid state.

9.8 Latent Heat

Latent heat is the amount of heat per unit mass transferred to or from a substance during change of state of the substance is called Latent heat.

$$L = \frac{Q}{m} \text{ or } Q = mL \quad \text{.....(9.11)}$$

(L is latent heat, Q is the amount of heat transferred to a substance and m is the mass of the substance), its SI unit is Jkg^{-1}.

Note: It is important to note that the transfer of latent heat occurs at constant temperature. To understand it comprehensively, take a look at this phenomenon. If heat is added to a given quantity of ice at $-10°C$, the temperature of ice increases until it reaches its melting point ($0°C$). At this temperature, the addition of more heat does not increase the temperature but causes the ice to melt, or changes its state. Once the entire ice melts, adding more heat will cause the temperature of the water to rise. A similar situation occurs during liquid-gas change of state at the boiling point. Adding more heat to boiling water causes vaporisation, without increase in temperature.

The value of L also depends on the pressure; Its value is usually given at standard atmospheric pressure. The latent heat for a solid-liquid state change, liquid-gas state, is called the latent heat of the fusion L_f (or heat of fusion). Similarly, the latent heat for a liquid-gas state change is called the latent heat of vaporisation L_v (or heat of vaporisation).

Example 9.3 When $0.30kg$ of ice at $0°C$ is mixed with $0.60kg$ of water at $60°C$ in a container, the resulting temperature is $8°C$. Calculate the heat of fusion of ice. ($s_{water} = 4186Jkg^{-1}K^{-1}$)

Solution: Heat lost by water $= ms_w(\theta_f - \theta_i)w = .6\ kg \times 4186Jkg^{-1}K^{-1} \times (60°C - 8°C)w$

Heat required to melt ice $= m_2L_f = .3\ kg \times L_f$

Heat required to raise temperature of ice water to final temperature $= m_2s_w\left(\theta_f - \theta_i\right)_I = .3\ kg \times 4186Jkg^{-1}K^{-1} \times (8°C - 0°C)$

Heat lost by water = Heat gained by ice to melt and further to gain temperature by $8°C$.

$$.6\ kg \times 4186Jkg^{-1}K^{-1} \times (60°C - 8°C) = .3\ kg \times L_f + .3\ kg \times 4186Jkg^{-1}K^{-1} \times (8°C - 0°C)$$

$$\Rightarrow L_f = 4.01 \times 10^5Jkg^{-1}.$$

Amila stopped teaching saying, “Today your professional teacher won't be able to cover more topics than this.” Arnold and Amila then departed for his home just after the informal class. The rest of them stayed for a bit and a gossip session began. The question circulating among them was “Is Amila financing us just due to her growing interest in them, especially Arnold, or is she also intending to do business here?”

Kimaya said, “For whatever reason she is supporting us financially, it doesn't matter much; we must appreciate her, as it would be a great facility for us.” “Yeah, and we would have more glory in our lives once the deal is finalized,” David supported.

*****

Aakash gently said, “Okay listen. The remaining topics in this chapter are not challenging. It’s easy for me to prepare them in my notebook, and then it will just be a matter of copying them for all of you.” Kimaya reacted, “We could photocopy the notes and put them in our notebooks.” Aakash agreed, “Yes, that would be the best.” They all departed then, most of them saying, “See you soon!”

Aakash’s prepared notes were as follows :

9.9 Heat Transfer

It is the process in which transfer of energy takes place from one part of a region to another due to temperature difference.

There are three different ways by which this transfer of energy takes place (I) Conduction (II) Convection and (III) Radiation

(I) Conduction: It is the mode of heat transfer that takes place between two adjacent parts of a body because of the temperature difference between them.

Let, one end of a metallic rod is put in a flame, the other end of the rod will be hot so quickly that it would be very uncomfortable to hold with bare hands. Here, heat transfer takes place by conduction from the hot end of the rod through its different parts to the other end.

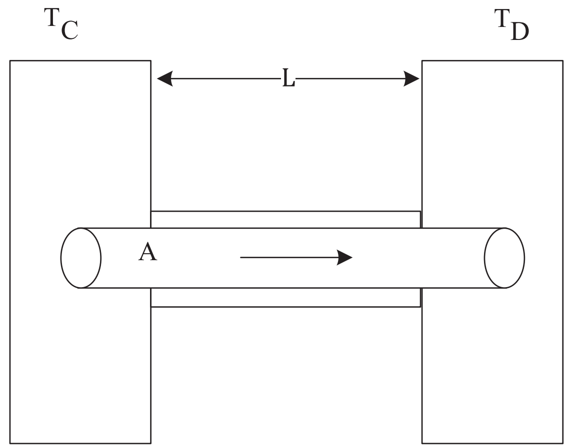


Fig 9.4

Now consider a metallic bar of length L and uniform cross-section A with its two ends maintained at different temperatures. This can be done, for example, by putting the ends in thermal contact with large reservoirs at temperatures, say, T_C and T_D, respectively (Figure 9.4). Suppose sides of the bar are fully insulated so that no heat is exchanged between the sides and the surroundings. After some time, a steady state is reached; the temperature of the bar decreases uniformly with distance from T_C to T_D; $(T_C > T_D)$. The reservoir at C supplies heat at a constant rate, which transfers through the bar and is given out at the same rate to the reservoir at D.

It is found experimentally that in this steady state, the rate of flow of heat (or heat current) H is proportional to the temperature difference $(T_C - T_D)$ and the area of cross-section A, and is inversely proportional to the length L.

$H = \frac{KA(T_C - T_D)}{L}$ ……(9.12)

The constant of proportionality K is called the thermal conductivity of the material. The greater the value of K for a material, the more rapidly it will conduct heat. The SI unit of K is $Js^{-1}m^{-1}K^{-1}$ or $Wm^{-1}K^{-1}$. The thermal conductivities values vary slightly with temperature, but can be considered being constant over a normal temperature range.

Example 9.4 A rod of copper of length $75cm$ and a rod of steel of length $125cm$ are joined together end to end. Both are of cross section with diameter $2cm$. The free ends of the copper and the steel rods are maintained at $100°C$ and $0°C$ respectively. What is the temperature of the steel -copper junction in the steady state of the system Assuming the rods are thermally insulated from the surrounding. (Thermal conductivity of steel $= 50.2 J s^{-1} m^{-1} K^{-1}$; and of copper $= 385 J s^{-1} m^{-1} K^{-1}$).

Solution: Let the temperature at the steel-copper junction be θ. We know that at the steady state the amount of heat flowing out from the steel rod is equal to the amount of heat flowing in through the steel rod is equal, thus

$$\frac{\Delta Q}{\Delta t} = \frac{K_{cu} A(100°C - \theta)}{75cm} = K_{steel} \frac{A\theta}{125} cm$$

$$\Rightarrow \frac{K_{cu} A(100°C - \theta)}{75cm} = K_{steel} \frac{A\theta}{125} cm$$

$$\Rightarrow \frac{100°C - \theta}{\theta} = \frac{75 K_{steel}}{125 K_{cu}} = \left(\frac{3}{5}\right) \times \left(\frac{46}{386}\right)$$

$$\Rightarrow \theta = 93°C.$$

Convection

It is the type of mode of heat transfer that happens in fluid (liquids and gases) and occurs due to movement of the matter itself.

To understand it clearly, consider the situation. When a fluid is heated, the particles near the heat source gain energy and become less dense because heating increases the kinetic energy of the molecules and decreases the intermolecular force of attraction. This causes them to rise, and cooler, denser fluid sinks to take their place, creating a continuous circulation or convection current.

Convection can be natural or forced. Convection. In natural convection, gravity plays an important part. When a fluid is heated from below, the hot part expands and, therefore, becomes less dense. Because of buoyancy, it rises and the upper colder part replaces it. This again gets heated, rises up and is replaced by the relatively colder part of the fluid. The process goes on.

This mode of heat transfer is evidently different from conduction. Convection involves bulk transport of different parts of the fluid. In forced convection, material is forced to move by a pump or by some other physical means. The common examples of forced convection systems are forced-air heating systems in home, the human circulatory system, and the cooling system of an automobile engine. In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection and maintaining it at a uniform temperature.

Natural convection is responsible for many familiar phenomena. During the day, the ground heats up more quickly than large bodies of water do. This occurs both because water has a greater specific heat capacity and because mixing currents disperse the absorbed heat throughout the great volume of water. The air in contact with the warm ground is heated by conduction. It expands, becoming less dense than the surrounding cooler air. As a result, the warm air rises (air currents) and the other air moves (winds) to fill the space-creating a sea breeze near a large body of water. Cooler air descends, and a thermal convection cycle is set up, which transfers heat away from the land. At night, the ground loses its heat more quickly, and the water surface is warmer than the land. As a result, the cycle is reversed (Figure 9.5).

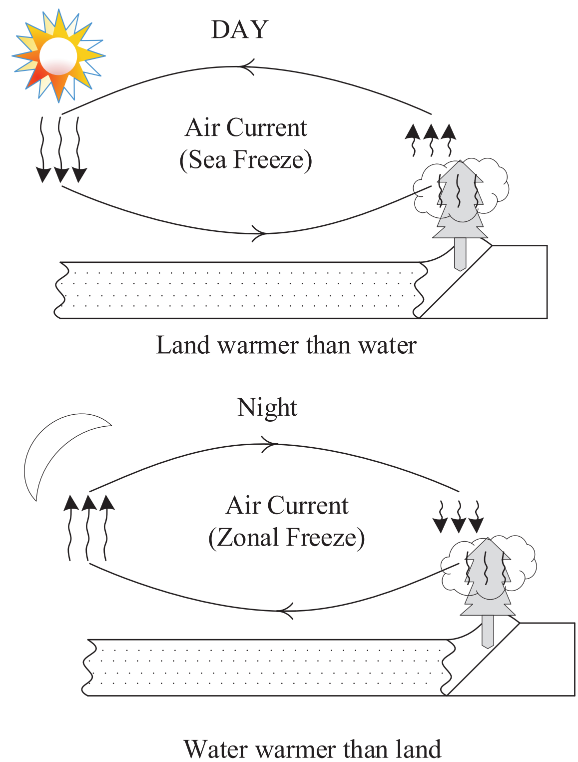


Fig 9.5

Radiation

It is the type of mode of heat transfer where no medium like air , water or any other type of matter is required to transfer the heat through, heat is transferred from a hotter body to colder body through electromagnetic waves.

The heat that transfers through electromagnetic waves comes under radiation. We receive sunlight on earth through the process of radiation as light (heat) emitted by the sun gets carried through electromagnetic waves, which travels in space and time.

9.10 Blackbody Radiation

In simple words we can say, the radiation emitted by a blackbody is called blackbody radiation.

It is important to know, what is a blackbody?

A body that absorbs all the radiation falling on it is called a blackbody. Such a body emits radiation at the fastest rate because good absorbers of radiation are also good emitters of radiation.

Emissive Power

It is the total thermal energy radiated per unit surface area per unit time from a body at a given temperature.

$E = \frac{Q}{At}$.....(9.13)

Absorptive Power

It is defined as the ratio of energy absorbed in a given instant of time to the radiant power incident on it at the same instant.

$$absorptive\ power = \frac{quantity\ of\ energy\ absorbed}{quantity\ of\ energy\ incident}$$

A body's absorptive power is defined at a given temperature.

9.11 Kirchoff's Law

Kirchoff's law states that the ratio of emissive power to absorptive power is the same for all bodies and it is equal to the emissive power of a blackbody at that temperature.

$$\frac{E(body)}{E(body)} = E\ (blackbody)$$

9.12 Wien's Displacement Law

The wavelength λ_m for which energy is the maximum, decreases with increasing temperature. The relation between λ_m and T is given by a law known as Wien's Displacement Law. $\lambda_m T = constant$..... (9.14) .The value of the constant (Wien's constant) is $2.9 \times 10^{-3} mK$. This law explains why the colour of a piece of iron heated in a hot flame first becomes dull red, then reddish yellow, and finally white hot. Wien's law is useful for estimating the surface temperatures of celestial bodies like the moon, sun and other stars.

9.13 Stefan Boltzmann Law

The total energy emitted per unit time by a blackbody of surface area A is given by: $u = \sigma A T^4$ ....(9.15) where T is the absolute temperature i.e. measured in kelvin, and σ is Stefan Boltzmann constant and is equal to $5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$. The equation (9.15) is called the Stefan Boltzmann Law.

A body which is not a blackbody, emits less energy than given by Equation (9.15). The energy emitted by such a body is given as: $u = e\sigma AT^4$……(9.16) where e is emissivity and e is always greater than zero (0) and less than one (1) for a non-black body and $e = 1$ for a blackbody.

A body at temperature T, with surroundings at temperatures T_s, emits as well as receives energy. For a perfect radiator, the net rate of loss of radiant energy is $u = \sigma A\ (T^4 - T_s^4)$

For a body with emissivity e, the relation modifies to $\sigma eA\ (T^4 - T_s^4)$

Example 9.5 A blackbody of surface area $10cm^2$ is heated to $127°C$ and is suspended in a room at temperature $27°C$. Calculate the initial rate of loss of heat from the body to the room.

Solution: For a blackbody at temperature T, the rate of emission is $u = A\sigma T^4$. When it is kept in a room at temperature T_0, the rate of absorption is $u_o = A\sigma T_0^4$.

The net rate of loss of heat is $u - u_0 = \sigma A\ (T^4 - T_0^4)$

Here $A = 10 \times 10^{-4}\ m^2, T = 400\ K$ and $T_0 = 300\ K$.

Thus, $u - u_0 = (5.67 \times 10^{-8} Wm^{-2}K^{-4})\ (10 \times 10^{-4} m^2)(400^4 - 300^4)K^4$

$= 0.99\ W$.

9.14 Newton's Law of Cooling

It states, the rate of loss of heat, $-\frac{dQ}{dt}$ of the body is directly proportional to the difference of temperature $\Delta T = (T_2 - T_1)$ of the body and the surroundings. The law is mostly applicable for small differences in temperature. Also, the loss of heat by radiation depends upon the nature of the surface of the body and the area of the exposed surface.

Thus, $-\frac{dQ}{dt} = k(T_2 - T_1)$ .....(9.17)

where k is a positive constant depending upon the area and nature of the surface of the body. Suppose a body of mass m and specific heat capacity s is at temperature T_2. Let T_1 be the temperature of the surroundings. If the temperature falls by a small amount dT_2 in time dt, then the amount of heat lost is $dQ = msdT_2$

∴ Rate of loss of heat is given by $\frac{dQ}{dt} = \frac{msdT_2}{dt}$ ….(9.18)

From Equation (9.17) and (9.18), we have $-\frac{msdT_2}{dt} = k\ (T_2 - T_1)$

$\frac{dT_2}{(T_2-T_1)} = -(k/ms)dt = -Kdt$ where $K = k/ms$

On integrating, $\log_e(T_2 - T_1) = -Kt + c \ or \ T_2 = T_1 + C'e^{-kt}$; where $C' = e^c$

Example 9.6 A pan filled with hot food cools from $94°C$ to $86°C$ in 2 minutes when the room temperature is at $20°C$. How long will it take to cool from $71°C$ to $69°C$?

Solution: The average temperature of $94°C$ and $86°C$ is $90°C$, which is $70°C$ above the room temperature. Under these conditions the pan cools $8°C$ in 2 minutes. Using Eq. (9.17), we have Change in temperature/Time = $K\Delta T$

$8°C/\ 2min = K(70°C\ )$ ……(i)

The average of $69°C$ and $71°C$ is $70°C$, which is $50°C$ above room temperature. K is the same for this situation as for the original.

$\frac{2°C}{Time} = K(50°C)$ …….(ii)

When we divide the above two equations, we have $8°C/2\ min\ /2°\ C/ time = K\ (70°C)/K\ (50°C) = Time = 0.7\ min = 42\ s$

Exercises

1. A worker fixing an aluminum rod on an iron pillar for a specific construction .The length of the iron and aluminum rod are $4.024m$ and $4.018m$, respectively at $32°C$. To what temperature should the silver be heated so as to perfectly fit in the pillar? $(\alpha = 2.5 \times 10^{-5}k^{-1})$
2. A steel scale $1m$ long is perfectly calibrated for a temperature of $27.0°C$. The length of a steel rod measured by this scale is found to be $63.0cm$ on a hot day when the temperature is $45.0°C$. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is $27.0°C$? Coefficient of linear expansion of steel $= 1.20 \times 10^{-5}K^{-1}$.
3. If the lower surface of a slab of stone of face area $3200cm^2$ and thickness $20cm$ is put to steam at $100°C$. A block of ice of mass $4.8gm$ at $0°C$ lies on the upper surface of the slab and it melts in one hour. Find the thermal conductivity of the stone. Latent heat of fusion of ice $= 3.36 \times 10^5 Jkg^{-1}$.
4. A bar of copper length $60cm$ and a bar of steel of length $120cm$ are joined together end to end. Both are of circular cross section with diameter $2cm$. The free ends of the copper and the steel bars are maintained at $100°C$ and $0°C$ respectively. The curved surfaces of the bars are thermally insulated. What is the temperature of the copper-steel junction? What is the amount of heat transmitted per unit time across the

junction? (Thermal conductivity of copper is $386 J s^{-1} m^{-1} {}^{\circ}C^{-1}$, and of the steel is $46 J s^{-1} m^{-1} {}^{\circ}C^{-1}$

5. Water falls from a height of $200m$ into a pool. Calculate the rise in temperature of the water assuming no heat dissipation from the water in the pool. (Take $g = 10\ m/s^2$, specific heat of water $= 4200 J/kg\ K$)
6. Two thin metallic spherical shells of radii r_1 and r_2 $(r_1 < r_2)$ are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature θ_1 and the outer shell at temperature $\theta_2 (\theta_1 < \theta_2)$. Calculate the rate at which heat flows radially through the material.
7. One end of a steel rod $(k = 46 J s^{-1} m^{-1} {}^{\circ}C^{-1})$ of length $1m$ is kept in ice at $0°C$ and the other end is kept in boiling water at $100°C$. The area of cross section of the rod is $.04\ cm^2$. Assuming that no heat is lost to the atmosphere, find the mass of the ice melting per second. (Latent heat of fusion of ice $= 3.36 \times 10^5 J kg^{-1}$).
8. Consider a rectangular sheet of solid material of length $l = 9\ cm$ and width $d = 4cm$. The coefficient of linear expansion is $\alpha = 3.1 \times 10^{-5} K^{-1}$ at room temperature and one atmospheric pressure. The mass of sheet $m = 0.1 kg$ and the specific heat capacity $C_v = 900 J kg^{-1} K^{-1}$. If the amount of heat supplied to the material is $8.1 \times 10^2\ J$ then change in area of the rectangular sheet is :
9. There are two vessels filled with an ideal gas where the volume of one is double the volume of another. The large vessel contains the gas at $8 kPa$ at $1000K$ while the smaller vessel contains the gas at $7\ kPa$ at $500K$. If the vessels are connected to each other by a thin tube allowing the gas to flow and the temperature of both vessels is maintained at $600K$, at steady state the pressure in the vessels will be (in kPa) (Hint- initial mass = final mass)
10. Match List-I with List-II

List-I	List-II
(A) Heat capacity of body	(i) $J kg^{-1}$
(B) Specific heat capacity of body	(ii) $J K^{-1}$
(C) Latent heat	(iii) $J kg^{-1} K^{-1}$
(D) Thermal conductivity	(iv) $J m^{-1} K^{-1} s^{-1}$

11. A brass rod of length $50cm$ and diameter $3.0mm$ is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at $250°C$, if the original lengths are at $40.0°C$? Is there a 'thermal stress' developed at the junction? The ends of the rod are free

to expand (Co-efficient of linear expansion of brass $= 2.0 \times 10^{-5} K^{-1}$, steel $= 1.2 \times 10^{5} K^{-1}$)

12. The coefficient of volume expansion of glycerine is $49 \times 10^{-5} K^{-1}$. What is the fractional change in its density for a $30°C$ rise in temperature?
13. A body cools from $80°C$ to $50°C$ in 5 minutes. Calculate the time it takes to cool from $60°C$ to $30°C$. The temperature of the surroundings is $20°C$.
14. A cup of coffee cools from $90°C$ to $80°C$ in t minutes when the room temperature is $20°C$. The time taken by the similar cup of coffee to cool from $80°C$ to $60°C$ at the same room temperature is:
15. The temperature of a body in air falls from $40°C$ to $24°C$ in 4 minutes. The temperature of the air is $16°C$. The temperature of the body in the next 4 minutes will be :

Answers

1. 91.73°C	2. 63.0136 cm, 63 cm	3. $2.8 \times 10^{-3} W m^{-1} °C^{-1}$	4. $94°C, 1.136\, J s^{-1}$
5. 0.48 K	6. $\frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}$	7. $5.5 \times 10^{-5} \frac{g}{s}$	8. $0.020\, cm^2$
9. $6\, kpa$	10. (A)-(ii), (B)-(iii), (c)-(i), (D)-(iv)	11. 0.336 cm, There is no thermal stress developed at the junction because the ends of the rod are free to expand	12. -0.0145
13. 10 min	14. $\left(\frac{13}{5}\right) t$	15. $\left(\frac{56}{3}\right) °C$	

Chapter - 10
Thermodynamics and Kinetic Theory of Gases

The final two weeks of their 11th grade schooling were left. The annual exam dates had already been announced. Early this morning Arnold went to Amila's Apartment (located in the city), by the bus, as he was invited by Amila regarding the deal.

*****

"I had a meeting with my group last night. We are able to contribute 15 Lakhs INR." Arnold uttered the words slowly to Amila." "That's ok, I will see the rest of the money but you attend today's class as it has been a long time you didn't attend." said Amila. "It doesn't impact anyway," Arnold responded, "We are hitting the book, understanding the modules comprehensively, and there is no drama of minimum attendance for attending the annual exam that is approaching."

"It looks like your life is at its best." Amila voiced. "But, there is also a contribution of yours in making it." Arnold praised. "Oh really!" exclaimed Amila. Arnold: "Really, before you we used to be a little stressed about attending every class thinking that if we missed the class we wouldn't be able to understand by our own. However, after your arrival in our lives even that tension is no longer as you are pro in the subject.

*****

Arnold stayed for a while and then Amila dropped him to the school by his car as the bus already went back to carry the rest of the guys to the city for today's schooling.

*****

"Welcome back, avengers! where have you all been?" Gita joked as the group entered the classroom. "I thought you all would come straight to appear for the examination." The rest of the class cracked into laughter. However, settling the noise she started teaching.

Thermodynamics is a branch of physics which involves study of energy and work of a system. It gives thermal laws involving heat, temperature, work and energy. It is a macroscopic science, It deals with bulk systems and does not go into the molecular constitution of matter.

In thermodynamics, a macroscopic science is one that is large enough to be described by macroscopic properties like temperature, pressure, and volume, without considering the behavior of individual atoms or molecules.

10.1 Thermal equilibrium

A state where two or more systems are in thermal contact and do not exchange heat energy, meaning they are at the same temperature is said to be in thermal equilibrium. The surrounding environment is insulated from the system (gas) or the wall of the container is adiabatic as shown in Figure 10.1.

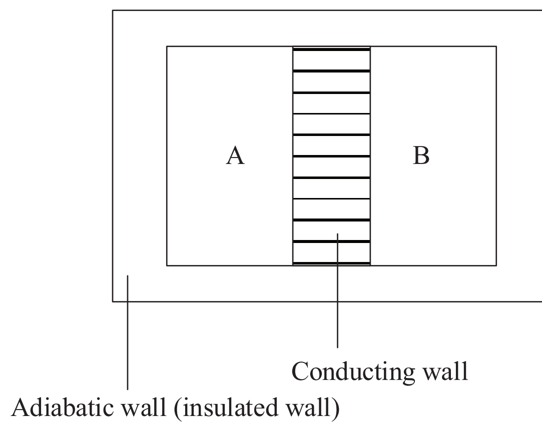


Fig 10.1

Thermal contact: It is physical interaction between two or more systems that allows transfer of heat (energy) between them.

Thermal equilibrium occurs when there is no temperature difference between different systems such as A and B.

10.2 Zeroth Law of Thermodynamics

It states if two systems are in thermal equilibrium with a third system then the two systems are in thermal equilibrium with each other.

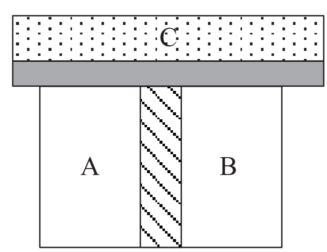


Fig 10.2 (a)

Explanation: Suppose two systems A and B, separated by an adiabatic wall, and each system is in contact with a third system C, via a conducting wall (Figure 10.2(a)). The states of the systems (i.e., their macroscopic variables like heat, pressure) will change until both A and B come to thermal equilibrium with C. Once this is achieved, suppose that the adiabatic wall between A and B is replaced by a conducting wall and C is insulated from A and B by an adiabatic wall (Figure 10.2(b)). It is found that the states of A and B change no further i.e. they

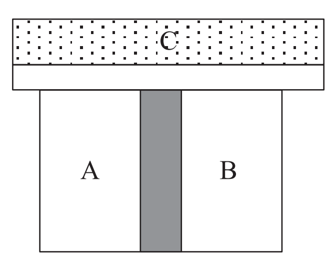


Fig 10.2 (b)

are found to be in thermal equilibrium with each other. This is the zeroth law of thermodynamics.

The Zeroth Law clearly suggests that when two systems A and B, are in thermal equilibrium, there must be a physical quantity that has the same value for both. This thermodynamic variable whose value is equal for two systems in thermal equilibrium is called temperature (T). Thus, if A and B are separately in equilibrium with $C, T_A = T_C$ and $T_B = T_C$. This implies that $T_A = T_B$, i.e. the systems A and B are also in thermal equilibrium. This is the concept of temperature.

10.3 Heat Work and Internal Energy

Heat: It is the energy transferred due to temperature difference between two objects. Temperature is a marker of the 'hotness' of a body. It determines the direction of flow of heat when two bodies are placed in thermal contact. Heat flows from the body at a higher temperature to the one at lower temperature. The flow stops when the temperatures equalise; the two bodies are then in thermal equilibrium.

Internal Energy: It is the total energy possessed by a system due to the random motion and interaction of its molecules. The concept of internal energy of a system is easy to understand. We know that every bulk system consists of a large number of molecules. Internal energy is simply the sum of the kinetic energies and potential energies of these molecules. It is denoted by $U \; and \; U$ is a macroscopic variable of the system. The crucial fact about internal energy is that it depends only on the state of the system, not on how that state was achieved.

Internal energy U of a system is an example of a thermodynamic 'state variable'–its value depends only on the given state of the system, not on the 'path' taken to arrive at that state. Thus, the internal energy of a given mass of gas depends on its state described by specific values of pressure, volume and temperature. Pressure, volume, temperature, and internal energy are thermodynamic state variables of the system (gas). If we neglect the small intermolecular forces in a gas, the internal energy of a gas is just the sum of kinetic energies associated with different random motions of the molecules.

Work

Work is either done on the system or by the system (like in mechanics). And the system consists of a large number of molecules. work is not a state variable rather it is the energy that is transferred to the system. But work always contributes to the internal energy of the system which is a state variable.

If work is done on the system, internal energy increases and the energy gets decreased if work is done by the system on the surroundings.

10.4 First law of Thermodynamics

We have seen that the internal energy U of a system can change through two modes of energy transfer: heat and work. Let $\Delta Q =$ Heat supplied to the system by the surroundings

$\Delta W =$ Work done by the system on the surroundings

$\Delta U =$ Change in internal energy of the system

The general principle of conservation of energy then implies that $\Delta Q = \Delta U + \Delta W$……. (10.1)

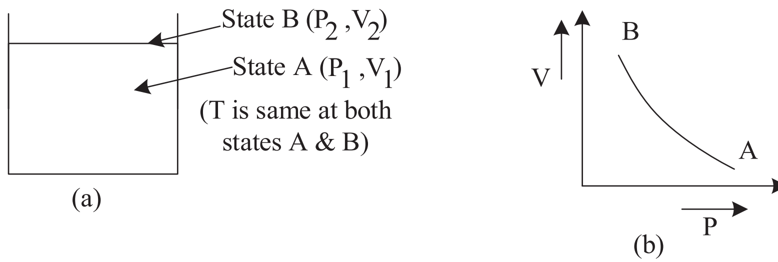
The energy (ΔQ) supplied to the system goes in partly to increase the internal energy of the system (ΔU) and the rest in work on the environment (ΔW)

Equation (10.1) is known as the First Law of Thermodynamics. It is simply the general law of conservation of energy applied to any system in which the energy transfer from or to the surroundings is taken into account.

Let us put Equation (10.1) in the alternative form $\Delta Q - \Delta W = \Delta U$…… (10.2) Now, the system may go from an initial state to the final state in a number of ways. For example, to change the state of a gas from (P_1, V_1) to (P_2, V_2), we can first change the volume of the gas from V_1 to V_2, keeping its pressure constant i.e. we can first go the state (P_1, V_2) and then change the pressure of the gas from P_1 to P_2, keeping volume constant, to take the gas to (P_2, V_2).

Alternatively, we can first keep the volume constant and then keep the pressure constant. Since U is a state variable, ΔU depends only on the initial and final states and not on the path taken by the gas to go from one to the other. However, ΔQ and ΔW will, in general, depend on the path taken to go from the initial to final states. From the First Law of Thermodynamics, Equation (10.2), it is clear that the combination $\Delta Q - \Delta W$, is, however, path independent. This shows that if a system is taken through a process in which $\Delta U = 0$ (for example isothermal expansion of an ideal gas), $\Delta Q = \Delta W$ i.e., heat supplied to the system is used up entirely by the system in doing work on the environment (surrounding).

If the system is a gas in a cylinder with a movable piston, the gas in moving the piston does work. Since force is pressure times area, and area times displacement is volume, work done by the system against a constant pressure P is $\Delta W = P\Delta V$ where ΔV is the change in volume of the gas. Thus, for this case, Equation (10.1) gives $\Delta Q = \Delta U + P\Delta V$……. (10.3)



The Fig. (a) shows a gas (system) in two states in a container
and Fig. (b) shows its graphical representation

"Mam, how would I say whether the work is done on the system or by the system," said David. Gita: "If ΔW is positive, we say work is done by the system on the surrounding and if ΔW is negative, we say work is done by the surrounding on the system." Aakash: "Surrounding,' can you elaborate it, what it means?" Gita: "Suppose if a container contains a particular gas as shown in the figure. Then here we would say the atmosphere is the surrounding and we have some pressure of gas in the container and there is some pressure of the atmosphere (called atmospheric pressure). And, if pressure of both the gas and the surrounding (the atmosphere) is equal, volume of the gas will be fixed and both atmosphere and gas will be said "are in thermal equilibrium." If atmospheric pressure is more of the gas then the atmosphere will push down the gas and eventually the volume contracts and work is done on the gas and if pressure of the gas is more of that atmosphere then the gas will push the atmosphere in upward direction, resulting the work is done by the system (the gas) and v of the gas expands. This is the basic concept behind it. It would also be detailed in the next class but I hope you all have already understood it." Gita uttered it at last and left the class

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The next formal session starts with the topic 'Thermodynamic Process'

Molar specific heat capacity – We know specific heat capacity $s = \dfrac{S}{m}$

Where $\boldsymbol{S}$ is heat capacity.

But when we deal with gas or liquid, the gas could be given directly in terms of mole instead of kg. And in that context we define a term molar specific heat capacity which is defined as 'The amount of heat $p$er unit mole absorbed or given off by the substance to change its temperature by one unit. It is denoted by C.

$$C = \frac{S}{\mu}$$

Where μ is the no of moles present in the given substance. S.I. unit of C is $Jmol^{-1}K^{-1}$.

But, additional parameters may be needed to define C as heat transfer can be done by keeping either pressure or volume constant. If the gas is held under constant pressure during the heat transfer then it is called the molar specific heat capacity at constant pressure and is denoted by C_P. And if the volume of gas is kept constant during the heat transfer, then the molar specific heat capacity is called molar specific heat capacity at constant volume and is denoted by C_v.

10.5 Thermodynamics Process

Quasi-static process

Consider a gas in thermal and mechanical equilibrium with its surroundings. The pressure of the gas in that case equals the external pressure and its temperature is the same as that of its surroundings. Suppose that the external pressure is suddenly reduced (say by lifting the weight on the movable piston in the container). The piston will accelerate outward. During the process, the gas passes through states that are not equilibrium states. The nonequilibrium states do not have well-defined pressure and temperature. In the same way, if a finite temperature difference exists between the gas and its surroundings, there will be a rapid exchange of heat during which the gas will pass through non-equilibrium states. In due course, the gas will settle to an equilibrium state with well-defined temperature and pressure equal to those of the surroundings.

Isothermal Process: A thermodynamic process in which temperature is fixed throughout is called isothermal process. The expansion of a gas in a metallic cylinder placed in a large reservoir of fixed temperature is an example of an isothermal process.

We now consider these processes in some detail. For an isothermal process (T fixed), the ideal gas equation gives $PV = constant$ i.e., pressure of a given mass of gas varies inversely as its volume. This is nothing but Boyle's Law. Suppose an ideal gas goes isothermally (at temperature T) from its initial state (P_1, V_1) to the final state (P_2, V_2). At any intermediate stage with pressure P and volume change from V to $V + \Delta V$ (ΔV small) $\Delta W = P\,\Delta V$

Taking ($\Delta V \rightarrow 0$) and summing the quantity ΔW over the entire process, $W = \int_{v1}^{v2} Pdv$

$= uRT \int_{v1}^{v2} \frac{dv}{v} = \frac{uRTlnv_2}{v_1}$ ……(10.4) where in the second step we have made use of the ideal gas equation $PV = \mu RT$ and taken the constants out of the integral. For an ideal gas, internal energy depends only on temperature. Thus, there is no change in the internal energy of an ideal gas in an isothermal process. The First Law of Thermodynamics then implies that heat supplied to the gas equals the work done by the gas: $Q = W$. Note from Equation (10.4) that for $V_2 > V_1, W > 0$; and for $V_2 < V_1, W < 0$. That is, in an isothermal expansion, the gas absorbs heat and does work while in an isothermal compression, work is done on the gas by the environment and heat is released.

Adiabatic Process

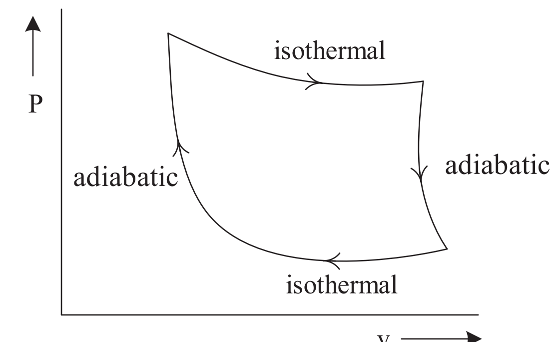


Fig 10.3

In an adiabatic process, the system is insulated from the surroundings and heat absorbed or released is zero. From Equation (10.1), we see that work done by the gas results in decrease in its internal energy (and hence its temperature for an ideal gas). We quote without proof (the result that you will learn in higher courses) that for an adiabatic process of an ideal gas. $PV^{\gamma} = const$ …… (10.5)

where γ is the ratio of specific heats (ordinary or molar) at constant pressure and at constant volume. $\gamma = \frac{C_p}{C_v}$

Thus if an ideal gas undergoes a change in its state adiabatically from $(P_1, V_1)\ to\ (P_2, V_2)$: $P_1 V_{\gamma}^{1} = P_2\ V_2^{\gamma}$…..(10.6) Figure 10.3 shows the $P - V$ curves of an ideal gas for two adiabatic processes connecting two isotherms.

We can calculate, as before, the work done in an adiabatic change of an ideal gas from the state (P_1, V_1, T_1) to the state (P_2, V_2, T_2).

$$W = \int_{v1}^{v2} Pdv$$

$$= constant \times \int_{v1}^{v2} \frac{dv}{v^{\gamma}} = constant \times \left.\frac{V^{-y+1}}{1-y}\right|_{v_1}^{v_2}$$

$$\frac{constant}{(1-y)} \times \left[\frac{1}{v_2^{y-1}} - \frac{1}{v_1^{y-1}}\right] \qquad \text{….. (10.7)}$$

From Equation (10.6) the constant is $p_1 v_1^y$ or $p_2 v_2^y$

$$w = \frac{1}{(1-y)} \times \left[\frac{p_2 v_2^y}{v_2^{y-1}} - \frac{p_1 v_1^y}{v_1^{y-1}}\right]$$

$$= \frac{1}{1-y}[P_2 v_2 - P_1 v_1] = \frac{\mu R(T_1 - T_2)}{y-1} \qquad \ldots (10.8)$$

Example A vessel of volume $8.0 \times 10^{-3} m^3$ contains an ideal gas at $300k$ and $200kPa$. The gas is allowed to leak till the pressure falls to $125kPa$. Calculate the amount of the gas (in moles) leaked assuming that the temperature remains constant.

Solution: As the gas leaks out, the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel is given by $\eta = \frac{pV}{RT}$. The number of moles in the vessel before the leakage is

$$\eta_1 = \frac{p_1 V}{RT}$$

And that after the leakage is

$$\eta_2 = \frac{p_2 V}{RT}$$

Thus, the amount leaked is

$$\eta_1 - \eta_2 = \frac{(p_1 - p_2)V}{RT}$$

$$= \frac{(200-125)\times 10^3 Nm^{-2} \times 8.0 \times 10^{-3} m^3}{(8.3 JK^{-1} mol^{-1}) \times (300K)}$$

0.24 mol

Example A vessel of volume $2000cm^3$ contains 0.1 mol of oxygen and 0.2 mol of carbon dioxide. If the temperature of the mixture is $300K$, find its pressure.

Solution: We have $p = \frac{\eta RT}{V}$

The pressure due to oxygen is

$$p_1 = \frac{(0.1mol)(8.3JK^{-1}mol^{-1})(300K)}{(2000 \times 10^{-6} m^{-3})} = 1.25 \times 10^5 pa.$$

Similarly, the pressure due to carbon dioxide is $p_2 = 2.50 \times 10^5 Pa$.

The total pressure in the vessel is

$$p = p_1 + p_2$$

$$= (1.25 + 2.50) \times 10^5 Pa = 3.75 \times 10^5 Pa.$$

Example A mixture of hydrogen and oxygen has volume $2000cm^3$, temperature $300K$, pressure $100kPa$ and mass $0.76g$. Calculate the masses of hydrogen and oxygen in the mixture.

Solution: Suppose there are η_1 moles of hydrogen and η_2 moles of oxygen in the mixture. The pressure of the mixture will be

$$p = \frac{\eta_1 RT}{V} + \frac{\eta_2 RT}{V} = (\eta_1 + \eta_2)\frac{RT}{V}$$

Or, $100 \times 10^3 Pa = (\eta_1 + \eta_2)\frac{(8.3JK^{-1}mol^{-1})(300K)}{(2000\times10^{-6}m^{-3})}$

Or, $\eta_1 + \eta_2 = 0.08mol.$ ... (i)

The mass of the mixture is

$$\eta_2 \times 2g\ mol^{-1} + \eta_2 \times 32g\ mol^{-1} = 0.76g$$

$$\eta_1 + 16\eta_2 = 0.38mol \quad \text{.... (ii)}$$

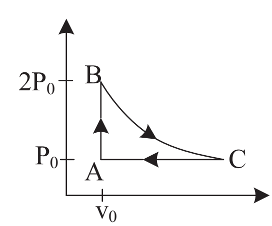
From (i) and (ii),

$\eta_1 = 0.06mol$ and $\eta_2 = 0.02mol$

The mass of hydrogen $= 0.06 \times 2g = 0.12g$ and the mass of oxygen $= 0.02 \times 32g = 0.64g$.

Example An ideal gas has pressure p_0, volume V_0 and temperature T_0. It is taken through an isochoric process till its pressure is doubled. It is now isothermally expanded to get the original pressure. Finally, the gas is isobarically compressed to its original volume V_0. (a) Show the process on a $p - V$ diagram. (b) What is the temperature in the isothermal part of the process? (c) What is the volume at the end of the isothermal part of the process?

Solution: (a) The process is shown in $p - V$ diagram in figure. The process starts from A and goes through $ABCA$.



(b) Applying $pV = \eta RT$ at A and B,

$$p_0V_0 = \eta RT_0$$

And $(2p_0)V_0 = \eta RT_B$

Thus, $T_B = 2T_0$

This is the temperature in the isothermal part BC.

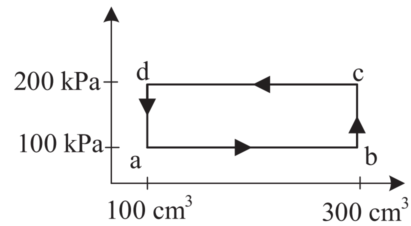
(C) As the process BC is isothermal, $T_C = T_B = 2T_0$.

Applying $pV = \eta RT$ at A and C,

$$p_0V_0 = \eta RT_0$$

And $p_0V_c = \eta R(2T_0)$

Or $V_c = 2V_0$



Example A thermodynamic system is taken through the cycle $abcda$ (figure). Calculate the work done by the gas during the parts ab, bc, ca and da.

Solution: (a) The work done during the part ab

$= \int_a^b \quad pdV = (100\ kPa) \int_a^b \quad dV$

$= (100\ kPa)\ (300\ cm^3 - 100\ cm^3)$

$= 20J.$

The work done during bc is zero as the volume does not change. The work done during cd

$= \int_a^b pdV = (200\ kPa)(100\ cm^3 - 300\ cm^3)$

$= -40J.$

The work done during da is zero as the volume does not change.

(b) The total work done by the system during the cycle $abcda$

$\Delta W = 20J - 40J = -20J.$

The change in internal energy $\Delta U = 0$ as the initial state is the same as the final state.

Isochoric process

In an isochoric process, V is constant. No work is done on or by the gas. From Equation (10.1), the heat absorbed by the gas goes entirely to change its internal energy and its temperature. The change in temperature for a given amount of heat is determined by the specific heat of the gas at constant volume.

Isobaric process

In an isobaric process, P is fixed. Work done by the gas is $W = P(V_2 - V_1) = \mu R(T_2 - T_1)$ .... (10.9) Since temperature changes, so does internal energy. The heat absorbed goes partly to increase internal energy and partly to do work. The change in temperature for a given amount of heat is determined by the specific heat of the gas at constant pressure.

Cyclic process

In a cyclic process, the system returns to its initial state. Since internal energy is a state variable, $\Delta U = 0$ for a cyclic process. From Equation (10.1), the total heat absorbed equals the work done by the system.

10.6 Second Law of Thermodynamics

The second law of thermodynamics states that in any spontaneous process, the total entropy of the universe (system + surrounding) always increases. This can be expressed as $\Delta S_{universe} > 0$ where ΔS is the change in entropy of the universe.

Entropy: It is a measure of the disorder or randomness in a system. A higher entropy value indicates a greater degree of disorder.

A spontaneous process is one that occurs without needing to be driven by an external force. It tends to proceed naturally. For example, melting of an ice is a spontaneous process because it does not require any external energy.

10.7 Reversible and Irreversible Process

Irreversible Process : An irreversible process in thermodynamics is one where the system and its surroundings cannot return to their initial states after the process is completed, without any change elsewhere in the universe. Essentially, the process cannot be reversed to restore the system and its environment to their exact original conditions.

Imagine some process in which a thermodynamic system (a gas) goes from an initial state i to a final state f. During the process the system absorbs heat Q from the surroundings and performs work W on it. Can we reverse this process and bring both the system and surroundings to their initial states with no other effect anywhere ?

Observation suggests that for most processes in nature this is not possible. The spontaneous processes of nature are irreversible. Several examples can be cited. The base of a vessel on an oven is hotter than its other parts. When the vessel is removed, heat is transferred from the base to the other parts, bringing the vessel to a uniform temperature (which in due course cools to the temperature of the surroundings). The process cannot be reversed; a part of the vessel will not get cooler spontaneously and warm up the base.

The free expansion of a gas is irreversible. The combustion reaction of a mixture of petrol and air ignited by a spark cannot be reversed. Cooking gas leaking from a gas cylinder in the kitchen diffuses to the entire room. The diffusion process will not spontaneously reverse and bring the gas back to the cylinder. The stirring of a liquid in thermal contact with a reservoir will convert the work done into heat, increasing the internal energy of the reservoir. The process cannot be reversed exactly; otherwise it would amount to conversion of heat entirely into work, violating the Second Law of Thermodynamics.

Irreversibility occurs mainly from two reasons : one, many processes (like a free expansion, or an explosive chemical reaction) take the system to non-equilibrium states; two, most processes involve friction, viscosity and other dissipative effects (example, a moving body coming to a stop and losing its mechanical energy as heat to the floor and the body; a rotating blade in a liquid coming to a stop due to viscosity and losing its mechanical energy with corresponding gain in the internal energy of the liquid). Since dissipative effects are present everywhere and can be minimised but not fully eliminated, most processes that we see in our practical life are irreversible.

Reversible Process: It is a process which can be reversed to return both the system and its surroundings to their original states without any net change to the universe. It is not possible in our practical life. It is just a hypothetical concept.

A thermodynamic process (state $i \rightarrow$ state f) is reversible if the process can be turned back such that both the system and the surroundings return to their original states, with no other change anywhere else in the universe. From the preceding discussion, a reversible process is an idealised notion. A process is reversible only if it is quasi-static (system in equilibrium with the surroundings at every stage) and there are no dissipative effects. For example, a quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is a reversible process.

10.8 Efficiency of Heat engine

It means how effectively a heat engine converts the heat energy it absorbs into useful mechanical work.

The efficiency denotes the proportion of the total heat input that is converted into useful work.

The efficiency of a heat engine (η) can be calculated as

$$\eta = \frac{work\ output(w)}{Heat\ input(Q_H)}$$

Or $\eta = 1 - \frac{heat\ rejected\ (Q_C)}{Heat\ input\ (Q_H)}$

Where η = efficiency of the heat engine

(It is also called thermal efficiency)

w = the useful work output generated by the engine.

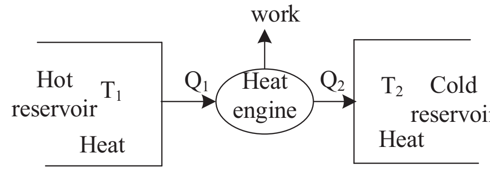
Q_H = Heat absorbed from the High-temperature source (hot reservoir)

$Q_c =$ Heat rejected to the low temperature sink (cold reservoir)

Efficiency is a dimensionless quantity.

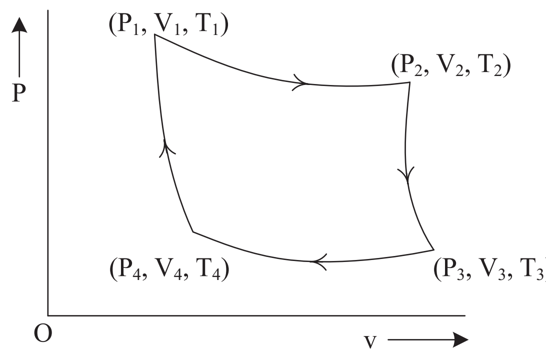
$T_1 > T_2$ (here $Q_1 = Q_H$

& $Q_2 = Q_c$)



10.9 Carnot Engine

A Carnot engine is a theoretical and idealized heat engine that works on the principles of the Carnot cycle. It is considered as the most efficient possible heat engine that can work between two given temperatures (according to the laws of thermodynamics)



Carnot Cycle Fig 10.4

Carnot efficiency formula

$\eta = 1 - \left(\frac{T_C}{T_H}\right)$

Carnot engine is a reversible engine. Most importantly it is the only reversible engine possible that operates between two reservoirs at different temperatures.

Every step of a Carnot cycle is shown in the figure 10.4

All the steps of the Carnot cycle can be reversed. It could be done this way, taking heat Q_2 from the cold reservoir at T_2, doing work w on the system, and transforming heat Q_1 to the hot reservoir as shown in Figure 10.5

An irreversible engine (I) coupled to a reversible refrigerator (R).

Here $Q_1 - w = Q_2$

There is one important fact known as Carnot's theorem, that in a Carnot cycle.

$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

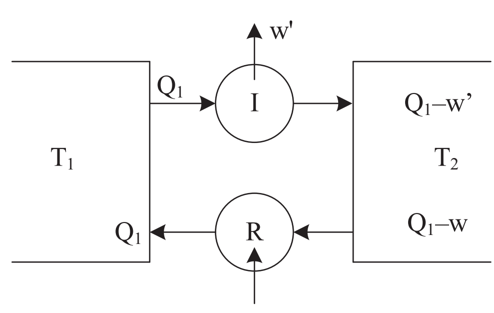


Fig 10.5

Example A heat engine operates between a cold reservoir at temperature $T_2 = 400k$ and a hot reservoir at temperature

T_1. It takes $300j$ of heat from the hot reservoir and delivers $240j$ of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be _____ k.

Solution: $Q_{in} = 300j, Q_{out} = 240J$

Work done = $Q_{in} - Q_{out} = 300 - 240 = 60J$

Efficiency = $\frac{w}{Q_{in}} = \frac{60}{300} = \frac{1}{5}$

And, Efficiency $= 1 - \frac{T_2}{T_1}$

$\Rightarrow \frac{1}{5} = 1 - \frac{400}{T_1} = \frac{4}{5}$

Therefore $T_1 = 500k$

Example 300 cal. of heat is given to a heat engine and it rejects 225cal. If source temperature is $227°C$, then the temperature of sink will be ______°C.

Solution: $\eta = \frac{w}{Q} = \frac{300-225}{300}$

$\Rightarrow \frac{75}{300} = 1 - \frac{T_L}{T_H}$

$\Rightarrow T_L = \frac{3}{4} T_H$

Therefore $\Rightarrow T_L = 102$°C.

Example In a Carnot engine, the temperature of reservoir is 527°C and that of sink is $200k$. If the work done by the engine when it transfers heat from reservoir to sink is $12000kj$, the quantity of heat absorbed by the engine from reservoir is ________ $\times 10^6 J$.

Solution: $\eta = 1 - \frac{T_2}{T_1}$

$= 1 - \frac{200}{800} = \frac{3}{4}$

$\therefore \eta = \frac{w}{Q_1}$

$\Rightarrow \frac{3}{4} = \frac{12000 \times 10^3}{Q_1}$

$\Rightarrow Q_1 = 16 \times 10^6 J.$

Example one mole of an ideal gas at 27°C is taken from A to B as shown in the given p_v indicator diagram. The work done by the system will be ______ $\times 10^{-1} J$. (Given $R = 8.3J/molek$, $L_n 2 = .6931$)

Solution: Here temperature remains constant during the process so for isothermal

$$w = \eta RT l_n \left(\frac{V_2}{V_1}\right)$$

$$= 1 \times 8.3 \times 300 \times l_n 2$$

$$= 17258 \times 10^{-1} J.$$

10.8 Behaviour of gases and equation of state of a perfect (ideal) gas.

In a gas, modules are far from each other and their mutual interactions are negligible except when two molecules collide. And in the case of a perfect gas, we suppose there is no collision of molecules.

A gas has certain characteristic like pressure, volume, temperature etc. And for a perfect gas, there is a relationship between p (pressure), V (volume). T (Temperature) and μ no of moles) exists that is

$pv = \mu RT$ ….. (10.10) as we read in chapter 9 (Equation 9.2)

$R = N_A K_B$ is a universal constant ….. (10.11)

$R = 8.314 jmol^{-1} k^{-1}$

T is absolute temperature (k) and N_A is Avogadro Number

k_B is the Boltzmann constant

Avogadro Number: It is the number of atoms, molecules, or ions in one mole of a substance. It's constant and is equal to 6.022×10^{23}.

$\mu = \frac{M}{M_0} = \frac{N}{N_A}$ (μ also denoted by η) …. 10.12

Where M is the mass of the gas containing N molecules, M_0 is the molar mass or one mole meaning mass of the gas containing N_A molecules.

Using equation (10.10) and (10.12)

$Pv = K_B NT$ or $P = K_B nT$ ….. (10.13)

Here n is the number density, i.e. number of molecules per unit volume.

A gas that follows equation (10.10) at all pressure and temperature is known as an ideal gas.

Note a gas shows ideal behaviour at low pressures and high temperatures.

In the ideal gas equation if we fix μ and T, we get

$pv = constant$ …. (10.14)

Equation 10.14 is known as Boyle's law. The equation also tells keeping μ & T fixed, pressure of a given mass varies inversely with volume.

10.9 Dalton's law of partial pressure:

If there is a mixture of non-interacting ideal gases: μ_1 moles of gas 1, μ_2 moles of gas 2 etc in a vessel of volume v at temperature T and pressure P. It is then found, the equation of state of the mixture is:

$pv = (\mu_1 + \mu_2 + \ldots)RT$ 10.15

Or $p = \mu_1 \frac{RT}{V} + \mu_2 \frac{RT}{V} + \ldots \ldots \ldots$

Or $P = P_1 + P_2 + \ldots$ (Where $P_1 = \frac{\mu_1 RT}{V}, P_2 = \frac{\mu_2 RT}{V}$ ... (10.16)

P_1 is the pressure that gas 1 would exert at the same temperature and volume that any other gas of the mixture is present.

And this is called partial pressure of the gas. Therefore, the total pressure of a mixture of ideal gases is the sum of partial pressure. It is called Dalton's law of partial pressures.

10.10 work done on compressing a gas

When a gas is compressed, work is done on the system (the gas) resulting in a positive value of work.

$w = -P\Delta V$ 10.17

Where w is work done

P is the external pressure applied to the gas and Δv is the change in volume $(V_{final} - V_{initial})$

Work is positive

Since the gas is being compressed (volume is decreasing), the work done is positive. It means the work is done on the system (the gas) by the surrounding

Work is negative

Since the gas is being decompressed (volume is increasing), the work done is negative.

It means the work is done by the system (the gas) to the surrounding.

Work done on a gas

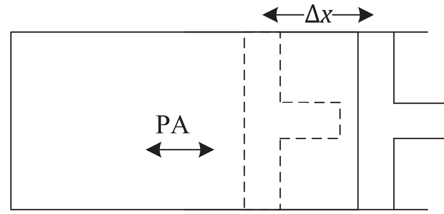


Fig 10.6

Consider a gas contained in a cylinder of cross sectional area A fitted with a movable piston (Fig 10.6). Let the pressure of the gas be p. The force exerted by the piston on the gas is pA in inward direction. Suppose the gas compress a little and the piston comes in by a small distance Δx. Then the work done on the gas by the piston is

$$\Delta w = (pA)(\Delta x) = p\Delta v \quad .... (10.18)$$

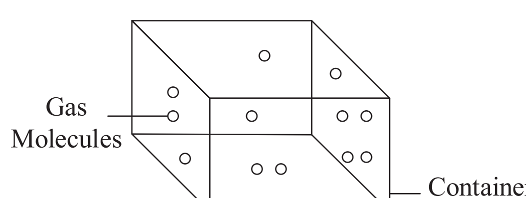
(that is simply Force $(PA) \times displacement\ \Delta x$)

10.11 Kinetic Theory of Gases

Assumptions of kinetic theory of gases

- All gases are made of molecules moving randomly in all directions.
- The size of a molecule is much smaller than the average separation between the molecules.
- The molecules exert no force on each other or on the walls of the container during collision.
- All collisions between two molecules or between a molecule and a wall are perfectly elastic. Also, the time spent during a collision is negligibly small.
- The molecules obey Newton's laws of motion.
- When a gas is left for sufficient time it comes to a steady state. The density and the distribution of molecules with different velocities are independent of position, direction and time.

10.12 Pressure of an ideal Gas



Consider a gas enclosed in a cube of side l. Take the axes x, y, z parallel to the sides of the cube as shown in the figure. Let a molecule with velocity (v_x, v_y, v_z) hits the wall parallel to yz plane of area $A(= l^2)$. Since the collision is elastic, the molecule rebounds with the same velocity; its y & z components of velocity don't change in the collision but the X component reverses sign. That is, the velocity after collision is $(-v_x, v_y, v_z)$. The Change in momentum of the molecule is $-mv_x - (mv_x) = -2mv_x$.

By the principle of conservation of momentum, the momentum imparted to the wall in the collision $= 2mv_x$.

The real gases available to us are a good approximation of an ideal gas at low density but it deviates from it when the density is increased.

Any sample of a gas is made of molecules. A molecule is the smallest unit having all the chemical properties of the sample. The observed behaviour of a gas results from the detailed behaviour of its large number of molecules.

The kinetic theory of gases attempts to develop a model of the molecular behaviour. Which should result in the observed behaviour of an ideal gas.

To find the force and pressure on the wall, we need to calculate momentum imparted to the wall per unit time in a small time interval Δt, a molecule with X −component of velocity v_x will hit the wall if it is within the distance $v_x \Delta t$ from the wall. That is, all molecules within the volume. $A v_x \Delta t$ only can hit the wall in time Δt.

But on the average, half of these are moving towards the wall and the other half away from the wall. Therefore, the number of molecules with velocity (v_x, v_y, v_z) hitting the wall in time Δt is $\frac{1}{2} A v_x \Delta t n$; where n is the number of molecules per unit volume. The total momentum transferred to the wall by these molecules in time Δt is $Q = (2mv_x)\left(\frac{1}{2} n A v_x \Delta t\right)$... (10.19)

The force on the wall is the rate of momentum transfer $Q/\Delta t$ and pressure is force per unit area:

$$P = \frac{Q}{A\Delta t} = nmv_x^2 \qquad \text{.... (10.20)}$$

Actually, all molecules in a gas don't have the same velocity; there is a distribution in velocities. The above equation, therefore, stands for pressure due to the group of molecules with speed v_x in the x −direction and n stands for the number density of that group of molecules. The total pressure is obtained by summing over the contribution due to all groups:

$$P = nm\overline{v_x^2}$$

Where $\overline{v_x^2}$ is the average of $\overline{v_x^2}$. Now the gas is isotropic, i.e. there is no preformed direction of velocity of the molecules in the vessel. Therefore by symmetry,

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

$$= \left(\frac{1}{3}\right)\left[\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}\right]$$

$$= \left(\frac{1}{3}\right)\overline{v^2} \qquad \text{.... (10.21)}$$

Where v is the speed and $\overline{v^2}$ denotes the mean of the squared speed.

Thus,

$P = \left(\frac{1}{3}\right) nm\overline{v^2}$ … (10.22)

10.13 Kinetic interpretation of Temperature

Equation 10.22 can be written as

$pv = \left(\frac{1}{3}\right) nvm\overline{v^2}$ …. (10.23)

$pv = \left(\frac{2}{3}\right) N_x \frac{1}{2} mv^2$ …. (10.24)

Where $N(= nv)$ is the no of molecules in the sample.

The quantity in the bracket is the average translational kinetic energy of the molecules in the gas. As the internal energy E of an ideal gas is purely kinetic,

$E = N\left(\frac{1}{2}\right) m\overline{v^2}$ …. (10.25)

Equation (10.24) then gives:

$pv = \left(\frac{2}{3}\right) E$ …. (10.26)

Now we delve into kinetic interpretation of temperature. Combining equation 10.26 with the ideal gas equation 10.10, we get

$E = \left(\frac{3}{2}\right) K_B NT$ …. (10.27)

Or $\frac{E}{N} = \frac{1}{2} m\overline{v^2} = \left(\frac{3}{2}\right) K_B T$ ….. (10.28)

i.e., the average kinetic energy of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas.

This is a fundamental result relating temperature, a macroscopic measurable parameter of a gas (a thermodynamic variable) to a molecular quantity, namely the average kinetic energy of a molecule. The two domains are connected by the Boltzmann constant. We note in passing that Equation 10.27 tells us that internal energy of an ideal gas depends only on temperature, not on pressure or volume. With this interpretation of temperature, kinetic theory of an ideal gas is completely consistent with the ideal gas equation and the several gas laws based on it.

For a mixture of non-reactive ideal gases, the total pressure gets contribution from each gas in the mixture. Equation 10.22 becomes

$$P = \left(\frac{1}{3}\right)\left[n_1 m\overline{v_1^2} + n_2 m\overline{v_2^2} + \ldots \ldots .\right] \qquad \ldots (10.28(b))$$

In equilibrium, the average kinetic energy of the molecules of different gases will be equal.

That is,

$$\frac{1}{2}m_1\overline{v_1^2} = \frac{1}{2}m_2\overline{v_2^2} = \left(\frac{3}{2}\right)K_B T \qquad \ldots\ldots (10.29)$$

So that

$$P = (n_1 + n_2 + \ldots \ldots)K_B T \quad \ldots. (10.30)$$

Which is Dalton's law of partial pressures.

From eq. (10.28), we can get an idea of the typical speed of molecules in a gas. At a temperature $T = 300k$, the mean square speed of a molecule in nitrogen gas is:

$$m = \frac{M_{N_2}}{N_A} = \frac{28}{6.02 \times 10^{26}}$$

$$= 4.65 \times 10^{-26} kg$$

$$\underline{v^2} = 3K_B \frac{T}{m} = (516)^2 m^2 s^{-2}$$

The square root of $\overline{v^2}$ is known as root mean square (rms) speed and is denoted by v_{rms}

(we can also write $\overline{v^2}$ as $< v^2 >$.)

$$v_{rms} = 516 ms^{-1}$$

The speed is of the order of the speed of sound in air, if follows from Eq. (12.19) that at the same temperature, lighter molecules have greater rms speed.

10.14 Law of Equipartition of Energy

The kinetic energy of a single molecule is

$$\varepsilon_t = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 \qquad \ldots (10.31)$$

For a gas in thermal equilibrium at temperature T the average value of energy denoted by $\langle\varepsilon_t\rangle$ is

$$\langle\varepsilon_t\rangle = \langle\frac{1}{2}mv_x^2\rangle + \langle\frac{1}{2}mv_z^2\rangle$$

$$= \frac{3}{2}k_B T \qquad (10.32)$$

Since there is no preferred direction eq. (10.32) implies

$\langle \frac{1}{2} m v_x^2 \rangle = \frac{1}{2} k_B T, \langle \frac{1}{2} m v_y^2 \rangle$

$= \frac{1}{2} K_B T, \langle \frac{1}{2} m v_z^2 \rangle = \frac{1}{2} K_B T$ ... (10.33)

A molecule free to move in space needs three coordinates to specify its location. If it is constrained to move in a plane it needs two; and if constrained to move along a line, it needs just one coordinate to locate it.

This can also be expressed in another way. We say it has one degree of freedom for motion in a line, two for motion in a plane and three for motion in space. The motion of a body as a whole from one point to another is called translational. Thus a molecule free to move in space has three translational degrees of freedom. Each translational degree of freedom contributes a term that contains square of some variable of motion, e.g. $\frac{1}{2} m v_x^2$ and $\frac{1}{2} m v_y^2, \frac{1}{2} m v_z^2$. In Eq. (10.33) we see that in thermal equilibrium, the average of such term is $\frac{1}{2} K_B T$.

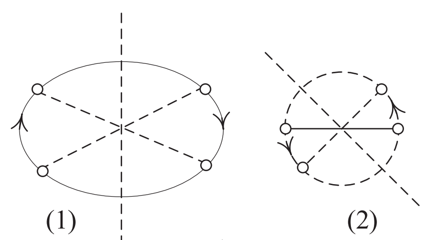


Fig 10.7

Molecules of a monatomic gas like argon have only translational degrees of freedom. But what about a diatomic gas such as O_2 or N_2? A molecule of O_2 gas has three translational degrees of freedom. But in addition it can also rotate about its centre of mass. Figure 10.7 shows the two independent axes of rotation 1 and 2, normal to the axis joining the two oxygen atoms about which the molecule can rotate. The molecule thus has two rotational degrees of freedom, each of which contributes a term to the total energy consisting of translational energy ε_t and rotational energy ε_r.

$$\varepsilon_t + \varepsilon_r = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2 + \frac{1}{2} I_1 w_1^2 + \frac{1}{2} I_2 w_2^2 \qquad (10.34)$$

Where w_1 and w_2 are the angular speeds about the axes 1 and 2 and I_1, I_2 are the corresponding moments of inertia. Note that each rotational degree of freedom contributes a term to the energy that contains the square of a rotational variable of motion.

We have assumed above that the O_2 molecule is a 'rigid rotator', i.e., the molecule does not vibrate. This assumption, though found to be true (at moderate temperatures) for O_2, is not always valid. Molecules, like C_O, even at moderate temperatures have a mode of vibration, i.e., its atoms oscillate along the interatomic axis like a one-dimensional oscillator and contribute a vibrational energy term ε_v to the total energy:

$$\varepsilon_v = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} k y^2$$

$\varepsilon = \varepsilon_t + \varepsilon_r + \varepsilon_v$ …. (10.35)

Where k is the force constant of the oscillator only y the vibrational co-ordinate.

Once again the vibrational energy terms in Eq. 10.35 contain squared terms of vibrational, variables of motion y and $\frac{dy}{dt}$.

Note down from the equation (10.35), one vibrational mode contributes two squared terms: Kinetic and potential energies.

We have seen that in thermal equilibrium at absolute temperature T, for each translational mode of motion, the average energy is $\frac{1}{2}K_BT$. The most attractive principle of classical statistical mechanics states that this is so far for each mode of energy: translational, rotational and vibrational. That is, in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to $\frac{1}{2}K_BT$. This is known as the law of equipartition of energy. Therefore, each translational and rotational degree of freedom of a molecule contributes $\frac{1}{2}K_BT$ to the energy, while each vibrational frequency contributes $2 \times \frac{1}{2}K_BT = K_BT,$ as a vibrational mode has both kinetic and potential energy modes.

10.15 Specific Heat Capacity

Monatomic Gases

The molecule of a monoatomic gas has only three translational degrees of freedom. Thus, the average energy of a molecule at temperature T is $\left(\frac{3}{2}\right)K_BT$. The total internal energy of a mole of such a gas is

$U = \frac{3}{2}K_BT \times N_A = \frac{3}{2}RT$ ….. (10.36)

The molar specific heat at constant volume, C_v is

$C_v(\text{monoatomic gas}) = \frac{dU}{dt} = \frac{3}{2}RT$ ….. (10.37)

For an ideal gas,

$C_P - C_v = R$ …. (10.38)

Where C_P is the molar specific heat at constant pressure. Thus, $C_P = \frac{5}{2}R$… (10.39)

That ratio of specific heats

$y = \frac{C_p}{C_v} = \frac{5}{3}$ … (10.40)

Diatomic Gases

A diatomic molecule considered as a rigid rotator, like a dumbbell, has 5 degrees of freedom: 3 translational and 2 rotational. Using the law of equipartition of energy, the total internal energy of a mole of such a gas is $U = \frac{5}{2}K_BT \times N_A = \frac{5}{2}RT$ .... (10.41)

The molar specific are then given by C_v (rigid diatomic)

$= \frac{5}{2}R, C_P = \frac{7}{2}R$ .... (10.42)

y (rigid diatomic) $= \frac{7}{5}$ .... (10.43)

If the diatomic molecule is not rigid but has in addition a vibrational mode then,

$U = \left(\frac{5}{2}K_BT + K_BT\right)N_A$

$= \frac{7}{2}RT$ .... (10.44)

$C_v = \frac{7}{2}R, C_P = \frac{9}{2}R, y = \frac{9}{7}R$ ... (10.45)

Polyatomic Gases

Generally a polyatomic molecule has 3 translational, 3 rotational degrees of freedom and a certain number (f) of vibrational modes. According to the law of equipartition of energy, for one mole of such a gas has

$U = \left(\frac{3}{2}K_BT + \frac{3}{2}K_BT + fK_BT\right)N_A$ .... (10.46)

i.e. $C_v = (3 + f)R, C_P = (4 + f)R,$ .... (10.47)

$Y = \frac{(4+f)}{(3+f)}$ .... (10.48)

Note that $C_P - C_v = R$ hold for any ideal gas, whether mono, di or polyatomic.

Example A diatomic gas $(y = 1.4)$ does $400J$ of work when it is expended isobarically. The heat given to the gas in the process is ______ J.

Solution: $w = p\Delta v = \eta R\Delta T = 400j$

$\therefore \Delta Q = \eta C_p \Delta T$

$= \eta \times \frac{7}{2}R \times \Delta T = \frac{7}{2} \times \eta \times R \times \Delta T$

$$= \frac{7}{2} \times 400 = 1400(\eta \times R \times \Delta T = 400)$$

Two students of the 3rd row were talking in their regional language, probably Kannad and it could be figured out from their expression that they were unhappy, struggling to understand many of the technical terms that were being taught rapidly this day. One of them asked Gita in a dual language (Hindi & English) "aap aaj ka total modules phir se samjha do, I didn't understand a single term today." As soon as he finished his statement, Aakash said "speak English, this is the common language here". But the boy responded harshly, "Fuc.. off." The word put not only Aakash but the whole class in a state of shock. Gita used to understand her class much better than anyone else. And she thought it must have happened due to his huge frustration over the terms that were not understood. So she later stabilized the class environment without scolding anyone.

When the class ended the boy met Aakash personally and apologized for that. Aakash just said ok, it happens sometimes you don't worry I can understand your situation particularly at that time. And Aakash walked to the bus parking side and Amila along with the other guys was also there. They went to a famous showroom of the city and finalized a bus of medium size, more parts of it were covered with glass Windows. Amila had a little talk with the guys regarding the final release date of the vehicle and finally it was decided that it would be in service after the annual exam (of 11th grade) which was so close, just after two weeks. Amila gave some advance money and booked it for release on 28th March that year.

And now they were in a kind of mood that the rest of the time before the exam they were going to spend it alone in their houses, no calls regarding the syllabus, no contact, no meeting. It seemed like they were trying to adapt themselves according to the vibes of exam-hall-time of the exam, what you have learnt during a particular academia. However, they were hopeful and expecting everything to be good and to be back even stronger in the business of study and adventure the very next year……..

Exercises

1. A monatomic gas having $\gamma = \frac{5}{3}$ is stored in a thermally insulated container and the gas is suddenly compressed to the $\left(\frac{1}{8}\right)t$ of its initial volume. The ratio of final pressure and initial pressure is: (γ is the ratio of specific heats of the gas at constant pressure and at constant volume)
2. Match List-I with List-II.

List-I	**List-II**
(A) Isothermal	(I) ΔW (work done) $= 0$
(B) Adiabatic	(II) ΔQ (supplied heat) $= 0$

(C) Isobaric (III) ΔU (change in internal energy) $\neq 0$

(D) Isochoric (IV) $\Delta U = 0$

Choose the correct answer from the options given below :

3. A gas is kept in a container having walls which are thermally non-conducting. Initially the gas has a volume of $800cm^3$ and temperature $27°C$. The change in temperature when the gas is adiabatically compressed to $200cm^3$ is : (Take $\gamma = 1.5$; γ is the ratio of specific heats at constant pressure and at constant volume)
4. During the melting of a slab of ice at $273K$ at atmospheric pressure :
5. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

   Assertion (A): With the increase in the pressure of an ideal gas, the volume falls off more rapidly in an isothermal process in comparison to the adiabatic process.

   Reason (R): In isothermal process, PV = constant, while in adiabatic process $PV_{\gamma} =$ constant. Here γ is the ratio of specific heats, P is the pressure and V is the volume of the ideal gas.

   In the light of the above statements, choose the correct answer from the options given below:

   (a) Both A and R are false
   (b) Both A and R are true and R is the correct explanation of A
   (c) A is true but R is false
   (d) both A and R are true but R is not correct explanation of A

6. The work done in an adiabatic change in an ideal gas depends upon only on ___________ . (a) change in its pressure (b) change in its temperature (c) change in its volume (d) none of these
7. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

   Assertion (A) : In an insulated container, a gas is adiabatically shrunk to half of its initial volume. The temperature of the gas decreases.

   Reason (R) : Free expansion of an ideal gas is an irreversible and an adiabatic process.

   In the light of the above statements, choose the correct answer from the options given below :

   (a) A is true but R is false
   (b) A is false but R is true
   (c) Both (A) and (R) are true

(d) Both (A) and (R) are false

8. A container of fixed volume contains a gas at $27°C$. To double the pressure of the gas, the temperature of gas should be raised to _____ $°C$.
9. The temperature of 1 mole of an ideal monatomic gas is increased by $50°C$ at constant pressure. The total heat added and change in internal energy are E_1 and E_2, respectively. If $\frac{E_1}{E_2} = \frac{x}{9}$ then the value of x is _________.
10. One mole of a monoatomic gas is mixed with three moles of a diatomic gas. The molecular specific heat of the mixture at constant volume is $\left(\frac{\alpha^2}{4}\right) RJ/molK$; then the value of α will be _________. (Assume that the given diatomic gas has no vibrational mode).
11. At a certain temperature, the degrees of freedom per molecule for gas is 8. The gas performs $150J$ of work when it expands under constant pressure. The amount of heat absorbed by the gas will be _________ J.
12. A block of ice of mass $120g$ at temperature $0°C$ is put in $300g$ of water at $25°C$. The $x\ g$ of ice melts as the temperature of the water reaches $0°C$. The value of x is _____________. (Use specific heat capacity of water = $4200Jkg^{-1}K^{-1}$, Latent heat of ice = $3.5 \times 10^5 Jkg^{-1}$)
13. The total internal energy of two mole monatomic ideal gas at temperature $T = 300K$ will be _____________ J. (Given $R = 8.31\, J/mol.K$)
14. A geyser heats water flowing at a rate of $2.0kg$ per minute from $30°C$ to $70°C$. If a geyser operates on a gas burner, the rate of combustion of fuel will be ____________ $g\ min^{-1}$. (Heat of combustion = $8 \times 10_3\, Jg^{-1}$, Specific heat of water = $4.2Jg^{-1}°C^{-1}$)
15. A heat engine operates with the cold reservoir at temperature $324K$. The minimum temperature of the hot reservoir, if the heat engine takes $300J$ heat from the hot reservoir and delivers 180 J heat to the cold reservoir per cycle, is _____________ K.
16. A monatomic gas performs a work of where Q is the heat supplied to it. The molar heat capacity of the gas will be _______________ R during this transformation. Where R is the gas constant.
17. A sample of gas with $\gamma = 1.5$ is taken through an adiabatic process in which the volume is compressed from $1200cm^3$ to $300\ cm^3$. If the initial pressure is $200\ kPa$. The absolute value of the work done by the gas in the process = ______________ J.
18. For an ideal heat engine, the temperature of the source is $127°C$. In order to have 60% efficiency the temperature of the sink should be ___________ $°C$. (Round off to the Nearest Integer)
19. 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it. The molar heat capacity of the gas during this

transformation is $\frac{xR}{8}$. The value of x is __________. (R = universal gas constant)

Answers

1. 32	2. A-iv, B-ii, C-iii, D-i	3. 300K	4. Positive work is done on the ice-water system on the atmosphere
5. (b)	6. b	7. b	8. 327
9. 15	10. 3	11. 750	12. 90
13. 7479	14. 42	15. 540	16. 2
17. 480	18. -113	19. 25	